$10^{th}$ Homework
Due: January $2^{nd}$, 2009

1. Consider a rigid body. Show that the angular momentum of the rigid body relative to any reference point $O$ is equal to the vectorial sum of the angular momentum of the center of mass of the body relative to the reference point $O$ and the angular momentum of the body relative to its center of mass.

2. Solve the following differential equations, given the initial conditions.

   (a) $y''(x) - 2y'(x) + y(x) = 0$ with $y(0) = 0$, $y'(0) = 1$.

   (b) $y''(x) + 2y'(x) + y(x) = 0$ with $y(0) = 0$, $y'(0) = 1$

   (c) $y''(x) + y(x) = 0$ with $y(0) = 0$, $y'(0) = 1$.

   (d) $y''(x) - y(x) = 0$ with $y(0) = 0$, $y'(0) = 1$

   where $y'(x) = \frac{dy}{dx}$ and $y''(x) = \frac{d^2y}{dx^2}$ (*Hint:* Assume a solution of the form $y = e^{\alpha x}$ and solve for alpha)

3. The damped harmonic oscillator satisfies the differential equation

   $$y''(t) + \gamma y'(t) + ky(t) = 0$$

   where the ' denotes differentiation with respect to $t$. Without explicitely solving the differential equation, show that

   $$\frac{d}{dt}\tilde{E} = 0$$

   where

   $$\tilde{E} = e^{\frac{\gamma}{m}t}\left[\frac{1}{2}my'(t)^2 + \frac{1}{2}ky(t)^2 + \frac{\gamma}{2}y(t)y'(t)\right]$$

4. A complex number is a number that can be written in the form $z = a + ib$ where $a$ and $b$ are some real numbers and $i^2 = -1$. $a$ is called the real part of $z$, i.e. $a = Re(z)$, and $b$ is called the imaginary part of $z$, i.e. $b = Im(z)$. If a complex number is given by $z = a + ib$ for some real numbers $a$ and $b$, then the complex conjugate of $z$ is defined as $z^* = a - ib$. The absolute value of $z$ is defined as $|z| = \sqrt{z^* z}$. The exponent of $z$ is given by

   $$e^z = e^{a+ib} = e^a\left(\cos(b) + i\sin(b)\right) \tag{1}$$

(a) Calculate the absolute values of $z = e^{ib}$, $z = 4 + i3$

(b) Show that if $z = 1/(a + ib)$, then $|z| = \sqrt{a^2 + b^2}$, $Re(z) = \frac{a}{a^2+b^2}$, $Im(z) = -\frac{b}{a^2+b^2}$ (*Hint:* Multiply and divide $z$ by $z^*$)

(c) For the given complex numbers $z$, find $r$ and $\theta$ such that $z = re^{i\theta}$

$z = 3 + i4$

$z = 3e^{i2} + e^{i\pi}$

$z = e^{2+i} + 2e^{1-i}$