Name and Surname: Student ID: Department: Signature:

## INSTRUCTIONS

There are 8 questions each worth 20 points. You should not answer all the questions. You should choose ONE question to answer from each of the pairs: (1,2), (3,4) and (7,8). You should answer the fifth AND the sixth questions, making a total of 5 questions. Mark the question that you have solved in the boxes below. Only the questions marked below will be graded.



- 1. Compare the three distributions that we have studied: microcanonical, canonical and grandcanonical. How do they differ? Under what conditions would you prefer one over the other? and why? (the last part is a personal question and there is no right or wrong. You have to justify yourself by answering to the "why" part)(20 points)
- 2. Describe the following concepts (Do not just write equation, you should explain what they are, how they are used, what is the difference between it and a similar concept, etc.): (4 points each, 20 points total)

La Chateliers principle

Ergodic Hypothesis

Entropy

Fermi Distribution

**Bose Distribution** 

3. Consider two subsystems of a larger system. Assume that the two subsystems can exchange energy, particles and that the volume that each occupies is distinct from the volume that the other subsystem occupies. If the total energy, total number of particles and the total volume is conserved, what are the conditions of equilibrium. (You should derive them. Just stating the conditions is not enough.) (20 points)

- 4. Consider a subsystem of a larger system consisting of only the particles of the closed system that are in states 1 or 2. The states have energies  $\epsilon_1$  and  $\epsilon_2$  respectively. What is the total number of particles in this subsystem if the larger system is a system of fermions? of boson? (20 points) (Hint: The energy levels are not degenerate, and the particles do not have any internal degeneracy. (20 points)
- 5. Consider the chemical reaction  $A + 2B \leftrightarrow 2C$  (the one presented by your friend in the class). What is the rate,  $r_f$ , of the reaction to go in the forward direction?(10 points) Using probability arguments, argue that  $r_f$  has to be of that form.(10 points) (Hint: for a reaction to occur, the particle that should react should be close together. The rate depends on how probable is it two find the reactants close enough) (20 points)
- 6. Obtain the velocity distribution of gas of classical particles whose energy momentum relation is  $\epsilon = cp$ , where p = mv and v is the magnitude of its velocity. Normalize your solution to N particles in a volume V. (20 points)
- 7. Calculate the rate of change of the temperature of a Van der Waals' gas undergoing a Joule Thompson process. (Hint: a Joule Thompson process is a process that changes the pressure of the gas keeping the heat function constant. Keep the specific heat of the gas as  $C_V$  and  $C_P$  whichever one is relevant. Your answer should contain only  $C_V, C_P, P, T$  only and should not contain any derivatives.)(20 points)
- 8. The kinetic energy for a particle of mass m is usually given as  $KE = \frac{p^2}{2m}$ . In reality, this is just the first term in the expansion of  $KE = \sqrt{m^2c^4 + p^2c^2} mc^2$  for  $p \ll mc$  where c is the speed of light. If one also keeps the second term in the expansion, then the kinetic energy becomes:

$$KE = \frac{p^2}{2m} - \frac{p^4}{8m^3c^2}$$

. Assuming that the correction due to this additional term is small, calculate the correction to the free energy F. Calculate the new specific heat. What is the condition on T for which the correction can be considered small? (20 points)

You can use the following formulas/definitions without deriving them:

$$dE = TdS - PdV + \mu dN$$

$$\begin{split} dF &= -SdT - PdV + \mu dN \\ dW &= TdS + VdP + \mu dN \\ d\Phi &= -SdT + VdP + \mu dN \\ F &= E - ST \; ; \; W = E + PV \; ; \; \Phi = E - ST + PV \\ S &= \ln \Delta \Gamma(E) \; ; \; \; \Delta \Gamma(E) = \Delta E \frac{\partial}{\partial E} \Gamma(E) \\ \ln N! &\simeq N \ln N - N \\ \int_0^\infty x^n e^{-x} &= n! \\ \beta &= \frac{1}{T}, \quad k = 1 \end{split}$$

The equation of state of a Van der Waals gas is:

$$\left(P + a\frac{N}{V}\right)\left(V - Nb\right) = NT$$

For anything else, you need to derive it.