PHYS 430 - Make-Up EXAM
29 MAY 2006

Name and Surname:
Student ID:
Department:
Signature:

1. (12 points)
   Fill in the blanks in the following statements with T, V, μ, E, N such that the statements are true: (12 points)

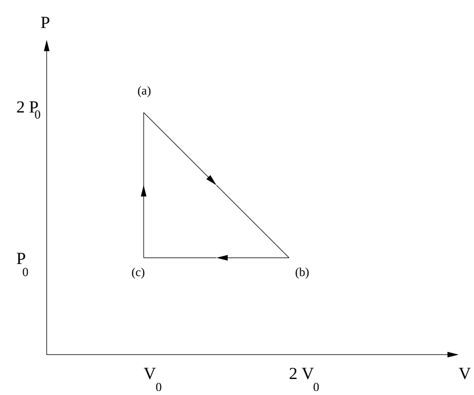
   (a) In the canonical ensemble, __________, __________, and __________ are fixed whereas __________ is fluctuating.

   (b) In the grandcanonical ensemble, __________ , __________ , and __________ are fixed and __________ and __________ are fluctuating

   (c) In the microcanonical ensemble, __________ , __________ and __________ are fixed.

2. (10 points)
   Consider an engine that works by extracting heat from the ocean and converting that heat into work. Is it possible? Explain. (10 points)

3. (30 points)
   Consider a reversible engine that has the PV diagram shown in the figure. The engine uses a monatomic ideal gas as its working substance. In each of the parts of the processes $(a) \rightarrow (b)$, $(b) \rightarrow (c)$ and $(c) \rightarrow (a)$, calculate the heat absorbed, heat released, work done (20 points). Calculate the net efficiency of the engine (10 points).



4. (40 points)
The energy momentum expression $\epsilon = \frac{p^2}{2m}$ is the first term in the expansion of the relativistic expression $\epsilon = \sqrt{p^2c^2 + m^2c^4} - mc^2$. For larger momenta, one needs to take into account higher order terms in the expansion. Including the first correction, the energy becomes

$$\epsilon = \frac{p^2}{2m} - \frac{p^4}{8m^3c^2} \tag{1}$$

Consider a gas of point like particles that have the energy momentum relation given by Eq. (1). If the gas is enclosed in a volume V and is at a temperature T, calculate the correction to the free energy, F of the gas due to relativistic effects. Assume that the relativistic correction is small(20 points). What is the correction to the pressure(10 points) and the specific heat C_V(10 points)

5. (10 points) (10 points)
Consider the subsystem of electrons in a metal that are in the i^{th} state which has the energy ϵ_i. Calculate Ω and P of this subsystem if the temperature and the chemical potential of the metal is T and μ respectively. What is the average number of electrons in this subsystem in terms of T and μ?

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
F &= E - ST \; ; \quad W = E + PV \; ; \quad \Phi = E - ST + PV \\
S &= \ln \Delta\Gamma(E) \; ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty dx x^n e^{-\gamma x} &= \frac{n!}{\gamma^{n+1}}, \qquad \int_0^R d\rho \rho e^{-\alpha\rho} = \frac{1}{\alpha^2}\left[1 - (1+\alpha R)e^{-\alpha R}\right] \\
\beta &= \frac{1}{T}, \qquad k = 1 \\
e^x &\simeq 1 + x, \quad x << 1; \quad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}; \quad \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \\
d^3w &= w^2 dw \sin\theta d\theta d\phi, (0 < w < \infty, \; 0 < \theta < \pi, \; 0 < \phi < 2\pi)
\end{aligned}$$

For anything else, you need to derive it.