# Sample Questions for the Phys430 Final

*The are questions that are possible to appear in the final exam on 27 May 2006. The final will be closed books and closed notes. The questions will be selected from the questions below, and other questions that you can find from the web page of the course, i.e. the homework and midterm questions of this term and the last term (these are also available from the web page.) You will not get exactly the same questions but very slightly modified questions (only some numbers will change). So in the final, if you do not solve the modified version, but the version appearing in my web page, you will not even get any partial credit from that question. For long questions, I will select only part of the questions to ask in the final.*

1. Show that

$$\frac{d^3p d^3q}{(2\pi\hbar)^3} = \frac{4\pi V}{(2\pi\hbar)^3} p^2 dp \tag{1}$$

   where the equality sign you obtain after integrating over $q$ and the direction of $\vec{p}$

2. Calculate the thermodynamical potentials $F$, $S$, $E$, $\mu$, $F$ of an ideal gas of pointlike particles confined to move in 2 dimesions on a surface $A$ if they have the energy-momentum relation $\epsilon = \frac{p^2}{2m}$ using
   (a) Micro Canonical Ensemble
   (b) Canonical Ensemble
   (c) Grand Canonicle Ensemble

3. Consider two small systems in contact. Show that in equilibrium:
   (a) If the two subsytems are allowed to exchange energy, the temperatures of both systems are the same
   (b) If both systems are seperated by a movable piston so that they exchange volume, but no other exchange is possible, the pressures of both systems are the same in equilibrium
   (c) If the two system are seperated by a membrane which only allows the exchange of particles, then the chemical potential of both systems are the same

4. Consider two particles which can be only in 2 states, one with energy $\epsilon_1$ and the other with energy $\epsilon_2$. If the system is at temperature $T$, write the partition function of the system if the particles are:

   (a) Distinguishable particles

   (b) Identical particles satisfying the Boltzman distribution

   (c) Identical particles which are fermions

   (d) Identical particles which are bosons.

   In each of these cases, calculate the energy $\langle E \rangle$ of the system and the fluctuations in the energy of the system, $\langle (\Delta E)^2 \rangle$

5. Consider a chain of arrows each of length $\ell$. Suppose you make a long chain of these stick by adding $N$ of these one at the end of the other. Moreover, assume that, each arrow can either point towards the left, or towards the right. If the total length of this chain is $L$ (towards the left), what is the entropy $S$ of this system? What is the force acting at the end of the chain if the temperature of the chain is $T$? $(0 = dE = TdS - FdL)$

6. Consider the two single particles states in a metal which have energies $\epsilon_i$ $(i = 1,\ 2)$ none of which is degenerate. If the metal is at temperature $T$ and has a chemical potential $\mu$, what is the average number of electrons in these states? What is the relative fluctuation in the number of particles in these states?

7. Consider a quantum harmonic oscillator that has a frequency $\omega$ (the energy levels are given by $\epsilon_n = \hbar\omega(n + \frac{1}{2})$). If the oscillator is at temperature $T$, calculate its free energy $F$. Calculate $E$ and $C_V$ for this harmonic oscillator.

8. Consider two object with specific heats $C_1$ and $C_2$ and which are at temperatures $T_1$ and $T_2 < T_1$ respectively. If the system is brought into equilibrium via a reversible process, calculate the work done and the final temperature of the systems when the system is brough to equilibrium. (In order to bring the system in equilibrium reversibly, you have to consider carnot cycles which take infinitesimal amount of heat from the hotter reservoir and dumps it into the colder reservoir doing work in the process.)

9. Consider a mixture of $N_1$ molecules of a diatomic ideal gas and $N_2$ molecules of a monatomic ideal gas. Calculate $C_V$, $C_P$ and $\gamma = \frac{C_P}{C_V}$ for this mixture.

10. Suppose that you have a gas of $N$ identical particles entrapped in a sphere of radius $R$ and are subject to a potential $V = \alpha r$ where $\alpha$ is some constant and $r$ is the distance from the center of the sphere. Assume that the energy of the particles is relted to the momentum through $\epsilon = \alpha p^n$.

    (a) Calculate the the density of particles as a function of the radius from the center. Properly normalize the density so that the integral of the density over the whole sphere gives $N$, the total number of particles.

    (b) Calculate the partition function of the system.

    (c) Calculate the pressure the gas exerts on the sphere surface.