2^{nd} **Midterm - December 1, 2007**

Name and Surname:
Student ID:
Department:
Signature:

Instructions: In each of the question, explain why you do each step. The questions might contain unnecessary information. If the question does not contain sufficient information, make any necessary assumptions. If you make unnecessary assumptions, you will lose points.

Discussion:

(Explain in your own words. You will lose 2 points for each mathematical relation that you write.)

1. What are the the laws of Kepler describing planetary motion ? What do they correspond to in the language of Newtonian dynamics? (15 points)

2. When do you have kinetic friction? static friction? When a sphere rolls without sliding, is there kinetic friction or static friction, why? Although there is neither the static nor kinetic friction in this case, no sphere could keep rolling until infinity on a surface in vacuum (no air resistance). Why does it slow down? (15 points)

Short Questions:

3. Consider a string that passes over a pulley and has two masses attached to its ends. Then the masses are let free to accelerate. If the tension at one side of the pulley is 5 N and that of the other end is 4 N, what is the net torque acting on the pulley? (5 points)

4. A mass of 1 kg that has a velocity 2.3 m/s hits another mass of 3.2 kg which is initially at rest. If they stick together after the collision, what is their final velocity? (5 points)

5. In nature, there are three dimensional fundamental constants. $\hbar$ (pronounced *h bar*) which has the dimension of angular momentum and has the value: $\hbar = 1.05 \times 10^{-34}\ Js$, the speed of light $c = 3.00 \times 10^{8}\ m/s$ and the Gravitational constant: $G_N = 6.667 \times 10^{-11}\ Nm^2/kg^2$. Using only these constants, derive a quantity that has the dimensions of mass. Using that the mass of a proton is $m_p = 1.67 \times 10^{-27}\ kg$, how many proton masses does that make? (5 points)

Explicit Calculation:

6. Consider a spherical planet of mass M and radius R. Assume that the mass density of the planet is uniform.
   (a) Calculate the gravitational self potential energy of the planet. (5 points)
   (b) Assume that a small tunnel is drilled that passes through the center of the planet. Ignoring the mass of the piece that is drilled out, write down the equation of motion of a small mass m that is left from one end of the tunnel. What kind of motion will this small mass make? (10 points)
7. Our unfortunate lover, let's give him a name and let's call him Ozkan, is still running away. Realizing that his plans to go to the moon will not work, he passes the cannon and arrives at a bridge over a frozen river. As he is passing over the bridge, it collapses and he finds himself on the ice facing towards the direction that he arrived, i.e. he is facing the father who is trying to reach him. He wants to change his orientation so that he wants to face the exactly opposite direction. How can he do that? Make an estimate. (15 points)
8. Ozkan manages to get him out of the frozen river, but he still needs to cross a rope bridge. The bridge looks quite old and rotten. He remembers having read somewhere that the ropes of the bridge can withstand a tension of at most 1000 N. If the two sides of bridge is separated by 5 m and the rope that makes the bridge is 6 m, will Ozkan succeed in getting to the other side? (Assume that Ozkan weighs 60 kg and ignore the weight of the bridge. $g = 9.81\ m/s^2$) (15 points)
9. In a model of the electron, the electron is treated as a small rotating sphere that has a radius $r_e = 2.818 \times 10^{-15}\ m$. Given that the mass of the electron is $m_e = 9.1 \times 10^{-31}\ kg$, its angular momentum $L = \hbar/2$, its electric charge $q = -1.6 \times 10^{-19}\ C$, find the velocity of the surface of the electron relative to its center of mass if the electron has a speed $v_e = 1.2 \times 10^5\ m/s$ (10 points)
10. Consider a chain on a table. The chain has a total mass of M and length L. Assume that the mass of the chain is uniform. There is a hole on the table, through which, one end of the chain is left free to slip down. Ignoring any friction, write down a differential equation that describes the motion of the end of the chain. (15 points).

Useful formulae:

You can use the following formula's without deriving them. For anything else, you need to derive it:

$$\begin{aligned} \vec{F} &= m\vec{a} \ , \quad \vec{a} = \frac{d\vec{v}}{dt} \ , \quad \vec{v} = \frac{d\vec{x}}{dt} \\ a_r &= \frac{v^2}{r} \end{aligned} \tag{1}$$

where a_r is the radial acceleration of an object making circular motion on a circle of radius r

$$I = \int dM d^2 \tag{2}$$

where dM is the mass of an infinitesimal volumes and d is the distance from the rotation access.

$$\begin{aligned} dV &= r^2 dr \sin\theta d\theta d\phi \\ &= \rho d\rho d\phi dz \\ &= dx dy dz \end{aligned} \tag{3}$$

where $r^2 = x^2 + y^2 + z^2 = \rho^2 + z^2$, $\cos\theta = z/r$, $\tan\phi = y/x$