6^{th} Homework
Due: 22 November 2007

1. Consider the three unit vectors $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ that were defined in the 2^{nd} homework. The position of a particle can be specified as $\vec{r} = r\hat{r}$. For a general motion, both r and $\hat{r}$ depend on time. The dependence of the unit vector $\hat{r}$ on time is due to the dependence of the angles θ and ϕ on time as the object moves. Express the acceleration and velocity of an object doing general motion in terms of the angles and their velocities and accelerations.

   (a) For a particle on a rigid body, that is making rotations around the z axis, what are the conditions on r, θ and ϕ coordinates and their derivatives? What is the velocity vector? Show that the magnitude of the velocity vector that you have calculated reduces to $v = \frac{d\phi}{dt}d = \omega d$ where $d = r\sin\theta$ is the distance of the point from the z axis. Show also the the velocity vector i perpendicular to $\hat{r}$ and $\hat{\phi}$

   (b) Uniform circular motion in the xy plane can be described as a motion with $\theta = \frac{\pi}{2}$, $\frac{dr}{dt} = 0$, $\frac{d^2\phi}{dt^2} = 0$. Show that with these assumptions, acceleration reduces to

   $$\vec{a} = -\omega^2 r\hat{r}$$

   where $\frac{d\phi}{dt} = \omega$

2. An automobile engine accelerates at a constant rate from 200 rev/min to 3000 rev/min in 7.0 s and then runs at constant speed.

   (a) Find the angular velocity and angular acceleration at $t = 0$ (just after acceleration begins) and at $t = 7$ s (just before acceleration ends).

   (b) A flywheel with a radius of 18 cm is attached to the shaft of the engine. Calculate the tangential and centripetal acceleration of a point on the rim of the flywheel at the times given above.

   (c) What angle does the net acceleration vector make with the radius at $t = 0$ and at $t = 7.0$ s? Draw diagrams showing the wheel and the acceleration vector at these times.

(H. C. Ohanian, "Physics," Pg319, Q13)

3. A hole of radius r has been drilled in a circular, flat plate of radius R. The center of the hole is at a distance d from the center of the circle. The mass of this body is M (before drilling). Find the moment of inertia for a rotation about an axis through the center of the circle, perpendicular to the plate. (H. C. Ohanian, "Physics," Pg321, Q30) (*Hint:* A hole of mass m is equivalent to an attached object of the same size an shape as the hole, but has a mass $-m$)

4. The Oerlikon Electrogyro bus uses a flywheel to store energy for propelling the bus. At each bus stop, the bus is briefly connected to an electric power line, so that an electric motor on the bus can spin up the flywheel to 3000 revolutions per minute. If the flywheel is a disk of radius 0.6 m and mass 1500 kg, and if the bus requires an average of 40 hp for propulsion at an average speed of 20 km/h, how far can it move with the energy stored in the rotating flywheel? (H. C. Ohanian, "Physics," Pg 322, Q40)

5. Two automobiles both of 1200 kg and both traveling at 30 km/h collide on a frictionless icy road. They were initially moving on parallel paths in opposite directions, with center-to-center distance (impact parameter) of 1.0 m. In the collision, the automobiles lock together, forming a single body of wreckage; the moment of inertia of this body about its center of mass is $2.5 \times 10^3\ kgm^2$

   (a) Calculate the angular velocity of the wreck.

   (b) Calculate the kinetic energy before the collision and after the collision. What is the change of kinetic energy?

   (H. C. Ohanian, "Physics," Pg350, Q31)

6. If the melting of the polar ice caps were to raise the water level on the Earth by 10 m, by how much would the day be lengthened? Assume that the moment of inertia of the ice in the polar caps is negligible (they are very near the axis), and assume that the extra water spreads out uniformly over the entire surface of the Earth (that is, neglect the area of the continents compared with the area of the oceans). The moment of inertia of the Earth (now) is $8.1 \times 10^{27}\ kgm^2$

7. In order to stabilize a ship against rolling, an inventor proposes that a large flywheel spinning at high speed should be mounted within the ship. The axis of the flywheel lies across the ship and the bearings are rigidly attached to the side of the ship.

   (a) If a wave hits the ship broadside and attempts to roll (capsize) the sip to the left, what will be the response of the ship? In answering this question, assume that the response of the ship is dominated by the effect of the flywheel. With words or with diagrams, carefully describe how the orientation of the ship will change.

   (b) If a wave hits the bow of the ship and attempts to push the bow to the left, what will be the response of the ship?

   (H. C. Ohanian, "Physics," Pg353, Q51)