PHYS 430 - TAKEHOME EXAM
January 2006

Name and Surname:
Student ID:
Department:
Signature:

1. Describe the equipartition theorem for a classical gas and demonstrate its application to a monatomic gas and a diatomic gas.

2. Calculate the entropy of a classical ideal gas using the canonical ensemble. Express your result in terms of N, V and T of the gas. Calculate its chemical potential μ

3. Consider a classical gas enclosed in a vertical cylinder of height H and cross sectional area A. The gas cylinder is in a uniform gravitational field. Calculate its free energy, F, entropy S, and energy E. Express your result in terms of N, H, A and T.

4. Consider a classical gas of N atoms. The atoms are either in the ground state or in the excited state whose energy is ϵ above the ground state. Calculate the free energy for this system. Express your result in terms of N, V and T.

5. In the system of the previous problem, how many atoms are in the ground state?

6. Consider a pendulum consisting of a point particle of mass m at the end of a massless string hanging from the ceiling. What is the average of the squared displacement of the particle.

7. Derive the velocity distribution, i.e. the number of particles that have speeds in the range from v to $v+dv$ for a classical relativistic gas whose energy momentum relation is given by $E = \sqrt{p^2c^2 + m^2c^4}$, where $p = mv(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ and show that the distribution is zero from $v \geq c$.

8. Consider two particles which can be in states 1, 2 or 3 having energies 0, ϵ, and 2ϵ respectively. Write down the partition function at a temperature T for this system in the canonical ensemble if the particles are boson. Do the same if they are fermions. (the single particle states are not degenerate).

9. Consider a classical gas whose energy momentum relation is given by $\epsilon = \alpha p^n$ for some constants α and n. Calculate the specific heat per particle of this gas as a function of the parameters of the problem. Show that one recovers the equipartition theorem result from $n = 2$

10. Calculate the Fermi energy for an electron gas confined to move along a 1-D wire of length L.

11. Calculate the Fermi energy for an electron gas confined to move in a volume V.

12. Show that if two reservoirs at temperatures T_1 and T_2 are in contact, requiring that the entropy must increase, means that energy can only flow from the hotter to the colder system. Using the same principle, show also that a system with negative temperature is "hotter" than a system with a positive temperature.

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

The equation of state of a Van der Waals gas is:

$$\left(P + a\frac{N}{V}\right)(V - Nb) = NT$$

For anything else, you need to derive it.