2^{nd} Homework
Due: 15 November 2007

1. If an artificial satellite, or some other body, approaches a moving planet on a hyperbolic orbit, it can gain some energy from the motion of the planet and emerge with a larger speed than it had initially. This slingshot effect has been used to boost the speeds of two voyager space crafts as they passed near Jupiter. Suppose that the line of approach of the satellite makes and angle θ with the line of motion of the planet and the line of recession of the satellite is parallel to the line of motion of the planet (the planet can be regarded as moving on a straight line during the time interval in question). The speed of the planet is u and the initial speed of the satellite is v (in the reference frame of the Sun).

   (a) Show that the final speed of the satellite is

   $$v' = u + \sqrt{v^2 + u^2 - 2uv\cos\theta}$$

   (b) Show that the satellite will not gain any speed in this encounter if $\theta = 0$ and show that the satellite will gain maximum speed if $\theta = 180°$.

   (c) If a satellite with $v = 3\ km/s$ approaches Jupiter at an angle $\theta = 30°$, what will be its final speed?

   (H. C. Ohanian, "Physics," Pg243, Q48) (*Hint:* Ignore the change in the velocity of the planter and go to a reference frame in which the planet is at rest.)

2. The solar wind sweeping past the earth consists of a stream of particles, mainly hydrogen ions of mass $1.7 \times 10^{-27}\ kg$. There are about 10^7 ions per cubic meter and their speed is $4 \times 10^5\ m/s$. What force does the impact of the solar wind exert on an artificial Earth satellite that has an area of $1.0\ m^2$ facing the wind? Assume that upon impact, the ions first stick to the surface of the satellite? (H. C. Ohanian, "Physics," Pg267, Q13)

3. Consider the solar wind whose properties are described in the previous problem. Assume that a satellite that has a total mass of 10 kg has a very large sail that traps the particles in the solar wind. What should

be the surface area (in units of m^2) of the sail such the the force of the solar wind will be enough to overcome the gravitational pull of the Sun?

4. A billiard ball of mass m and radius R moving with speed v on a smooth, frictionless table collides elastically with an identical stationary billiard ball glued firmly to the surface of the table.

   (a) Find a formula for the angular deflection suffered by the moving billiard ball as a function of the impact parameter b (defined in Figure 11.17). Assume the billiard balls are very smooth so that the force during contact is entirely along the center-to-center line of the balls.

   (b) Find a formula for the magnitude of the momentum change suffered by the billiard ball.

   (c) find a formula for the magnitude of the momentum change suffered by the billiard ball.

   (H. C. Ohanian, "Physics," Pg294, Q35)

5. Because of the rotation of the Earth, about its axis, a stone resting on a tower, say, the equator has an eastward velocity in an inertial reference frame centered on the Earth but not rotating with it. In this reference frame, the foot of the tower also has an eastward velocity but slightly smaller than that of the stone.

   (a) Show that if the height of the tower is h, the difference in the magnitude of these velocities is ωh, where ω is the angular velocity of rotation of the Earth.

   (b) If the stone drops from the tower, its excess eastward velocity will cause it to land at some distance from the foot of the tower. Show that it will land at a distance of about $wh\sqrt{2h/g}$ eastward of the foot of the tower. Evaluate for $h = 30\ m$.

   (H. C. Ohanian, "Physics," Pg319, Q8)

6. Consider a semi-sphere who is free to rotate around an axis on its flat surface and passing through its center. Calculate its moment of inertia. its center of

7. Consider the creation of a pion in a collision between two protons. The reaction is described by the equation:

$$p + p \to p + p + \pi^0$$

If one of the protons is initially stationary, what minimum kinetic energy must the other proton have to make this reaction possible? The mass of the pion is 2.4×10^{-28} kg. Pretend that the Newtonian formulas for kinetic energy remain valid (even though the speed of the proton is near the speed of light) (H. C. Ohanian, "Physics," Pg295, Q44)