PHYS 430 - Statistical Thermodynamics
29 MAY 2006

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

In your calculations explain your steps. Just writing equations without any explanation why you do those manipulations will loose you points.

1. (20 points)
   Comment of the following concepts (4 points each):
   (a) Entropy
   (b) Irreversible process
   (c) Negative Temperatures
   (d) The three laws of thermodynamics
   (e) Statistical Mechanics vs. Thermodynamics

2. (20 points)
   Draw the PV diagram of a Carnot engine working between 2 heat reservoirs at temperatures T_1 and $T_2 < T_1$. If the Carnot engine uses an ideal gas as a working medium. Show by explicit calculation that the efficiency of the Carnot cycle is given $\eta = 1 - \frac{T_2}{T_1}$

3. (20 points)
   Calculate the Free energy of an ideal gas whose energy momentum relation is given by $\epsilon = \frac{p^2}{2m}$. Express your result in terms of N the number of particles, T, the temperature of the gas, and V the volume of the container.

4. (20 points)
   Consider a chain of arrows each of length ℓ. Suppose you make a long chain of these stick by adding N of these one at the end of the other. Moreover, assume that, each arrow can either point towards the left, or towards the top. If the length of this chain towards the left is L_x , what is the entropy S of this system? What is the x component of the force acting at the end of the chain if the temperature of the chain is T? ($0 = dE = TdS - F_x dL_x - F_z dL_z$)

5. Consider a system composed of two quantum harmonic oscillators of the same frequency ω. The energy levels of the system is given by $E_{n_1,n_2} = \hbar\omega(n_1+n_2+1)$. Calculate its free energy. Obtain E and C_V for this system.

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
F &= E - ST \; ; \quad W = E + PV \; ; \quad \Phi = E - ST + PV \\
S &= \ln \Delta\Gamma(E) \; ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty dx x^n e^{-\gamma x} &= \frac{n!}{\gamma^{n+1}}, \qquad \int_0^R d\rho \rho e^{-\alpha\rho} = \frac{1}{\alpha^2}\left[1 - (1+\alpha R)e^{-\alpha R}\right] \\
\beta &= \frac{1}{T}, \qquad k = 1 \\
e^x &\simeq 1 + x, \quad x << 1; \quad \sum_{n=0}^\infty x^n = \frac{1}{1-x}; \quad \sum_{n=0}^\infty \frac{x^n}{n!} = e^x \\
d^3 w &= w^2 dw \sin\theta d\theta d\phi, (0 < w < \infty, \; 0 < \theta < \pi, \; 0 < \phi < 2\pi)
\end{aligned}$$

For anything else, you need to derive it.