8^{th} HOMEWORK
Due May 8, 2008

1. In the following equations, solve for A and ϕ in terms of the other constants using phasor diagrams

   (a) $A\cos(\omega t - \phi) + 3A\sin(\omega t - \phi) = \sin(\omega t)$

   (b) $\sqrt{2}\cos(\omega t - \phi) - A\sin(\omega t - \phi) = \cos(\omega t)$

   (c) $\pi\cos(\omega t - \phi) + \sqrt{3}A\sin(\omega t - \phi) = 2\sin(\omega t)$

2. A sheet of aluminum is being pushed between the poles of a horseshoe magnet. Describe the direction of flow of the induced current, or eddy currents, in the sheet. Explain why there is a strong friction force that opposes the motion of the sheet.

3. A circuit consists of a resistor connected in series to a battery; the resistance is 5Ω and the emf of the battery is $12\ V$. The wire (of negligible resistance) connecting these circuit elements are laid out along a square of $20\ cm \times 20\ cm$. The entire circuit is placed face on in an oscillating magnetic field. The instantaneous value of the magnetic field is

$$B = B_0 \sin \omega t \tag{1}$$

   with $B_0 = 0.15\ T$ and $\omega = 360 radian/s$.

   (a) Find the instantaneous current in the resistor.

   (b) Find the average power dissipated in the resistor

4. What is the natural frequency for an LC circuit consisting of a $2.2 \times 10^{-6} F$ capacitor and a $8.0 \times 10^{-2}\ H$ inductor?

5. Consider and RC circuit consisting of a capacitor and a resistor in series. The capacitor is initially charged

   (a) Show that Kirchhoff's rule leads to the following equation for this system:

$$R\frac{dQ}{dt} + \frac{1}{C}Q = 0 \tag{2}$$

(b) Verify that the solution of this equation is

$$Q = Q_0 e^{-t/RC} \tag{3}$$

where Q_0 is the initial charge at time $t = 0$.

(c) Show that the current is

$$I = -\frac{Q_0}{RC} e^{-t/RC} \tag{4}$$

(d) Suppose that the resistance is 3.0 Ω. Suppose that the capacitance of the capacitor is $1.0 \times 10^{-4} F$ and that the initial voltage across its terminals is 3.0 V. For this special case, plot the current as a function of time. What is the current at the initial instant? After how many seconds will the current have dropped to one-half of its initial value.

6. An LCR series circuit has $L = 0.5\ H$, $C = 2.0 \times 10^{-5} F$, and $R = 10\ \Omega$. At time $t = 0$, the capacitor is fully charged with a voltage at 20 V across its plates.

(a) What is the initial energy in the circuit?

(b) What is the percentage loss of energy per period?

(c) At what time after $t = 0$ will the energy in the circuit have fallen to one-half of its initial value? One-tenth of its initial value?

(d) Plot the energy as a function time for the time interval $0\ s \leq t \leq 0.1\ s$