- In previous weeks, the alternatives have been mutually exclusive; only one could be selected.
- If the projects are not mutually exclusive; they are called independent projects.
- We learned the criteria to select from several independent projects; it is possible to select any number of them.
- But in real life, there is always some upper limit on the amount of capital available for investment.
- Therefore, the Investment Capital must be distributed among the viable independent investment opportunities.
- The technique applied is called the Capital Budgeting Method; determines the economically best rationing of initial investment capital among independent projects.
- Capital Budgeting Method is an application of the *PW* method.

- The term project is used to identify each independent option.
- The term bundle is used to identify a collection of independent projects.
- A contingent project is one that has a condition placed upon its acceptance or rejection.
  1. A cannot be accepted unless B is accepted.
  2. A can be accepted instead of B, but not both.
- A dependent project is one that must be accepted or rejected based on the decisions of other projects.
  1. B must be accepted if both A and C are accepted.
  2. These complicating conditions are handled by creating packages of related projects in such a way to ensure the independency with the remaining projects.

Characteristics of Capital Budgeting Problem

1. Several independent projects are identified, and net cash flow estimates are available
2. Each project is either selected entirely or not; partial investment into a project is not available.
3. A stated budgetary constraint restricts the total amount available for investment. Budget constraint may also be present for other years. The investment limit is denoted by b.
4. The objective is to maximize the return on investments using a measure of worth, usually the PW value.

Selection Guide for Capital Budgeting

Accept projects with the best *PW* values determined at MARR over the projected life, provided the investment capital limit is not exceeded.

- Equal service assumption is not valid in capital rationing
- There is no life cycle of a project beyond its estimated life
- But there is an implied reinvestment assumption:
  - All positive net cash flow of a project are reinvested at the MARR from the time they are realized until the end of the longest-lived project.

- n projects with same expected life, don't exceed the budget limit b
- First, we need to formulate all possible mutually exclusive BUNDLES
  - Do-nothing project
  - one project at a time
  - two project at a time
  - ⋮
  - all n projects together
- Total number of mutually exclusive alternatives $=2^n$
- Then the *PW* for each bundle is determined at MARR.
- The bundle with the largest PW value is selected.

Now let's develop all possible mutually exclusive alternatives for four projects given below; when the budget limit $b = 15,000$.

Project	Initial Investment
A	$-10,000
B	-5,000
C	-8,000
D	-15,000

Bundle	Projects involved	Total Investment
1	Do nothing	0
2	A	-10,000
3	B	-5,000
4	C	-8,000
5	D	-15,000
6	AB	-15,000
7	BC	-13,000

The formal procedure to solve a capital budgeting problem using *PW* analysis:

1. Develop all mutually exclusive bundles with a total initial investment that does not exceed the capital limit b.
2. Sum the net cash flows for all projects in each bundle j for each year t from 1 to the expected project life n_j; to find NCF_{jt} . Refer to the initial investment of bundle j at time $t = 0$ as NCF_{j0}.
3. Compute the present worth value PW_j for each bundle at the MARR.

$$PW_j = \sum_{t=1}^{t=n_j} NCF_{jt}(P/F, i = MARR, t) - NCF_{j0}$$

4. Select the bundle with the numerically largest PW_j value.

In Class Work 14

Our company has $20,000 to allocate next year to new projects. Any or all of the five projects in the table below may be accepted. Each project has an expected life of 9 years. Select the projects to invest if a minimum of 15% return is desired.

Project	Initial Investment	Annual Net CF	Project Life
A	$-10,000	$2,870	9
B	-15,000	2,930	9
C	-8,000	2,680	9
D	-6,000	2,540	9
E	-21,000	9,500	9

We have $b = 20,000$ to select one bundle that will maximize the PW.

Bundle j	Projects Involved	Initial Investment NCF_{j0}	Annual Net CF NCF_j	PW_j
1	A	$-10,000	$2,870	$3,694
2	B	-15,000	2,930	-1,019
3	C	-8,000	2,680	4,788
4	D	-6,000	2,540	6,120
5	AC	-18,000	5,550	8,482
6	AD	-16,000	5,410	9,814
7	CD	-14,000	5,220	10,908
8	DN	0	0	0

$$PW_j = \sum_{t=1}^{t=9} NCF_{jt}(P/F, i, t) - NCF_{j0}$$
$$= NCF_j(P/A, 15\%, 9) - NCF_{j0}$$

Invest $14,000 into projects C and D.

- Usually the projects do not have the same expected life
- *PW* method for capital budgeting problem evaluates each project over the period of the longest-lived project, n_L.
- All positive net cash flow of a project are assumed to be reinvested at the MARR from the time they are realized until the end of the longest-lived project (from year n_j through year n_L)
- Therefore, the use of LCM of lives or longest-lived life, n_L is not necessary, each project's *PW* is calculated over its own life n_j

In Class Work 15

For $MARR = 15\%$ per year and $b = \$20,000$, select from the following independent projects.

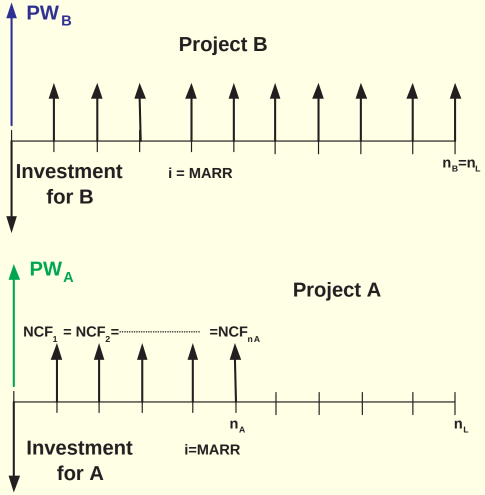
Project	Initial Investment	Annual Net CF	Project Life
A	$-8,000	$3,870	6
B	-15,000	2,930	9
C	-8,000	2,680	5
D	-8,000	2,540	4

We have $b = 20,000$ to select one bundle that will maximize the PW.

Bundle j	Projects Involved	Initial Investment NCF_{j0}	Net Cash Flows Year t	Net Cash Flows NCF_{jt}	PW_j
1	A	$-8,000	1-6	$3,870	$6,646
2	B	-15,000	1-9	2,930	-1,019
3	C	-8,000	1-5	2,680	984
4	D	-8,000	1-4	2,540	-748
5	AC	-16,000	1-5	6,550	7,630
			6	3,870	
6	AD	-16,000	1-4	6,410	5,898
			5-6	3,870	
7	CD	-16,000	1-4	5,220	235
			5	2,680	
8	DN	0		0	0

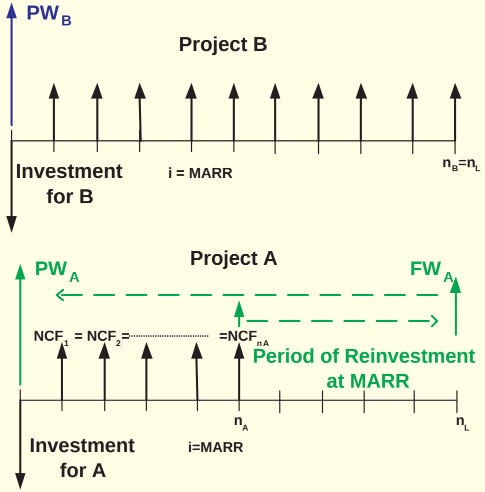
Invest $16,000 into projects A and C.

Now, we can verify that all positive net cash flow of a project are reinvested at the MARR from the time they are realized until the end of the longest-lived project (from year n_j through year n_L).



$$PW_{Bundle} = PW_A + PW_B$$

Now, we can verify that all positive net cash flow of a project are reinvested at the MARR from the time they are realized until the end of the longest-lived project (from year n_j through year n_L).



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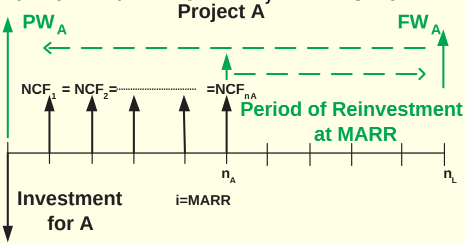

$$
\begin{aligned}
PW_j &= NCF_j(F/A, MARR, n_j)(F/P, MARR, n_L - n_j)(P/F, MARR, n_L) \\
&= NCF_j \frac{(1+i)^{n_j} - 1}{i}(1+i)^{n_L - n_j} \frac{1}{(1+i)^{n_L}} \\
&= NCF_j \frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}} \\
&= NCF_j(P/A, MARR, n_j)
\end{aligned}
$$

- Can be formulated using integer programming
- **Maximize:** Sum of PW of net cash flows of independent projects
- **Constraints:**
  - Sum of initial investments can not exceed budget limit b
  - Each project is completely selected or not
- **Decision Variable:** x_k: project k is selected or not
- k represent each independent project not bundle.

$$\begin{aligned} &\textit{Maximize} && \sum_{k=1}^{n} PW_k x_k \\ &\textit{Subject to} && \\ & && \sum_{k=1}^{n} NCF_{k0} x_k \leq b \\ & && x_k = 0 \textit{ or } 1 \textit{ for } k = 1, 2, \ldots, n \end{aligned}$$

Now, the IP formulation for the In Class Work 15;
$k = 4$, and $b = 20,000$.

$$\begin{aligned} \textit{Maximize} \quad & 6,646x_1 - 1,019x_2 + 984x_3 - 748x_4 \\ \textit{Subject to} \quad & \\ & 8,000x_1 + 15,000x_2 + 8,000x_3 + 8,000x_4 \leq 20,000 \\ & x_1, x_2, x_3 \textit{ and } x_4 = 0 \textit{ or } 1 \end{aligned}$$

Solution: $x_1 = x_3 = 1$ and $x_2 = x_4 = 0$ with objective value of 7,630.

Additional Constraints

1. If projects $a, b, \ldots, j$ in $k = 1, \ldots, n$ are mutually exclusive
   - $x_a + x_b + \cdots + x_j \leq 1$
2. Project c can only be accepted if project d is accepted
   - $x_c - x_d \leq 0$
3. If projects e and f are strict compliments
   - $x_e - x_f = 0$