PHYS 113 - Make-up EXAM
8 January 2008

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

In answering the questions, use correct signs and units. Explain your steps. You will lose points for the lack of explanation and use of wrong units/signs.

1. **Short Questions** Answer the following questions. Use correct vector signs and correct dimensions. (15 points)
   (a) Assume that the force acting on a mass of 1 kg is given by $\vec{F} = (3\ N)\hat{x} - (1\ N)\hat{y}$. If the particle has an initial velocity $\vec{v}_0 = (1\ m/s)\hat{y}$, and is at the point $\vec{r} = 0$, what is the position as a function of time? (5 points)
   (b) Consider a mass m attached at the end of a massless rope is under the influence of gravity. The mass is slightly displaced from the equilibrium position and let free. Write down the equation governing the subsequent motion of the mass and find the frequency of oscillations. (5 points)
   (c) Consider two objects of identical masses. Assume that one of the objects is at rest and the other one is moving at a speed of 4 m/s. If two masses undergo a completely elastic head on collision, what will be the final speeds of the masses? (5 points)
2. Consider a mass m at rest on an inclined plane whose inclination angle is θ. Assume that the static friction coefficient is μ_s and the kinetic friction coefficient is μ_k. (15 points)
   (a) What is the critical angle θ_c at which the mass starts to slide? (5 points)
   (b) For $\theta > \theta_c$, what is the work done by friction as a function of time? (5 points)
   (c) What is the work done by gravity as a function of time? (5 points)
3. Consider a pulley of mass m and radius R. Suppose that there is a massless chord going around the pulley and at the ends of the rope, two masses m_1 and m_2 are attached. (13 points)

(a) Calculate the moment of inertia of the pulley. (Assume that the mass of the pulley is distributed uniformly and it has the shape of a cylinder of height h)(3 points)

(b) Draw the free body diagrams for each of the masses and the pulley. Show the forces explicitly. (3 points)

(c) Write down the equations governing the motions of each mass and the pulley (3 points)

(d) Solve for the position of each one of the masses (4 points)

4. Consider the following $P-V$ diagram of a process involving a monatomic ideal gas. The gas is first expanded isothermally from (P_1, V_1) until (P_2, V_2), then it is let to expand adiabatically from (P_2, V_2) until (P_3, V_3), in the third stage it is compressed from V_3 to V_1 at constant pressure, and in the final stage, its pressure is increased from P_3 up to P_1 at constant volume. Calculate the net work done and the efficiency of the engine. (20 points)

5. It is believed that, during the formation of a star, while the gas cloud collapses, the gravitational energy released heats up the particles making the star. The mass of the Sun is $m_S = 1.98 \times 10^{30}\ kg$ and its radius is $r_S = 695500\ km$. (20 points)

   (a) Calculate the amount of energy (in Joules) released while the sun collapses from a cloud of radius $2r_S$ to the present radius. (Assume that the gas cloud always has a uniform density during the process, and the particles making the cloud has a mass of $3.0 \times 10^{-27} kg$)($G_N = 6.67300 \times 10^{-11} m^3 kg^{-1} s^{-2}$) (10 points)

   (b) If the Sun was an ideal gas, what would have been its final temperature at the end of this process if its initial temperature is negligible? (10 points)

6. Consider a bucket filled with a liquid of mass density ρ. On the side of the bucket, there is a small hole at a distance h from the top level of the liquid. Outside the bucket, there is a square plate of width L covering the hole such that the center of the square coincides with the hole on the bucket. Calculate the angle the plate makes with the side of the bucket as water gets out of the bucket. (Ignore the variation of the level of the liquid, i.e. keep h constant, the liquid makes elastic collisions with the plate and the angle that the water beam makes with the perpendicular to the plate is the same before and after collision). (20 points)

Some possibly useful formulae: Moment of Inertia:

$$I = \sum_i m_i d_i^2 \tag{1}$$

where m_i and d_i are the mass and distance from rotation axis of mass i.

Ideal Gas:

$$\begin{aligned} PV = Nk_BT &= nRT \\ PV^\gamma &= \mathit{constant} \text{ in an adiabatic process} \\ W &= \int_{V_i}^{V_f} PdV \\ \Delta E &= \Delta Q - \Delta W \\ E &= \frac{3}{2}k_BT \end{aligned} \tag{2}$$

where $P$is the pressure, V is the volume, T is the temperature of the gas. N is the number of particles and n is the number of moles of particles making the gas. $R = 8.4 \times J/K/mol$, $k_B = 1.4 \times 10^{-23}\ J/K$ and $\gamma = 5/3$ for a monatomic ideal gas.

Gravitational potential energy of two point masses m_1 and m_2 separated by a distance R:

$$U = -G_N \frac{m_1 m_2}{R} \tag{3}$$

Some approximations (valid for $|x| << 1$)

$$\begin{aligned} \sin x &\simeq x \\ \cos x &\simeq 1 - \frac{1}{2}x^2 \\ (1+x)^n &\simeq 1 + nx \\ \log(1+x) &\simeq x \\ e^x &\simeq 1 + x \end{aligned} \tag{4}$$

Bernoulli's Equation:

$$\frac{1}{2}\rho v^2 + \rho g z + p = \mathit{constant} \text{ (along a streamline)} \tag{5}$$