PHYS 430 - MAKEUP EXAM
January 2006

Name and Surname:
Student ID:
Department:
Signature:

1. Consider a hypothetical system where N classical non-interacting particles are confined to move only on a surface of area A. Suppose that the energy momentum relation of the particles making the system is given by $\epsilon = \alpha p^n$ where α is some positive constant and p is the magnitude of the momentum of the particle. a) If $n = 2$, what does the equipartition theorem say about the energy of this system? b) In the canonical ensemble, calculate the entropy of the system if the system has temperature T.
(20 points)

2. Consider any system of your choice. Write the starting equations in studying the system if your are using a) Microcanonical Distribution b) Canonical Distribution c) Grandcanonical Distribution. Describe (do not calculate) the subsequent steps in calculating the thermodynamic potentials.

3. Derive the energy density for radiation in equilibrium with itself. (For light to reach equilibrium, there has to be some matter interacting with it. In this problem, we are not interested in the interaction with the matter.)

4. Consider a gas of fermions bound to move on the surface separating two different materials (a 2D electron gas). If there are N electrons in an area A, calculate the Fermi energy of this system.

5. Consider N particles subject to an external 3D harmonic oscillator potential, $\frac{1}{2}\alpha r^2$, where r is the distance between the particle and the center of the potential. If the system is at a temperature T, what is the free energy of the system.

6. Calculate the specific heat at constant pressure for th Van der Waals Gas(VDW). To achieve this, you can do as follows:

   Express $\left(\frac{\partial C_V}{\partial V}\right)_T$ in terms of the derivatives of the equation of state only. Using the equation of state of the Van der Waals Gas, show that this derivative is zero. Hence, prove the C_V for the van der Waals gas might depend only on the temperature of the gas, and is independent of the volume.

Consider a Van der Waals gas of N molecules. At constant temperature, consider the limit $V \to \infty$. You have just shown that by changing the volume at constant temperature, the specific heat does not change. At this limit, the VDW gas becomes an ideal gas whose specific heat is known. Hence the specific heat of the VDW gas at constant volume is the same as the specific heat of the ideal gas.

Now that you know the specific heat at constant volume, derive the relation between the specific heat at constant volume and the specific heat at constant pressure. Using this result, you can calculate the specific heat at constant pressure.

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

The equation of state of a Van der Waals gas is:

$$\left(P + a\frac{N}{V}\right)(V - Nb) = NT$$

For anything else, you need to derive it.