PHYS 113 - FINAL EXAM
17 January 2008

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Read the questions carefully. Among the given information, not all of them might be necessary. In such a case, choose the relevant information. If there is not enough information, make necessary assumptions. State clearly your work and any assumptions that you make, if any. Explain your calculations. The total points of the exam is 120 points including 20 bonus points.

1. Solve the following short questions.(5 points each, 30 points total)

   (a) If the position of a particle is described by the position vector $\vec{r}$ where

   $$\frac{\vec{r}(t)}{1m} = (3 + 4\tilde{t})\hat{x} + (5\tilde{t^3})\hat{y} + 5\hat{z} \tag{1}$$

   where $\tilde{t} = \frac{t}{1sec}$, find the instantaneous acceleration at time $t = 2\,\text{sec}$

   (b) A man is standing 3 m from the center of a straight road. A car which has a mass of 800 kg is initially at a distance of 15 km from the observer. If the car moves with a constant speed of 60 km/hr along the center line of the road, approaching the observer, calculate the angular momenta of the car with respect to the observer as a function of time. Express your answer in units of kgm^2/s

   (c) Consider a massless beam which has two point like masses attached to the ends. If the length of the beam is 1 m and the masses are each 1 kg, calculate the moment of inertia of the beam with respect to rotations around an axis perpendicular to the beam and passing through the middle.

   (d) The human skeleton can be considered as a system of levers. The muscles create the forces necessary to move the body parts by contracting. Estimate the force that need to be delivered to your elbow in order to hold a mass of 5 kg such that your elbow makes a right angle.

(e) Consider a conservative force field of the form

$$\vec{F} = -\frac{k}{r^n}\hat{r} \tag{2}$$

where k and n are some positive constants. What is the corresponding potential energy? (Take a point at infinity as your reference point and assume that it has zero potential energy)

(f) Consider two identical cars each having a mass of 800 kg. One of the cars is stationary and the other car is moving with a velocity 50 km/hr. If the cars collide and stick together, what is the maximum possible velocity after collision?

2. Consider an ideal gas enclosed in an isolated cylinder of volume V and cross sectional area A. On top of the gas, there is a frictionless piston of mass M. Initially the system is at rest,i.e., the weight of the piston is balanced by the pressure of the gas. If the piston is pushed down slightly (much smaller than the height of the cylinder), write down the equation governing the subsequent motion of the piston. Describe the motion. (For the ideal gas, the adiabatic equation is $PV^\gamma = 0$. Assume $\gamma = \frac{5}{3}$)(20 points)

3. Consider a circular loop rotating vertically at a constant angular velocity ω in a uniform gravitational field. Assume that there is a bead of mass m on the ring that can slide freely on the loop. What is the angle that the line joining the center of the loop to the bead makes with the vertical? (20 points)

4. In the class we have shown that the standing waves that can be formed on a string with fixed ends has the property that the total length of the string had to be a multiple of half the wavelength of the wave. If, instead of an open string with fixed ends, a loop would be used, the condition would have been that the circumference of the loop should be an integer multiple of a full wavelength. In quantum mechanics, the particles also have wave properties. In a simple model, treat an electron orbiting around a single proton, as a wave moving on a string (there is no physical string but the orbit acts as a string). The wavelength of the electron is related to its momentum through $\lambda = \frac{2\pi\hbar}{p}$ where $\hbar$ is a constant. (50 points)

   (a) The force keeping the electron around the proton is similar in form the the gravitational force. The potential energy of the

electron and the force felt by the electron are given as

$$U = -k\frac{e^2}{r}$$
$$\vec{F} = -k\frac{e^2}{r^2}\hat{r} \tag{3}$$

where k is some constant and e is the electric charge of the electron. Assume that the electron has a circular orbit of radius r. Show that the momentum and the radius of the orbit are related through $p = \sqrt{mke^2/r}$ where m is the mass of the electron. (*Hint:* Treat the electron as a point like particle (not a wave) and write the centripetal force.)(10 points)

(b) Using the momentum obtained, show that the total energy of the electron in such a circular orbit is $E = -k\frac{e^2}{2r}$. (10 points)

(c) Using the momentum that is calculated in part (a), and using that the circumference of the orbit should be a multiple of the electron wavelength, show that the radius of the orbit can only take the values of the form

$$r_n = \frac{\hbar^2}{mke^2}n^2 \tag{4}$$

where n is some positive integer. (10 points)

(d) Using the result of part (b), what is the corresponding energy of the electron whose orbit has a radius r_n? (10 points)

(e) What is the angular momentum of this electron (treating the electron as a point like particle that has the momentum calculated and orbiting a circular orbit with radius r_n)(10 points)