Name and Surname:
Student ID:
Department:
Signature:

Each question worth 20 points. You should show your work. You will lose points if you do not put the right units and put the vector signs for vectors.
Time: 150 mins

1. Consider two gravitating identical bodies of masses M each, seperated by a distance R. Consider a smaller mass $m << M$ placed at the equilibrium point between the two larger masses. Suppose the smaller mass is displaced slightly by an amount x from the equilibrium position in the direction (i) perpendicular or (ii) parrallel to the line joining the the larger masses. Find the equations governing the subsequent motion of the particle. Along which direction is the position of the particle is a stable point for small disturbances? (*Hint:* For $x << 1$, $(1+x)^n \simeq 1+nx+\frac{1}{2}n(n-1)x^2$ for all values of n)

2. Consider a football player who kicks a piece of rock of mass m with a speed v at an angle θ with respect to the ground. When the piece of rock is at its topmost position, it breaks into two pieces of equal mass and one of the pieces falls straight down. How far away from the player does the other piece fall? What should be θ to maximize this range?

3. Consider a sphere of mass M and radius R, rotating with angular frequency w about an axis parallel to the ground. The sphere is left to the ground with zero initial translational speed. Due to the friction with the ground, as it slips, the sphere will start gaining translational speed. Calculate the distance the sphere travels before it starts rolling motion without slipping. (Let μ_s and μ_k denote the coefficients of static and kinetic friction respectively)

4. Consider a platform of mass M at rest on a frictionless surface. On top of this platform, assume that there is a spring with spring constant k, placed horizontally. At one end of the spring, there is attached a body of mass m, and the other end is fixed on the platform. At $t = 0$, the mass is displaced horizontally from the equilibrium position by a distance A and then released with zero initial velocity. Write an expression for the subsequent motion of the platform. (*Hint:* In order to write an expression, you first have to chose a reference frame and place a coordinate grid on that frame. In your answer, you have to specify which coordinates and which reference frame you use. Ignore the friction forces between the platform and the body.)

5. A fat window cleaner of mass $2M$ is standing in the middle of an elevator. His elevator consist of a uniform, flat piece of wood of length L and mass M attached to the chords at two ends. The other end of the chords are attached to the top of the building. He has placed his bucket of water on one of his sides midway between him and the edge of his platform. If the bucket has mass m, find the tension on each of the cords.