Makeup - January 18, 2010

Name and Surname:
Student ID:
Department:
Signature:

You should show your work. You will lose points if you do not show your work. If the question asks you to find a vector, you should find both its magnitude and direction. The questions can contain unnecessary information of insufficient information. If the question contains insufficient information, make necessary assumptions. Any unnecessary assumptions will cost you points.

Discussion

Answer the following question with words only. You do not need to give a quantitative answer, a qualitative answer is enough. You will lose points if you use equations.

1. Explain the following concepts (2 points each)
   (a) Inductance
   (b) emf
   (c) Ferromagnet
   (d) Magnetization
   (e) Polarization
2. In the class, we have discussed that in a conducting media, the electromagnetic waves decay exponentially. What happens to the missing energy? (You should also discuss how that effect is included in the Maxwell's equation) (10 points)

Explicit Calculations:

In answering following problems, show your steps in detail. You should also explain why you do a specific step.

3. Consider an infinite plane on whose surface there is a uniform surface charge density $\vec{K}$. Choose x such that it points in the direction of the current, and z axis to be perpendicular to the plane. Assume also that there is an infinite straight wire that carries a current I. Assume that the current I runs parallel to the surface charge density and is at a distance d from the plane. Find the magnetic field at an arbitrary point in space. Sketch the magnetic field lines. (20 points)

4. Consider a square prism that has sides $L \times L \times h$. Assume that the electrostatics potential at the upper square is fixed at V_0 and all the sides are fixed at $V = 0$ potential. Find the electrostatic potential at any point inside the prism. (20 points)

5. Starting from Maxwell's equations in a linear medium with magnetic permeability μ and dielectric constant ϵ, (i) derive the boundary conditions that electric and magnetic fields should satisfy at the boundary connecting two different media. (10 points) (ii) the reflection and transmission coefficients of a wave that hits a perfect conductor perpendicularly(10 points). (*Hint:* Inside a perfect conductor, both the electric and magnetic fields are zero.)

6. Consider a coaxial cable made up of a conducting cylinder or inner radius a and locate at its axis, another cylindrical wire of radius b. The space $b < r < a$ is filled with a dielectric material of dielectric constant ϵ. The coaxial cable operates in TEM mode, i.e. both the electric and magnetic fields are perpendicular to the wave-vector, which is parallel to the axis of the cable. Choose the z axis to be along the axis of the coaxial cable.

   (a) The electric and magnetic fields can be written as

   $$\begin{aligned} \vec{E} &= \vec{E}_0(x,y)e^{-i(\omega t - kz)} \\ \vec{B} &= \vec{B}_0(x,y)e^{-i(\omega t - kz)} \end{aligned} \tag{1}$$

   where $\vec{E}_0$ and $\vec{B}_0$ are vectors perpendicular to the z axis. What are the equations satisfied by the $\vec{E}_0$ and $\vec{B}_0$ fields? (10 points)

   (b) What should be the relation between k and ω such that the electromagnetic fields given in Eq. 1 satisfy the Maxwell's equations? (10 points)

   (c) What are the possible solutions (modes) for electric and magnetic fields? What are their linear momentum density? (20 points)

Some useful formulas:

Associated Legendre Polynomials $P_l^m(x)$ are solutions of the differential equation:

$$\frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right] + \left[l(l+1) - \frac{m^2}{1-x^2}\right]y = 0 \tag{2}$$

that are finite for $-1 \leq x \leq 1$. Bessel functions $J_n(x)$ are solutions of the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0 \tag{3}$$

that are finite at $x = 0$.