PHYS 430 SUPPLEMENTARY EXAMINATION FOR GRADUATION
June 22, 2007

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary.

1. Consider a gas of point like non-interacting particles. Assume that the relative separation between the particles is sufficiently large so that you do not need to make a distinction between fermions and bosons. If the system has temperature T, calculate the free energy, F, the entropy S, the specific heat per particle c_V and the chemical potential μ of the system.

2. Consider a heat engine that has the PV diagram shown in Fig. 1. Calculate the efficiency of the heat engine.

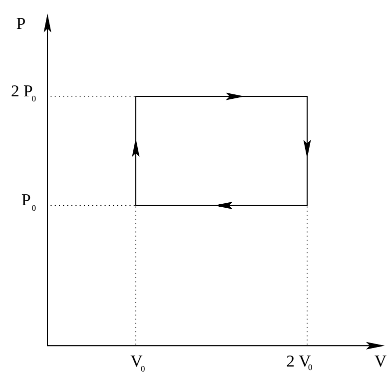

Figure 1: Figure for Q2

3. What are the three laws of thermodynamics?

4. Consider two particles which can be only in 2 states, one with energy ϵ_1 and the other with energy ϵ_2. If the system is at temperature T, write the partition function of the system if the particles are:

   (a) Distinguishable particles

   (b) Identical particles satisfying the Boltzmann distribution

   (c) Identical particles which are fermions

   (d) Identical particles which are bosons.

5. Consider a quantum harmonic oscillator that has a frequency ω (the energy levels are given by $\epsilon_n = \hbar\omega(n + \frac{1}{2})$). If the oscillator is at temperature T, calculate its free energy F. Calculate E and C_V for this harmonic oscillator.

6. Show that
$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$
for any thermodynamical system (10 points). Using this result and the equation of state of a Van der Waals gas:
$$\left(P + a\left(\frac{N}{V}\right)^2\right)(V - Nb) = NT$$
show that for the Van Der Waals' gas(10 points),
$$\left(\frac{\partial E}{\partial V}\right)_T = a\left(\frac{N}{V}\right)^2$$

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \ ; \quad \Omega = F - \mu N \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N
\end{aligned}$$

$$
\begin{aligned}
\int_0^\infty dx x^n e^{-x} &= n! \\
\int_{-\infty}^\infty dx \int_{-\infty}^\infty dy &= \int_0^{2\pi} d\phi \int_0^\infty d\rho\rho, \qquad \rho^2 = x^2 + y^2, \qquad \tan\phi = \frac{x}{y} \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

For anything else, you need to derive it.