2^{nt} Midterm - December 19, 2009

Name and Surname:
Student ID:
Department:
Signature:

You should show your work. You will lose points if you do not show your work. The questions can contain unnecessary information of insufficient information. If the question contains insufficient information, make necessary assumptions. Any unnecessary assumptions will cost you points.

Discussion

Answer the following question with words only. You do not need to give a quantitative answer, a qualitative answer is enough. You will lose points if you use equations.

1. What are the phenomena that lead to (i) paramagnetism ($B_T > B_{applied}$), (ii) diamagnetism ($B_T < B_{applied}$) and (iii) ferromagnetism. Explain in detail. If these materials are placed in a uniform magnetic field, sketch the magnetic field lines near these materials.

Short questions

2. Suppose you have an infinite straight wire that carries a current I. Near this wire, there is a square loop of side length L such that two sides of the loop are parallel to the wire, and the plane containing the loop contains the wire. If the current increases at a constant rate what is the induced current on the wire? (Assume that the loop has total resistance R and conductivity σ),

3. What is the magnetic field inside a toroid whose cross-section is a square? Assume that the side of the square is L, n is the number of turns per unit length and the distance between the center of the square and the toroid is R.

Explicit Calculations:

In answering following problems, show your steps in detail. You should also explain why you do a specific step.

4. Consider a conducting sphere that is divided into two equal hemi-spheres. The upper hemi-sphere is fixed at a potential V_0 and the lower hemisphere

at a potential $-V_0$. If this sphere is rotating at an angular frequency of ω around an axis passing through the centers of both hemispheres, what is the magnetic dipole moment of the system? (The magnetic dipole moment of a current running through a wire in a plane is $\vec{\mu} = I\vec{a}$).

5. Classically, the electron can be modeled as a rotating sphere of radius R_e.

   (a) Assume that the charge of the electron is distributed uniformly on the surface of the electron. The classical radius R_e of the electron is such that the energy stored in the electric field is equal to the mass of the electron. Find R_e.

   (b) Suppose that the mass of the electron is distributed uniformly over its volume. What is the angular frequency ω such that if the electron is spinning around an axis passing through its center, the angular momentum of the electron is $\frac{\hbar}{2}$ (The angular momentum of a point particle with respect to the chosen center of coordinates is $\vec{L} = \vec{r} \times \vec{p}$).

   (c) For this frequency, what is the total magnetic dipole moment of the electron?

   (d) Find the ratio $\frac{|\vec{\mu}|}{|\vec{L}|}$, for this electron.

6. What is the self inductance of a wire that makes a circle of radius R?

Some useful formulas:

Associated Legendre Polynomials $P_l^m(x)$ are solutions of the differential equation:

$$\frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right] + \left[l(l+1) - \frac{m^2}{1-x^2}\right] y = 0 \tag{1}$$

that are finite for $-1 \leq x \leq 1$. Bessel functions $J_n(x)$ are solutions of the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0 \tag{2}$$

that are finite at $x = 0$.