2^{nd} **Midterm - April 19, 2008**

Name and Surname:
Student ID:
Department:
Signature:

You should show your work. You will lose points if you do not put the right units and put the vector signs for vectors. For vector quantities, you have to express both the magnitude and the direction. The maximum points that you can get is 120. 20 points are bonus points.

Mathematical Tools:

1. If $\vec{B} = y\hat{x}$, calculate the integral $\oint_S \vec{B} \cdot d\vec{S}$ if S is (i)a sphere of radius 1 centered at origin, and (ii) a cube of side length 1 and such that one corner is at the origin and three sides are along the positive x, y and z axes. (10 points)

2. If $\vec{B} = y\hat{x} - x\hat{y}$, calculate the integral $\oint_{\mathcal{C}} \vec{B} \cdot d\vec{l}$ if $\mathcal{C}$ is (i) a circle of radius 1 and centered at the origin, and (ii) a square of side length 1 and two sides on the positive x and y axes. (10 points)

3. Consider a point like object that carries an electric charge of 1 C. For the given pair of velocity and magnetic fields, calculate the magnetic force that the particle feels (the given velocities are in units of m/s and the magnetic field is in units of T): (4 points each, 12 points total)

   (a) $\vec{v} = 4\hat{x} - 2\hat{y}$, $\vec{B} = 3\hat{x} + 4\hat{y} + \hat{z}$

   (b) $\vec{v} = \hat{x} + \hat{y} + \hat{z}$, $\vec{B} = -2y\hat{x} + x\hat{y}$

   (c) $\vec{v} = \hat{x} - \hat{y} + \hat{z}$, $\vec{B} = -2y\hat{x} + x\hat{y} + 3\hat{z}$

Short Questions:

NOTE: The electric and magnetic fields are vector quantities.

4. What is the magnetic field created by an infinitely long wire lying along the z axes and carrying a current I in the positive z direction? (15 points)

5. Consider a capacitor in a circuit. The capacitor is made of two large sheets of cross-section area A separated by a distance d. If at a given instant, the current entering the capacitor is I, what is the rate of change $\frac{d\vec{E}}{dt}$ of the electric field between the plates? (20 points)

6. What is the magnetic field inside an ideal solenoid that carries a current I and has n turns per unit length? (15 points)

Long Calculations:

7. Consider a sphere of radius R and mass M that carries a charge Q uniformly distributed over its volume. Assume that the sphere rotates around the z axes with an angular velocity ω. (30 points)

   (a) Calculate the angular momentum of the sphere in terms of R, M and ω. (10 points)

   (b) Calculate the magnetic dipole moment created by the sphere (15 points)

   (c) Using the result of part (a), solve for ω in terms of the angular momentum and then eliminate ω in the expression for the magnetic dipole moment. (5 points)

8. Consider an ideal solenoid that has n turns per unit length. Each turn has a radius R. Assume that the current inside the solenoid is given as: $I = I_0 \cos(\omega t)$ where I_0 and ω are some constants. The current creates a changing magnetic field inside the solenoid which in turn induces an electric field. Calculate the energy stored in the magnetic field created by the current (10 points) and also in the electric field created by this changing magnetic field (15 point).

Useful formulae:

On a sphere or radius R, the surface element can be written as: $d\vec{S} = R\sin\theta d\theta d\phi\hat{r}$, on a plane parallel to the xy plane, $d\vec{S} = dxdy\hat{z}$; on a plane parallel to xz plane, $d\vec{S} = -dxdz\hat{y}$; on a plane parallel to the yz plane, $d\vec{S} = dydz\hat{x}$. (Note that the orientation of the surface element can change depending on the problem)

The infinitesimal volume element:
In Cartesian coordinates: $dV = dxdydz$
In cylindrical coordinate: $dV = \rho d\rho d\phi dz$ where $x = \rho\cos\phi$ and $y = \rho\sin\phi$
In spherical coordinates: $dV = r^2 dr\sin\theta d\theta d\phi$ where $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = t\cos\theta$

Electric force:

$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}\hat{r} \tag{1}$$

Magnetic force:

$$\vec{F}_M = \frac{\mu_0}{4\pi}qq'\frac{\vec{v}\times(\vec{v}'\times\hat{r})}{r^2} \tag{2}$$

Electric and magnetic fields satisfy (the laws that we have seen up to now)

$$\begin{aligned}
\oint \vec{E}\cdot d\vec{S} &= \frac{Q}{\epsilon_0} \\
\oint \vec{E}\cdot d\vec{L} &= -\frac{d\Phi_B}{dt} \\
\oint \vec{B}\cdot d\vec{S} &= 0 \\
\oint \vec{B}\cdot d\vec{l} &= \mu_0 I
\end{aligned}$$

The magnetic field created by a small segment of an charge carrying wire:

$$d\vec{B} = \frac{\mu_0}{4\pi}I\frac{d\vec{l}\times\hat{r}}{r^2} \tag{3}$$

The moment of inertia of an object for rotations around the z axis:

$$I = \int \rho(x^2+y^2)dV \tag{4}$$

where ρ is the mass density and the integral is over the whole volume of the object.