

- Economic evaluation of an alternative requires **cash flow estimates** over a stated time period and a **criterion** for selecting the best alternative
- The alternatives are developed from project proposals to accomplish a stated purpose
- Some projects can be economically or technologically viable, some are not
- After the viable projects stand out among the others, these are the alternatives to evaluate
- In formulating alternatives;
  - 1 **Mutually Exclusive:** Only one of the viable projects can be selected
  - 2 **Independent:** More than one viable project may be selected

- **do-nothing** (DN) option can be another alternative; no new costs or revenues.
- Mutually exclusive alternatives compete with each other, best alternative is selected
- Independent projects compete with the do-nothing alternative instead of another project
- If there are  $m$  independent projects; one, two or more of them can be selected. ( $2^m$  selections!)
  - Let we have  $m = 3$ , and three independent alternatives are  $\{A, B, C\}$
  - Then we have  $2^3 = 8$  possible selections:
  - $\{DN, A, B, C, AB, AC, BC, ABC\}$
  - Assumption: No budget limit!

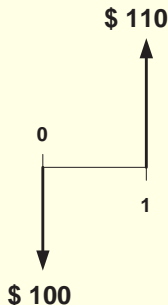
- It is important to recognize the nature or type of alternatives before starting evaluation
- Cash flows determine whether the alternatives are **revenue-based** or **service-based**
  - 1 **Revenue-Based:** Each alternative generates cost and revenue cash flow estimates and possibly savings
  - 2 **Service-Based:** Each alternative has only cost cash flow estimates
    - Revenues or savings are not dependent on alternatives
    - Revenues or savings are assumed to be equal among alternatives
    - Mostly public (government) initiatives
    - Evaluation on based on cost estimates
- All alternatives evaluated in an engineering economy study must be of the same type

Basic procedure for applying the **Present Worth (PW)** criterion:

- 1 Determine the interest rate ( $i$ ) that the firm wishes to earn on its investments.  $i = \text{MARR}$  is usually given to us.
- 2 Estimate the service life ( $n$ ) of the project
- 3 Estimate the cash inflow for each period over the service life
- 4 Estimate the cash outflow for each period over the service life
- 5 Determine the net cash flows ( $A_n$ ) for each period (net cash flow = cash inflow - cash outflow)
- 6 Find the present worth of each net cash flow at **MARR**. Add up these present-worth figures; their sum is defined as the projects *PW*:

$$\begin{aligned}
 PW(i) &= \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \dots + \frac{A_N}{(1+i)^N} \\
 &= \sum_{n=0}^N \frac{A_n}{(1+i)^n} = \sum_{n=0}^N A_n(P/F, i, n)
 \end{aligned}$$

- $P$  value or now called **Present Worth**  $PW$  is calculated using MARR for each alternative.
- When the alternatives have equal lives, while carrying out the  $PW$  analysis for **MUTUALLY EXCLUSIVE ALTERNATIVES**:
  - 1 **One Alternative**: Calculate  $PW$  at MARR. If  $PW \geq 0$ , the requested MARR is met or exceeded and the alternative is financially viable.
  - 2 **Two or more Alternatives**: Calculate  $PW$  of each alternative at MARR. Select the alternative with the  $PW$  value that is **numerically largest**. (not absolute value)



$$F = P(1+i) \quad \longrightarrow \quad i = 10\%$$

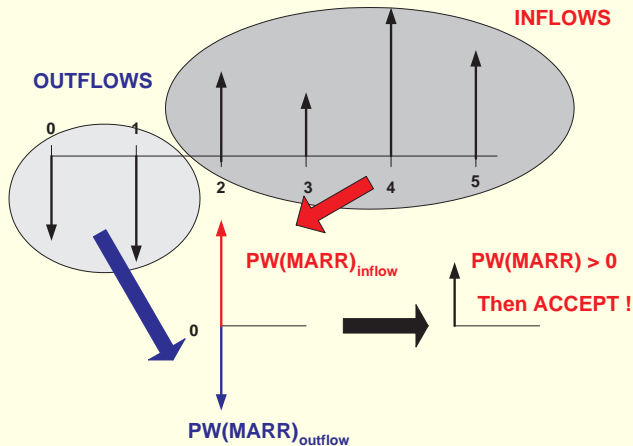
$$PW = -100 + 110/1.1 = 0$$

Let MARR=20%

$$PW = -100 + 110/1.2 = -8.33$$

**This investment's return rate is < 20%**

- *PW* analysis is popular because future costs and revenue estimates are all transformed into **equivalent dollars now**; all future cash flows are converted into present dollars
- Makes easier to determine the economic advantage of one alternative over another



## Example

$PW_1$	$PW_2$	Selected One
\$-1500	\$-500	2
-500	1000	2
2500	-500	1
2500	1500	1

If the projects are **INDEPENDENT** (and revenue projects), the selection guideline is as follows:

- 1 For one or more independent projects, select all projects with  $PW \geq 0$  at the MARR

## Example

$PW_1$	$PW_2$	Selected One
\$-1500	\$-500	DN
-500	1000	2
2500	-500	1
2500	1500	1,2

## Example

Perform a present worth analysis of equal-service machines with the costs shown below, if the MARR is 10% per year. Revenues for all three alternatives are expected to be the same.

	Electric- Powered	Gas- Powered	Solar- Powered
First cost,\$	\$-2500	\$-3500	-6000
Annual Operating cost, \$	-900	-700	-50
Salvage value, \$	200	350	100
Life, years	5	5	5

- Service alternatives
- Salvage value considered as a **negative cost**, so + sign used
- *PW* of each alternative is calculated at  $i = 10\%$ , for  $n = 5$  years.

## Example

	Electric- Powered	Gas- Powered	Solar- Powered
First cost,\$	\$-2500	\$-3500	-6000
Annual Operating cost, \$	-900	-700	-50
Salvage value, \$	200	350	100
Life, years	5	5	5

$$\begin{aligned}
 PW_E &= -2500 - 900(P/A, 10\%, 5) + 200(P/F, 10\%, 5) \\
 &= -2500 - 900(3,7908) + 200(0.6209) = \$ - 5787.54
 \end{aligned}$$

$$\begin{aligned}
 PW_G &= -3500 - 700(P/A, 10\%, 5) + 350(P/F, 10\%, 5) \\
 &= -3500 - 700(3,7908) + 350(0.6209) = \$ - 5936.245
 \end{aligned}$$

$$\begin{aligned}
 PW_S &= -6000 - 50(P/A, 10\%, 5) + 100(P/F, 10\%, 5) \\
 &= -6000 - 50(3,7908) + 100(0.6209) = \$ - 6127.45
 \end{aligned}$$

Select **Electric-powered** machine with  $PW_E = \$ - 5787.54$

When the present worth method is used to compare **mutually exclusive** alternatives that have different lives, the procedure is same as before except:

- *PW* of alternatives must be compared over the same number of years and end at the same time
- A fair comparison can be made only when *PW* values represent costs (and receipts) associated with **equal service**
- The equal-service requirement can be satisfied by either of two approaches:
  - 1 Compare the alternatives over a period of time equal to **least common multiple** (LCM) of their lives
  - 2 Compare the alternatives using a **study period of length of  $n$  years**, which does not necessarily take into consideration the useful lives of the alternatives. (**Planning horizon approach**)

- In either case, MARR is used to calculate the PW of each alternative
- The LCM approach makes the cash flows for all alternatives extend to the same time period
- The LCM approach requires some assumptions to be made about subsequent life cycles of the alternatives:
  - 1 The service provided by alternatives will be needed for the LCM of years or more
  - 2 The selected alternative will be repeated over each life cycle of the LCM in exactly the same manner
  - 3 The cash flow estimates will be the same in every life cycle
- The LCM requires that the estimated salvage values be included in each life cycle

- For the study period approach, a time horizon is chosen over which the economic analysis is conducted
- Only the cash flows during this time period is considered relevant to the analysis
- All cash flows occurring beyond the study period are ignored
- An estimated market value at the end of the study period must be made.
- Study period approach is often used in replacement analysis
- The time horizon chosen may be relatively short
- Also useful when LCM of alternatives yields unrealistically long time horizons

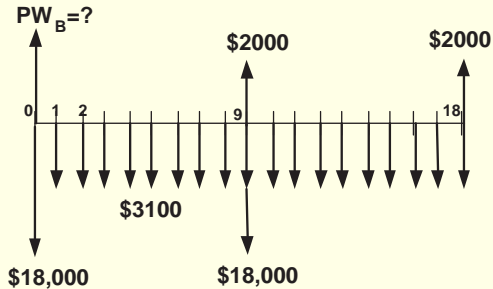
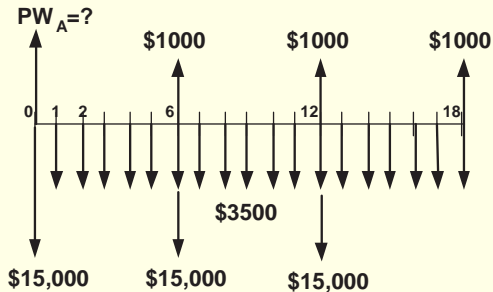
## In Class Work 6

Eric Forman, a project engineer is assigned to start up a new office in Wisconsin. Two lease options are available:

	Location A	Location B
First cost, \$	\$-15,000	\$-18,000
Annual lease cost, \$ per year	-3,500	-3,100
Deposit return, \$	1,000	2,000
Lease term, years	6	9

- Determine which lease option should be selected on the basis of a present worth comparison if MARR is 10% per year.
- If a study period of 5 years is used and the deposit returns are not expected to change, which location should be selected
- Which location should be selected if a 6-year study period and a deposit return of \$6,000 after 6 years at location B is used.

a)



$$\begin{aligned}PW_A &= -15,000 - 3,500(P/A, 10\%, 18) + 1,000(P/F, 10\%, 6) \\ &\quad - 15,000(P/F, 10\%, 6) - 15,000(P/F, 10\%, 12) \\ &\quad + 1,000(P/F, 10\%, 12) + 1,000(P/F, 10\%, 18)\end{aligned}$$

$$\begin{aligned}PW_A &= -15,000 - 3,500(8.2014) + 1,000(0.5645) \\ &\quad - 15,000(0.5645) - 15,000(0.3186) + 1,000(0.3186) \\ &\quad + 1,000(0.1799) = \$ - 55,888.4\end{aligned}$$

$$\begin{aligned}PW_B &= -18,000 - 3,100(P/A, 10\%, 18) + 2,000(P/F, 10\%, 9) \\ &\quad - 18,000(P/F, 10\%, 9) + 2,000(P/F, 10\%, 18) \\ &= -18,000 - 3,100(8.2014) + 2,000(0.4241) \\ &\quad - 18,000(0.4241) + 2,000(0.1799) = \$ - 49,850\end{aligned}$$

**Location B is selected.**

**b)** For a 5-year study period no cycle repeats are necessary.

$$\begin{aligned}
 PW_A &= -15,000 - 3,500(P/A, 10\%, 5) + 1,000(P/F, 10\%, 5) \\
 &= -15,000 - 3,500(3,7908) + 1,000(0,6209) \\
 &= \$ - 27,647
 \end{aligned}$$

$$\begin{aligned}
 PW_B &= -18,000 - 3,100(P/A, 10\%, 5) + 2,000(P/F, 10\%, 5) \\
 &= -18,000 - 3,100(3,7908) + 2,000(0,6209) \\
 &= \$ - 28,509.7
 \end{aligned}$$

**Location A is selected.**

**c)** For a 6-year study period, deposit return of \$6,000 for location B

$$\begin{aligned}
 PW_A &= -15,000 - 3,500(P/A, 10\%, 6) + 1,000(P/F, 10\%, 6) \\
 &= -15,000 - 3,500(4.3553) + 1,000(0,5645) \\
 &= \$ - 29,679
 \end{aligned}$$

$$\begin{aligned}
 PW_B &= -18,000 - 3,100(P/A, 10\%, 6) + 6,000(P/F, 10\%, 6) \\
 &= -18,000 - 3,100(4.3553) + 6,000(0.5645) \\
 &= \$ - 28,114
 \end{aligned}$$

**Location B is selected.**

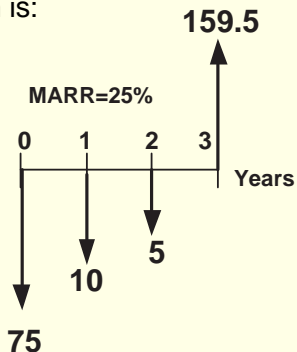
- Future Worth (*FW*) of an alternative can be found:
  - 1 Directly from the cash flows by determining the future worth value
  - 2 multiplying *PW* value by the *F/P* factor at
- MARR
- Therefore it is an extension of *PW* analysis
- Used when the prime goal is to maximize the future wealth of the company
- Also used if the asset will be sold or traded before its expected life is reached
  - *FW* value estimates the alternative's worth at the time sale or disposal
- Selection guidelines are same as *PW*
  - 1 One alternative:  $FW \geq 0$  means MARR is met
  - 2 For two or more mutually exclusive: Select the numerically largest

## Example

A British food distribution corporation purchased a Canadian food store chain for 75 million dollars three years ago. There was a net loss of \$10 million at the end of year 1 of ownership. Net cash flow is increasing with an arithmetic gradient of \$+5 million per year starting the second year. This means that breakeven net cash flow was achieved this year. Because of the heavy debt financing used to purchase the Canadian chain, the international board of directors expects a MARR of 25% per year from any sale.

- 1 The British corporation has just been offered \$159.5 million by a French company for the ownership of the Canadian chain. Use *FW* analysis to decide whether the MARR is met at this selling price or not.
- 2 If the British corporation continues to own the chain, what selling price must be obtained at the end of the year 5 to make the MARR.

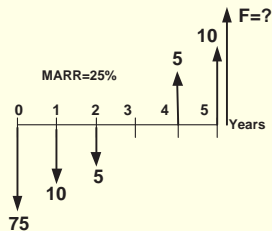
- ① Cash flow diagram is:



$$\begin{aligned}
 FW_3 &= -75(1.25)^3 - 10(1.25)^2 - 5(1.25) + 159.5 \\
 &= -8.86 < 0, \text{ MARR of 25\% not met}
 \end{aligned}$$

1 **<0, MARR of 25% not met**

2 cash flow diagram is:



$$FW_3 = -75(1.25)^5 - 10(1.25)^4 - 5(1.25)^3 + 5(1.25) + 10 + F$$

$$0 = F - 246.81$$

$$F = \$246.81 \text{ million}$$

**at least to meet MARR of 25%**

- Capitalized Cost (CC) is the **present worth** of an alternative that will last **forever**
- Public sector projects such as bridges, dams, railroads fall into this category
- CC is derived from  $P = A(P/A, i\%, n)$  as  $n \rightarrow \infty$

$$\begin{aligned} P &= A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] \\ &= A \left[ \frac{1 - \frac{1}{(1+i)^n}}{i} \right] \end{aligned}$$

- As  $n \rightarrow \infty$ , the expression inside the bracket becomes  $1/i$ , and the symbol CC replaces  $P$
- $CC = \frac{A}{i} = \frac{AW}{i}$

The cash flows in a capitalized cost calculation are usually of two types:

- 1 Recurring
- 2 Nonrecurring

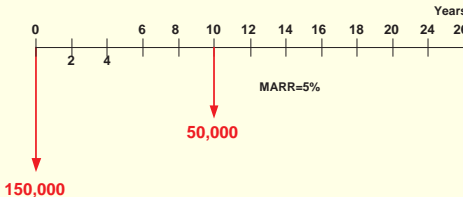
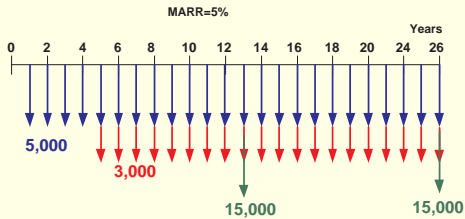
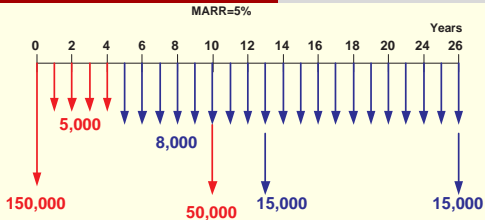
The following procedure assists in calculating the CC for an infinite sequence of cash flows.

- 1 Draw a cash flow diagram for all nonrecurring cash flows and at least two cycles of all recurring cash flows
- 2 Find the present worth of all nonrecurring amounts. This will be their CC value
- 3 Find the equivalent uniform annual worth ( $A$ ) through one life cycle of all recurring amounts. Add this to all other uniform amounts occurring in years 1 through infinity and the result is total equivalent uniform annual worth ( $AW$ )
- 4 Divide the  $AW$  obtained at step 3 by  $i$  to obtain a CC value
- 5 Add the CC values obtained in steps 2 and 4

## In Class Work 7

A new software for the purpose of monitoring property tax is installed. The installation cost is \$150,000 and there is an additional cost of \$50,000 after 10 years. The annual software contract cost is \$5,000 for the first 4 years and \$8,000 thereafter. In addition, there is expected to be a recurring major upgrade cost of \$15,000 every 13 years. Assume that MARR is 5%. If the new software will be used for the indefinite future, find the equivalent value:

- 1 now
- 2 for each year thereafter



2)

$$\begin{aligned} CC_1 &= -150,000 - 50,000\left(\frac{1}{(1.05)^{10}}\right) \\ &= \$ - 180,695 \end{aligned}$$

3)

$$\begin{aligned} A_1 &= -15,000(A/F, 5\%, 13) \\ &= -15,000(0.05646) = \$ - 847 \end{aligned}$$

4)

$$\begin{aligned} CC_2 &= \frac{-3,000}{0.05}\left(\frac{1}{1.05^4}\right) \\ &= \$ - 49,362 \end{aligned}$$

$$CC_3 = A_1 + A_2 = \frac{-5,000 - 847}{0.05} = \$ - 116,940$$

$$5) CC_T = -180,695 - 49,362 - 116,940 = \$ - 346,997$$

$$b) A = Pi = -346,997(0.05) = -17,350$$

## Example

Two alternative sites are considered for a bridge to cross a river in **New York**.

- 1 A suspension bridge will cost \$50 million with annual inspection and maintenance costs of \$35,000. In addition, the concrete deck would have to be resurfaced every 10 years at a cost of \$100,000. The cost of purchasing right-of-way is expected to be \$2 million for the suspension bridge.
- 2 A truss bridge and approach roads are expected to cost \$25 million and annual maintenance costs of \$20,000. The bridge has to be painted every 3 years at a cost of \$40,000. In addition, the bridge would have to be sandblasted every 10 years at a cost of \$190,000. The cost of purchasing right-of-way is expected to be \$15 million for the truss bridge.

Compare the alternatives on the basis of their capitalized cost if the interest rate is 6% per year.

- To compare two or more alternatives on the basis of capitalized cost, find  $CC_T$  for each alternative.
  - $CC_T$  represents the total present worth of financing and maintaining an alternative **forever**
  - Alternative with the smallest capitalized cost should be selected
- ①  $CC_1 = -50 - 2 = \$ - 52$  million

$$\begin{aligned}
 A_1 &= -100,000(A/F, 6\%, 10) \\
 &= -100,000(0.07587) = \$ - 7587 \\
 CC_2 &= \frac{A_1 + A_2}{i} = \frac{-7,587 - 35,000}{0.06} \\
 &= -709,783 \\
 CC_S &= CC_1 + CC_2 = \$ - 52,71 \text{ million}
 \end{aligned}$$

$$1 \quad CC_S = \$ - 52,71 \text{ million}$$

$$2 \quad CC_1 = -25 - 15 = \$ - 40 \text{ million}$$

$$\begin{aligned} A_1 &= -190,000(A/F, 6\%, 10) \\ &= -190,000(0.07587) = \$ - 14,415 \end{aligned}$$

$$\begin{aligned} A_2 &= -40,000(A/F, 6\%, 3) \\ &= -40,000(0.31411) = \$ - 12,564 \end{aligned}$$

$$\begin{aligned} CC_2 &= \frac{A_1 + A_2 + A_3}{i} = \frac{-14,415 - 12,564 - 20,000}{0.06} \\ &= -782,983 \end{aligned}$$

$$CC_T = CC_1 + CC_2 = \$ - 40,78 \text{ million}$$

**Truss Bridge is selected.**

In present worth analysis, we assume that all the funds in a firm's treasury can be placed in investments that yield a return equal to **MARR**. We may view these funds as an **investment pool**.

