Final Take-Home Exam
Due: January 14, 2009

Instructions

In the following questions, be sure to explain clearly your reasoning. You are allowed to ask and discuss the problems with anybody, but your answer should be your own understanding of the problem. In the second question, you are not allowed to ask yourself the same question as any of your friends, and neither any question that we have discussed during the lectures. All the questions worth 20 points.

Questions

1. Draw the lowest order Feynman diagrams for the following transitions:

$$\begin{aligned} B_s &\quad \to D_s^- \nu \ell \\ \pi^0 &\quad \to 2\gamma \\ \pi^+ &\quad \to \ell^+ \nu \\ W^+ &\quad \to \ell^+ \nu \\ B^0 &\quad \to \bar{B}^0 \end{aligned} \tag{1}$$

2. Ask yourself a question and answer it using the knowledge you gained during these lectures. The question should not include long calculations and should be based on explaining some of the observations mentioned in PDG listings. You will be graded mainly depending on the originality of your question and answer, i.e. if you just change the names or numbers in the question we have discussed in the class, or, the question is a trivial question, or if (even if it is just a coincidence) your question is very similar to your friends question, you will not get any points.

3. $SU(3)$ symmetry tells us that the mesons made up of a quark and an anti-quark can be grouped into an octet and a singlet (we will consider only the lightest quarks u, d and s). This classification is independent of the other quantum numbers, such as the spin, angular momentum, or radial, of the meson. This means that for any set of quantum numbers

not related to the flavor, there should be an octet and a singlet. During the lectures we have discussed the pseudoscalar octet and singlet and the vector octet and singlet. These are the lowest $l = 0$, $s = 0$ and $l = 0$, $s = 1$ states. If the quark model is correct, there should also be radial excitation of the $l = 0$ and $s = 0$ pseudoscalar mesons (an octet and a singlet). Which ones of these can you identify in the PDG tables? (note that all the quantum numbers of the radial excitations are the same as the ground states)

4. In the class, we have discussed the mixing between mesons. Consider the pseudoscalar mesons η and η'. In the exact $SU(3)$ limit, they can be identified with the octet and the singles states as: $\eta = \eta_8$ and $\eta' = \eta_0$ where

$$\begin{aligned}\eta_8 &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \\ \eta_0 &= \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)\end{aligned} \tag{2}$$

Breaking of the $SU(3)$ symmetry, will cause mixing between the states η_8 and η_0, and hence the physical η and η' states will be

$$\begin{aligned}\eta &= \eta_8 \cos\theta + \eta_0 \sin\theta \\ \eta' &= \eta_0 \cos\theta - \eta_8 \sin\theta\end{aligned} \tag{3}$$

where θ is the mixing angle and $\theta = 0$ in the $SU(3)$ limit. Another possible basis states to work with is the so called flavor basis, which is defined as

$$\begin{aligned}\eta_q &= \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \\ \eta_s &= \bar{s}s\end{aligned} \tag{4}$$

In terms of the flavor basis, the physical η and η' states can be written as

$$\begin{aligned}\eta &= \eta_q \cos\phi + \eta_s \sin\phi \\ \eta' &= \eta_q \sin\phi - \eta_s \cos\phi\end{aligned} \tag{5}$$

Answer the following question for the η and η' mesons:

(a) What is the angle ϕ in the exact $SU(3)$ limit?

(b) To find the value of ϕ that is realized in nature, consider the decays $J/\psi \to \eta^{(\prime)}\rho$. Using the OZI rule, argue that in this decay only the η_q component of η or η' is produced, the production of η_s component is OZI suppressed.

(c) Show that in the approximation that one neglects the production of the η_s component, the amplitudes for the decays satisfy

$$\frac{A(J/\psi \to \eta\rho)}{A(J/\psi \to \eta'\rho)} = \cot\phi \tag{6}$$

(d) Find the widths for these decays from PDG, and relating the ratio of the widths to the ratio of the amplitudes, find ϕ. (Difference in the phase space due to different masses of η and η' will be important).

(e) What is the percent deviation of ϕ from the exact $SU(3)$ symmetry expectation?

5. In the absence of weak interactions, show that the vector mesons ρ^0, ω and ϕ have zero magnetic moment. For this purpose, note that if they had a magnetic moment, $V \to V\gamma$ transition could happen. Argue that this transition can not happen (note that this decay is not allowed kinematically, but this does not mean that this transition can not happen virtually. You have to use symmetry arguments to show that even such a virtual transition is not possible). Show also that ϕ can not decay into a photon and a ρ or ω even if these decays are kinematically allowed. What are this limits for these decays (look-up at the PDG)? Estimate the relevant weak coupling from these limits.