Some basic tax terms and relations useful in engineering economy studies are explained here.

- Gross Income, GI: is the total income realized from:
  - revenue-producing sources, and
  - sale of assets, royalties, license fees.
- Income tax: is the amount of taxes that must be delivered to the government.
  - Taxes are actual cash flows.
- Operating Expenses, E: are all corporate costs incurred in the transaction of business.
  - These expenses are tax deductible for corporations.
  - Annual Operating Costs, Maintenance and Operations Costs are examples.
- Taxable Income, TI: is the amount upon which taxes are based. For corporations, depreciation is tax deductible.

$$\begin{aligned} TI &= \textit{Gross Income} - \textit{Expenses} - \textit{Depreciation} \\ &= GI - E - D. \end{aligned}$$

- Tax Rate, T: is a percentage of TI owed in taxes.

$$\begin{aligned} Taxes &= (Taxable\ Income)x(applicable\ tax\ rate) \\ &= (TI)(T). \end{aligned}$$

- Net Profit After Taxes, $NPAT$: is the amount remaining each year when income taxes are subtracted from taxable income.

$$\begin{aligned} NPAT &= Taxable\ Income - Taxes \\ &= TI - (TI)(T) \\ &= (TI)(1 - T). \end{aligned}$$

- There are many other tax bases for taxes other than income:
  - Sales Tax,
  - Value-added Tax,
  - Import Tax,
  - Property Tax.

- Net Cash Flow, *NCF*: was defined as cash inflows minus cash outflows for a period.
- The annual *NCF* was used in *PW*, *FW*, *AW* and *ROR* methods.
- Now we are expanding our terminology in the presence of taxes and depreciation.
- *NCF* is replaced by the term Cash Flow Before Taxes, *CFBT*.
- We will introduce the term Cash Flow After Taxes, *CFAT*.
- Both *CFBT* and *CFAT* are actual cash flows.

Analysis with *CFAT*

Once *CFAT* estimates are developed, the economic evaluation is performed using the same periods and selection guidelines applied previously, only this time, *CFAT* estimates are utilized.

$CFBT$ must include the initial capital investment and salvage value for the years in which they occur.

$$\begin{aligned} CFBT &= Gross\ Income - Expenses - Initial\ Investment + Salvage \\ &= GI - E - P + S. \end{aligned}$$

- $CFAT = CFBT - taxes$
- $CFAT = GI - E - P + S - (GI - E - D)(T)$
- Depreciation is not an actual cash flow, therefore;
  - depreciation is not a negative cash flow in $CFAT$,
  - Only used in the calculation of taxes.
- TI value can be negative, and the associated negative income tax is considered as a tax savings for the year.
  - Assumption: Negative tax will offset taxes for the same year in other income producing areas of corporation

In Class Work 20

We are planning to build 35 facilities throughout the nation. Each facility is expected to cost \$550,000 initially with a salvage value of \$150,000 after 5 years. Straight-line depreciation will be utilized with a recovery period of 5 years. We estimate that each facility will bring \$200,000 in revenue and \$90,000 in costs. Using a tax rate of 35%, tabulate the *CFBT* and *CFAT* estimates for the following 5 years for a facility.

$$\begin{aligned} CFBT &= Gross\ Income - Expenses - Initial\ Investment + Salvage \\ &= GI - E - P + S. \end{aligned}$$

Year	GI	E	P and S	CFBT
0			$550,000	$-550,000
1	$200,000	$90,000		$110,000
2	$200,000	$90,000		$110,000
3	$200,000	$90,000		$110,000
4	$200,000	$90,000		$110,000
5	$200,000	$90,000	150,000	$260,000

$$CFAT = GI - E - P + S - (GI - E - D)(T)$$

$$D_t = \frac{B - S}{n} = \frac{550,000 - 150,000}{5} = \$80,000$$

Year	GI	E	P and S	D	TI	Taxes	CFAT
0			$550,000				$-550,000
1	$200,000	$90,000		80,000	$30,000	$10,500	$99,500
2	$200,000	$90,000		80,000	$30,000	$10,500	$99,500
3	$200,000	$90,000		80,000	$30,000	$10,500	$99,500
4	$200,000	$90,000		80,000	$30,000	$10,500	$99,500
5	$200,000	$90,000	150,000	80,000	$30,000	$10,500	$249,500

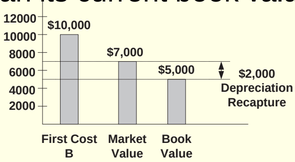
Next, we will see the tax applications when the facility is sold for a value different than its salvage value.

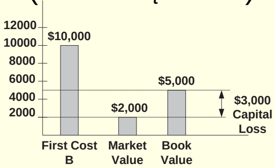
$$TI = GI - E - D + DR + CG - CL, \text{ where}$$

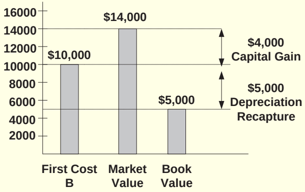
- DR: Depreciation recapture
- CG: Capital Gain
- CL: Capital Loss

When an asset is disposed, due to tax obligations we must check if we have the following gains or loses:

1. Depreciation Recapture, *DR*: When the asset is sold for more than the current book value ($DR = MV - BV_t$).
2. Capital Gain, *CG*: When the asset is sold for more than its first cost ($CG = MV - B$).
3. Capital Loss, *CL*: When the depreciable asset is disposed of for less than its current book value ($CL = BV_t - MV$).







- When a currently installed asset (the defender) is challenged with possible replacement, the effect of taxes can have an impact on the decision to replace or not.
- Before-tax and after-tax *AW* values may significantly differ.
- There may be tax considerations in the year of possible replacement due to Depreciation Recapture, Capital Gain, and Capital Loss.
- Same replacement study technique with *CFAT* estimates.

In Class Work 21

A power company purchased coal extraction equipment 3 years ago for $600,000. Management has discovered that it is technologically outdated now. New equipment has been identified. If the market value of $400,000 is offered as trade-in for the current equipment, perform a replacement study using

1. a before-tax MARR of 10% per year, and
2. a 7% per year after-tax MARR.

Assume an tax rate of 34%. Use straight line depreciation with $S = 0$ for both alternatives

	Defender	Challenger
Market Value,$	400,000	
First Cost, $		1,000,000
Annual Cost, $	-100,000	-15,000
Recovery Period, years	8 (originally)	5

1. When taxes are not considered:

$$CFBT = GI - E - P + S.$$

DEFENDER

Age	Year	E	P and S	CFBT
3	0		$400,000	$-400,000
4	1	$-100,000		$-100,000
5	2	$-100,000		$-100,000
6	3	$-100,000		$-100,000
7	4	$-100,000		$-100,000
8	5	$-100,000	0	$-100,000

$$AW_D = -100,000 - 400,000(A/P, 10\%, 5) = \$ - 205,520.$$

1. When taxes are not considered:

$$CFBT = GI - E - P + S.$$

Challenger

Age	Year	E	P and S	CFBT
	0		$1,000,000	$-1,000,000
	1	$-15,000		$-15,000
	2	$-15,000		$-15,000
	3	$-15,000		$-15,000
	4	$-15,000		$-15,000
	5	$-15,000	0	$-15,000

$$AW_C = -15,000 - 1,000,000(A/P, 10\%, 5) = \$ - 278,800.$$

Retain the defender for 5 more years.

1. When taxes are not considered, defender is retained.
2. When taxes are considered:

$$CFAT = GI - E - P + S - (GI - E - D)(T)$$
$$D_t = \frac{B-S}{n} = \frac{600,000-0}{8} = \$75,000$$

DEFENDER

Age	Year	E	P and S	D	TI	Taxes	CFAT
3	0		$400,000				$-400,000
4	1	$100,000		75,000	$-175,000	$-59,500	$-40,500
5	2	$100,000		75,000	$-175,000	$-59,500	$-40,500
6	3	$100,000		75,000	$-175,000	$-59,500	$-40,500
7	4	$100,000		75,000	$-175,000	$-59,500	$-40,500
8	5	$100,000		75,000	$-175,000	$-59,500	$-40,500

$$AW_D = -40,500 - 400,000(A/P, 7\%, 5) = \$ - 138,056.$$

1. When taxes are not considered, defender is retained.
2. When taxes are considered:

$$
\begin{aligned}
CFAT &= GI - E - P + S - (GI - E - D)(T) \\
D_t &= \frac{B - S}{n} = \frac{1,000,000 - 0}{5} = \$200,000 \\
BV_3 &= \$600,000 - 3(75,000) = 375,000 \\
DR_3 &= 400,000 - 375,000 = 25,000
\end{aligned}
$$

CHALLENGER

Age	Year	E	P and S	D	TI	Taxes	CFAT
	0		$1,000,000		$25,000	$8,500	$-1,008,500
	1	$15,000		200,000	$-215,000	$-73,100	$58,100
	2	$15,000		200,000	$-215,000	$-73,100	$58,100
	3	$15,000		200,000	$-215,000	$-73,100	$58,100
	4	$15,000		200,000	$-215,000	$-73,100	$58,100
	5	$15,000	0	200,000	$-215,000	$-73,100	$58,100

$$AW_C = 58,100 - 1,008,500(A/P, 7\%, 5) = \$ - 187,863.$$

Retain the defender for 5 more years.

- The required after-tax MARR is established using:
  - the market interest rate,
  - tax rate,
  - average cost of capital.
- CFAT estimates are used to calculate *PW*, *AW* at after-tax MARR.
- Selection guidelines are same as before:
  1. One Project: *PW* or *AW* ≥ 0, the project is financially viable, since after-tax MARR is met or exceeded.
  2. Two or more alternatives: Select the alternative with the best *PW* or *AW* value.
- Equal Service Life assumption still needed!

Example

We have two construction options - stucco on metal lath (S) and bricks (B) - each have about the same noise transmission loss. We have estimated the first costs and the after-tax savings each year for both designs. Use *CFAT* values and after-tax MARR of 7% per year to determine which is economically better.

Plan S		Plan B	
Year	CFAT	Year	CFAT
0	$-28,800	0	$-50,000
1-6	5,400	1	14,200
7-10	2,040	2	13,300
10	2,792	3	12,400
		4	11,500
		5	10,600

First develop *AW* for alternatives using CFAT at after-tax MARR:

$$
\begin{aligned}
AW_S &= \big[-28,800 + 5,400(P/A,7\%,6) \\
&+ 2,040(P/A,7\%,4)(P/F,7\%,6) \\
&+ 2,792(P/F,7\%,10)\big](A/P,7\%,10) = \$422 \\
AW_B &= \big[-50,000 + 14,200(P/F,7\%,1) + \ldots \\
&+ 10,600(P/F,7\%,5)\big](A/P,7\%,5) = \$327
\end{aligned}
$$

Both plans are financially viable, select plan S.

$$
\begin{aligned}
PW_S &= -28,800 + 5,400(P/A,7\%,6) \\
&+ 2,040(P/A,7\%,4)(P/F,7\%,6) \\
&+ 2,792(P/F,7\%,10) = \$2,964 \\
PW_B &= -50,000 + 14,200(P/F,7\%,1) + \ldots \\
&+ 10,600(P/F,7\%,10) = \$2,297
\end{aligned}
$$

Both plans are financially viable, select plan S (LCM is 10 years).

- Same ROR methods as before.
- A *PW* or *AW* relation is developed and set to 0 to solve for i^*:

$$0 = \sum_{t=1}^{t=n} CFAT_t(P/F, i^*, t)$$

$$0 = \sum_{t=1}^{t=n} CFAT_t(P/F, i^*, t)(A/P, i^*, n)$$

- There is an approximating relation:

$$Before - tax\ ROR = \frac{after - tax\ ROR}{1 - T}$$

- If the decision concerns the economic viability of a project, and the resulting ROR from the approximating relation is close to after-tax MARR, then a detailed after-tax analysis should be performed.

Example

A fiber optics manufacturing company operating in Hong Kong has spent $50,000 for a 5-year-life machine that has a projected $20,000 annual CFBT and annual depreciation of $10,000. The company has a T of 40%.

1. Determine the after-tax rate of return,
2. Approximate the before tax return.

1. $CFAT_0 = -50,000$. For years 1 through 5:

$$\begin{aligned} CFAT &= GI - E - P + S - (GI - E - D)(T) \\ &= CFBT - (CFBT - D)(T) \\ &= 20,000 - (20,000 - 10,000)(0.4) = \$16,000 \\ 0 &= -50,000 + 16,000(P/A, i^*, 5) \rightarrow i^* = 18.03\% \end{aligned}$$

1. after-tax rate of return:

$$0 = -50,000 + 16,000(P/A, i^*, 5) \rightarrow i^* = 18.03\%$$

2. Using the approximating relation:

$$\begin{aligned} Before - tax\ ROR &= \frac{after - tax\ ROR}{1 - T} \\ &= \frac{0.1803}{1 - 0.4} = 30.05\% \end{aligned}$$

The actual i^* using the ROR equation:

$$0 = -50,000 + 20,000(P/A, i^*, 5) \rightarrow i^* = 28,65\%$$

An after-tax rate of return evaluation performed on two or more alternatives:

- must utilize *PW* or *AW* relation to determine incremental i^* of the incremental CFAT series between two alternatives.
- Selection Guideline:
  - Select the alternative that requires largest initial investment, provided the extra investment is justified ($\Delta i^* >$ after-tax MARR).
- Equal Service Assumption is required.
  - Use LCM of lives when *PW* relation is used for incremental CFAT series.
  - *AW* analysis is performed on actual CFAT series for one life cycle.

Example

There are two options, and we want to implement one of these systems. Expected after-tax MARR is 20%.

Years	0	1	2	3	4
System 1 CFAT, $	-100,000	-35,000	-30,000	-20,000	-15,000
System 2 CFAT, $	-130,000	-20,000	-20,000	-10,000	-5,000

Years	0	1	2	3	4
Incremental CFAT (2-1), $1,000	-30	15	10	10	10

$$0 = -30 + 15(P/F, \Delta i^*, 1) + 10(P/A, \Delta i^*, 3)(P/F, \Delta i^*, 1)$$
$$\Delta i^* = 20.10\%$$

Select higher investment, system 2 since $\Delta i^* = 20.10\% > 20\% =$ after-tax MARR .