1^{st} **Midterm - March 21, 2007**

Name and Surname:
Student ID:
Department:
Signature:

You should show your work. You will lose points if you do not put the right units and put the vector signs for vectors.

Discussion:

Each question is 15 points. Any equation you use will cost you 2 points.

1. Argue that, in equilibrium, there can not be any electric field inside a metal. Neither inside the bulk of the metal, nor in any cavity inside the metal. Argue also that the electric field on the surface of the metal has to be perpendicular to the metal.

2. Explain action-at-a-distance and how the field concepts is used to avoid action-at-a-distance.

Short Questions:

3. Express the following units in terms of kilograms, meters and seconds: (10 points)

   - Coulomb
   - Ampere
   - Volt
   - Farad

4. Starting from the Gauss' Law, show that the electric field of a point charge q is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \tag{1}$$

   where r is the distance from the point charge and $\hat{r}$ is the radial unit vector (10 points)

Calculations:

5. Consider a linear semi-infinite positive charge distribution along the x-axis, and a linear semi-infinite negative charge distribution along the y

axis.Assume that the linear charge densities are $+\lambda$ and $-\lambda$ respectively. Calculate the electric field at an arbitrary point (x_0, y_0) where $x_0, y_0 > 0$. (20 points).

6. Consider a sphere that has a uniform mass density ρ and a positive charge density σ. The sphere has a radius R and at a point on the surface is attached a rope of length $2R$. The other end of the rope is attached to a vertical wall that has a uniform positive surface charge density λ. Under the action of both the gravity of the Earth and the electrostatic repulsion between the wall, the sphere will come to equilibrium when the rope makes an angle θ with the wall. Calculate θ (20 points) (First find a relation between ρ, σ, λ and R such that if it is satisfied, the sphere will touch the wall. If it is not satisfied, the sphere will start hanging in air. Find θ for both of these cases.)

7. Consider two identical dipoles. Each of the dipoles consist of one $+q$ and one $-q$ charge separated by a distance l. Suppose the dipoles are separated by a distance $R >> l$, and denote the vector connecting one dipole to the other one as $\vec{R}$. Calculate the energy of this two dipole configuration. For this aim, follow the following procedure:

   - First consider the potential energy created by a single dipole at a position $\vec{R}$. Express this energy in terms of the polarization $\vec{p}$ of the dipole. (Remember that the polarization vector is a vector whose magnitude is $|\vec{p}| = ql$ and whose direction points from the negative charge to the positive charge.) Express your answer in vector notation, in terms of $\vec{p}$ and $\vec{R}$. Use scalar or vector products of vectors as is suitable (10 points)
   - Secondly, consider a dipole in an external electrostatic potential. Express the total potential energy of the dipole. Express your answers in terms of vectors and the derivatives of the potential. (10 points)
   - Combining the two results, express the total potential energy of the system. Ignore the self potential energies of the dipoles. (Note that the potential energy of the dipole system is the amount of work that you have to do in order to bring the second dipole to its final position from infinity). Do not assume any particular orientation of neither any of the dipoles not for the their separation vector $\vec{R}$. (10 points)