8^{th} Homework
Due: 27 December 2007

1. Two masses m_1 and m_2 are joined by a spring of spring constant k. Show that the frequency of vibration of these masses along the line connecting them is
$$\omega = \sqrt{\frac{k}{\mu}}$$
where $\mu = \frac{m_1 m_2}{m_1+m_2}$ is called the reduced mass of the system. (*Hint:* The CM remains at rest.) (H.C. Ohanian, "Physics," pg364, Q20)

2. In windup clocks a strong torsional spring is used to store mechanical energy. Suppose that each week a clock requires four full turns of the winding key to keep running. The initial turn requires a torque of 0.30 $N.m$ and the final turn a torque of 0.45 $N.m$
   (a) What amount of mechanical energy do you store in the spring when winding the clock?
   (b) What is the consumption of mechanical power by the clock?
   (c) What is the torsional spring constant?

   (H. C. Ohanian, "Physics," Pg.365, Q.28)

3. To test that the acceleration of gravity is the same for a piece of iron and a piece of brass, an experimenter takes a pendulum of length 1.800 m with an iron bob and another pendulum f the same length with a brass bob and starts them swinging in unison. After swinging for 12 min, the two pendulums are no more than one-quarter of a (one way) swing out of step. What is the largest difference between the values of g for iron and for brass consistent with these data? Express your answer as a fractional difference. (H. C. Ohanian, "Physics," Pg365, Q51)

4. Consider a bar of steal. Calculate the velocity of transverse waves, and also of longitudinal waves. Assume that the waves have low amplitudes. (*Hint:* On a string, the restoring force is provided by the tension, i.e. due to tension, the string tries to remain straight. In this case, it will be due to nonzero bulk modulus or shear viscosity that the bar will try to remain straight).

5. The end of a long string of mass density μ is knotted at the beginning of another long string of mass density μ' (the tensions in these strings are equal). A Harmonic wave travels along the first string toward the knot. This incident wave will be partially transmitted into the second string and partially reflected. The frequencies of all these waves are the same. With the knot at $x = 0$, we can write the following expressions for the incident, reflected and transmitted waves:

$$\begin{aligned} y_1 &= A_{in}\cos(kx - \omega t) \\ y_2 &= A_{ref}\cos(kx + \omega t) \\ y_3 &= A_{trans}\cos(k'x - \omega t) \end{aligned}$$

Show that

$$\begin{aligned} A_{ref} &= \frac{k - k'}{k + k'}A_{in} = \frac{\sqrt{\mu} - \sqrt{\mu'}}{\sqrt{\mu} + \sqrt{\mu'}}A_{in} \\ A_{trans} &= \frac{2k}{k + k'}A_{in} = \frac{2\sqrt{\mu}}{\sqrt{\mu} + \sqrt{\mu'}}A_{in} \end{aligned} \quad (1)$$

(*Hint:* At $x = 0$, the displacement of the string must be continuous, $y_1 = y_2 = y_3$; if not, the string would break at the knot. Furthermore, the slope of the string must be continuous, $dy_1/dx + dy_2/dx = dy_3/dx$; if not, the string would have a kink and the (massless) knot would receive an infinite acceleration.) (H.C. Ohanian, "Physics," Pg. 387, Q14)

6. A guitar player attempts to tune his instrument perfectly with the help of a tuning fork. If the guitar player sounds the tuning fork and a string on his guitar simultaneously, he perceives beats at a frequency of 4 per second. The tuning fork is known to have a frequency of 294 Hz. What fractional increase (or decrease) of the tension of the guitar string is required to bring the guitar in tune with the tuning fork? From the available information, can you tell whether an increase or decrease of tension is required? (H. C. Ohanian, "Physics," Pg387, Q21)

7. A light wave of wavelength $5.0\times10^{-7}\ m$ strikes a mirror perpendicularly. The reflection of the wave by the mirror makes a standing wave with a node at the mirror. At what distance from the mirror is the nearest antinode? The nearest node? (H. C. Ohanian, "Physics," Pg389, Q.34)