3^{rd} HOMEWORK

Due March 31, 2008

1. For the following vector pairs $(\vec{A}, \vec{B})$, calculate i) $\vec{A} \cdot \vec{B}$, ii) $\vec{A} \times \vec{B}$, and iii) the angle between $\vec{A}$ and $\vec{B}$

   (a) $\vec{A} = 3\hat{i} + 2\hat{j} - 5\hat{k}$, $\vec{B} = \sqrt{2}\hat{i} + \pi\hat{j} + e\hat{k}$

   (b) $\vec{A} = \hat{i} + \hat{j} - \hat{k}$, $\vec{B} = 11.23\hat{i} + 2.7\hat{j} - 3.1\hat{k}$

   (c) $\vec{A} = \hat{i} + \hat{j} - \hat{k}$, $\vec{B} = \hat{i}$

   (d) $\vec{A} = 3\hat{i} + 3\hat{j} - 3\hat{k}$, $\vec{B} = \hat{i} - \pi\hat{j}$

   (e) $\vec{A} = 1.23\hat{i} + 4.56\hat{j} - 7.89\hat{k}$, $\vec{B} = 0.98\hat{i} + 7.65\hat{j} - 4.32\hat{k}$

   (f) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = -\hat{i} - \hat{j} - \hat{k}$

2. Evaluate the following indefinite integrals. Show your work and do not look up integral tables:

   (a) $\int d\theta \frac{1}{\cos\theta}$

   (b) $\int d\theta \frac{1}{\sin\theta}$

   (c) $\int d\theta \frac{1}{\tan\theta}$

   (d) $\int dx \frac{1}{a^2+x^2}$

   (e) $\int dx \frac{1}{a^2-x^2}$

   (f) $\int dx \frac{1}{(a^2+x^2)^{\frac{1}{2}}}$

   (g) $\int dx \frac{1}{(a^2-x^2)^{\frac{1}{2}}}$

3. You want to make a resistor of 1 Ω out of a carbon rod of diameter 1.0 mm. How long a piece of carbon do you need?

4. A lightning rod of iron has a diameter of 0.80 cm and a length of 0.50 m. During a lightning stroke, it carries a current of 1.0×10^4 A. What is the potential drop along the rod?

5. A solid truncated cone is made of a material of resistivity ρ. The cone has a height h, a radius a at one end, and a radius b at the other end. Derive a formula for the resistance of the cone.

6. The air of the atmosphere has a slight conductivity due to the presence of a few free electrons and positive ions.

   (a) Near the surface of the Earth, the atmospheric electric field has a strength of about 100 V/m and the atmospheric current density is 4×10^{-12} A/m^2. What is the resistivity?

   (b) The potential difference between the ionosphere (upper layer of the atmosphere) and the surface of the Earth is 4×10^5 V. What is the total resistance of the atmosphere? (Hint: For the purpose of this problem, you may assume that the Earth is flat.)

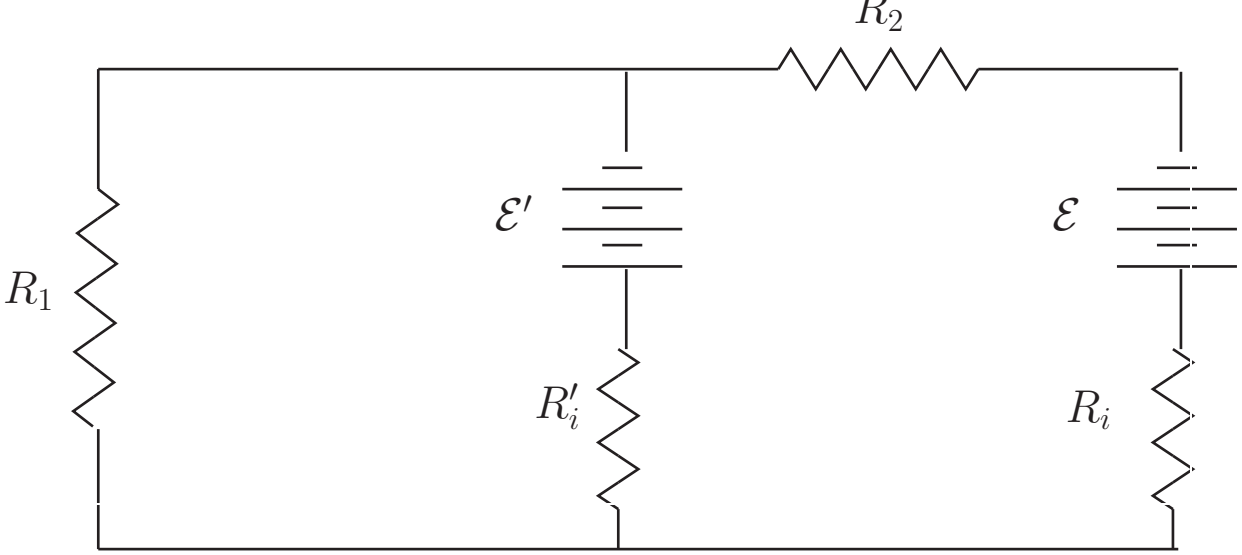

Figure 1:

7. Two batteries with internal resistances are connected as shown in Figure 1. Given that $R_1 = 0.50\ \Omega$, $R_2 = 0.20\ \Omega$, $\mathcal{E} = 12.0\ V$,$\mathcal{E}' = 6.0\ V$, $R_i = 0.025\ \Omega$, and $R_i' = 0.020\ \Omega$, find the currents in the resistances R_1 and R_2