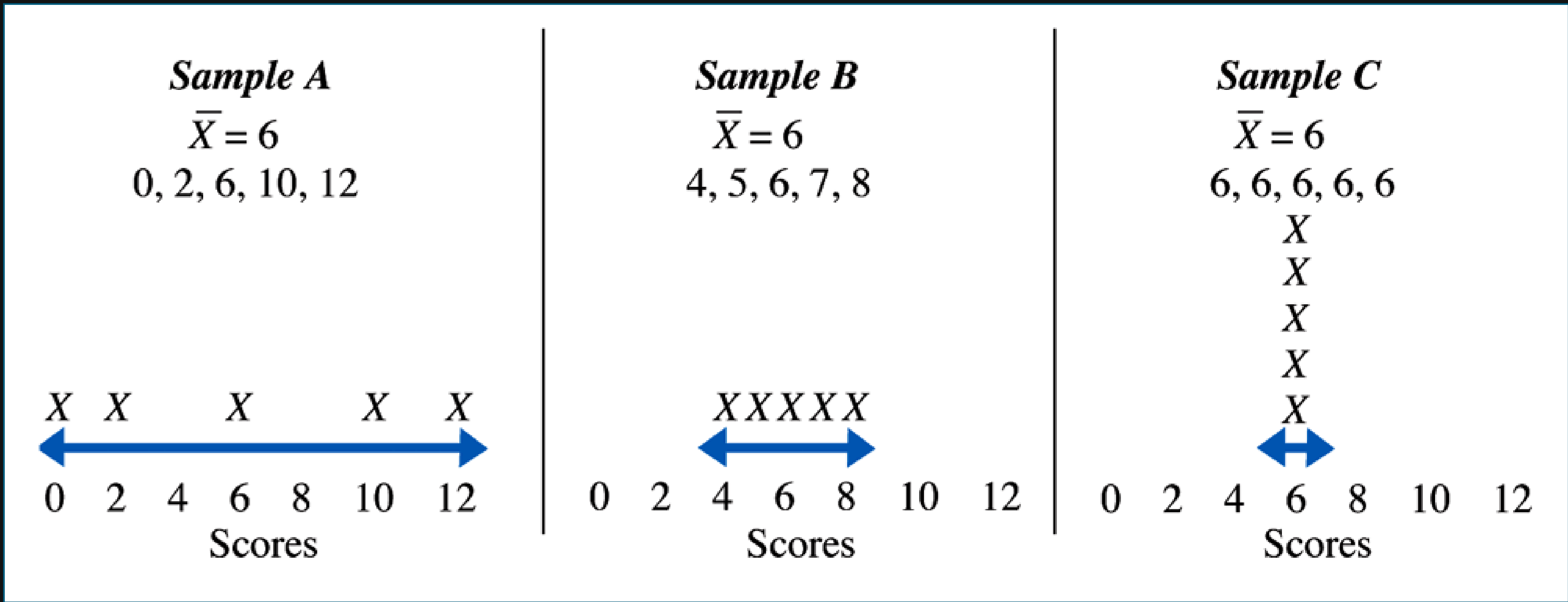
VARIABILITY:
Range
Variance
Standard Deviation

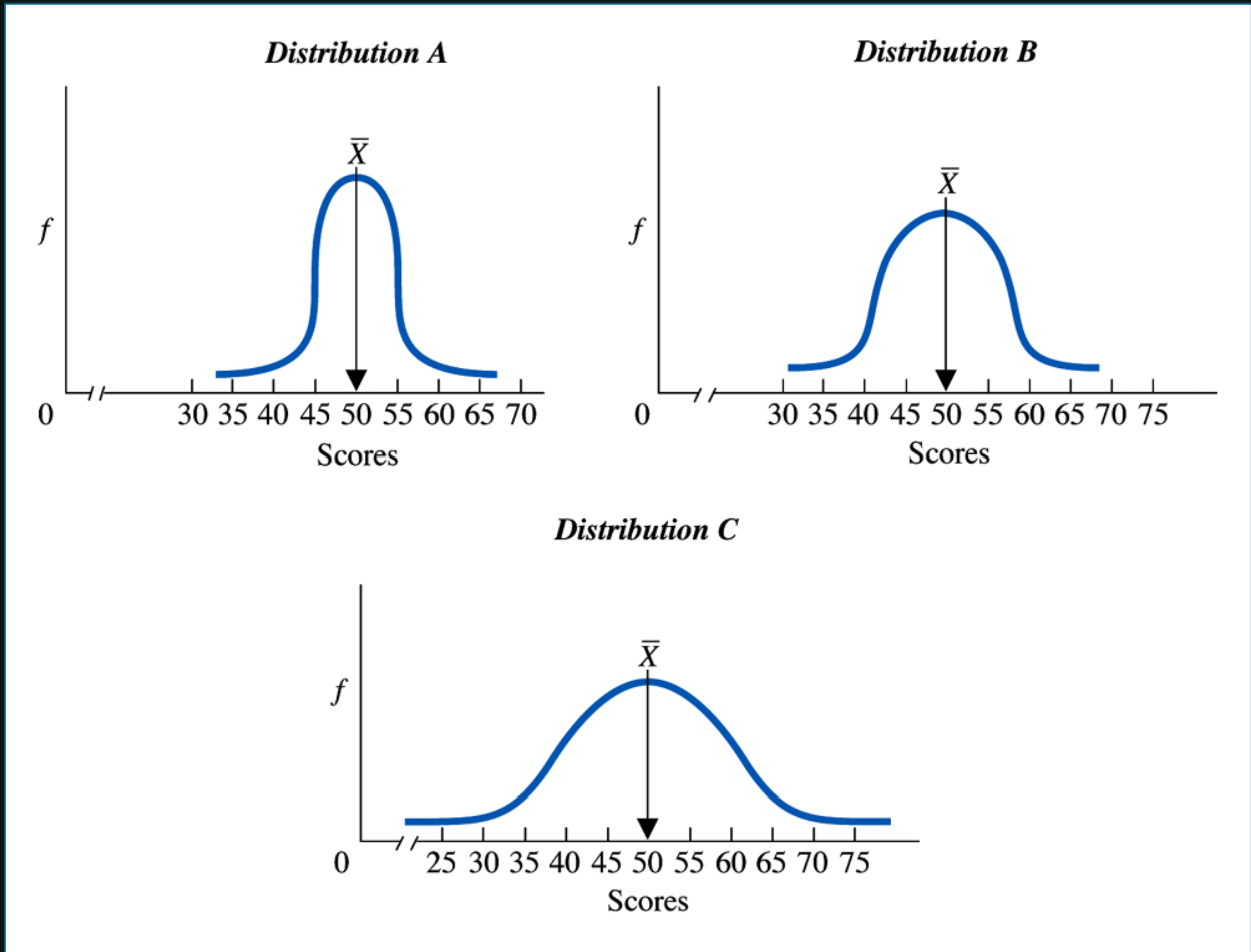
Measures of Variability

Describe the extent to which scores in a distribution *differ* from each other.

Distance Between the Locations of Scores in Three Distributions



Three Variations of the Normal Curve



The Range, Variance, and Standard Deviation

The Range

- The **range** indicates the distance between the two most extreme scores in a distribution

Range = highest score – lowest score

Variance and Standard Deviation

- The *variance* and *standard deviation* are two measures of variability that indicate how much the scores are spread out around the mean
- We use the mean as our reference point since it is at the center of the distribution

The Sample Variance and the Sample Standard Deviation

Sample Variance

- The ***sample variance*** is the average of the squared deviations of scores around the sample mean
- Definitional formula

$$S_X^2 = \frac{\Sigma(X - \overline{X})^2}{N}$$

Sample Variance

- Computational formula

$$S_X^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N}$$

Sample Standard Deviation

- The ***sample standard deviation*** is the square root of the sample variance
- Definitional formula

$$s_X = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}}$$

Sample Standard Deviation

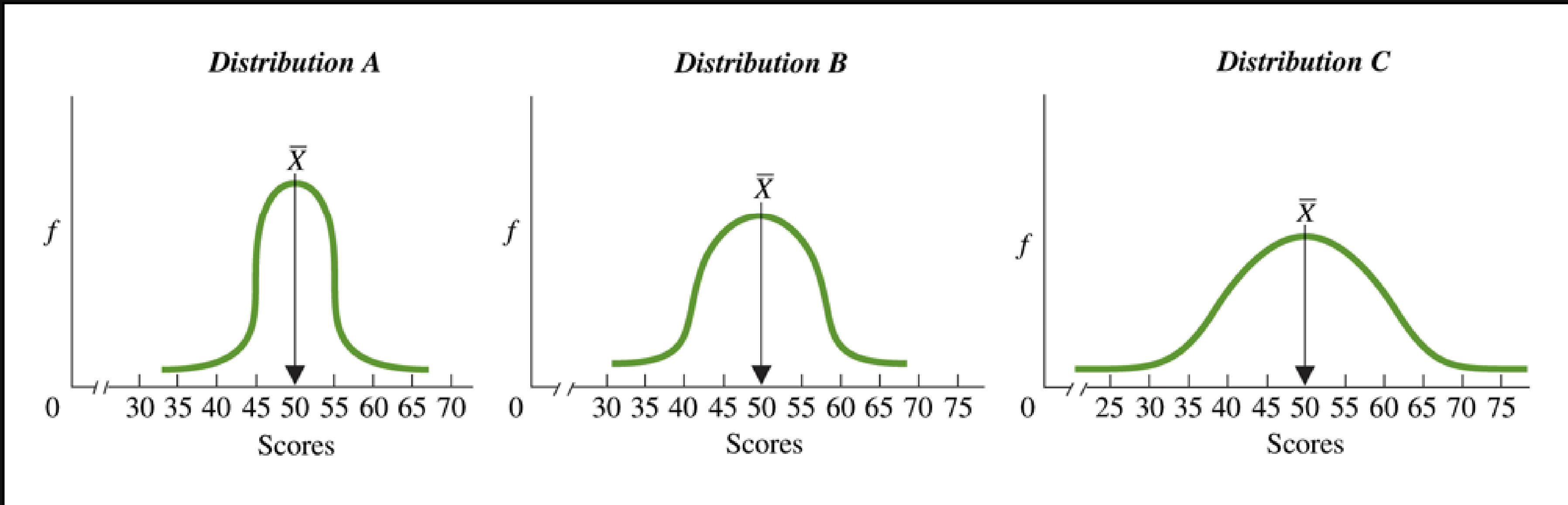
- Computational formula

$$S_X = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N}}$$

The Standard Deviation

- The *standard deviation* indicates the "average deviation" from the mean, the consistency in the scores, and how far scores are spread out around the mean

Normal Distribution and the Standard Deviation



Normal Distribution and the Standard Deviation

Approximately 34% of the scores in a perfect normal distribution are between the mean and the score that is one standard deviation from the mean.

Standard Deviation and Range

For any roughly normal distribution, the standard deviation should equal about one-sixth of the range.

The Population Variance and the Population Standard Deviation

Population Variance

- The ***population variance*** is the true or actual variance of the population of scores.

$$\sigma_X^2 = \frac{\Sigma(X - \mu)^2}{N}$$

Population Standard Deviation

- The ***population standard deviation*** is the true or actual standard deviation of the population of scores.

$$\sigma_X = \sqrt{\frac{\Sigma(X - \mu)^2}{N}}$$

The Estimated Population Variance and The Estimated Population Standard Deviation

Estimating the Population Variance and Standard Deviation

- The sample variance (S_X^2) is a **biased estimator** of the population variance.
- The sample standard deviation (S_X) is a **biased estimator** of the population standard deviation.

Estimated Population Variance

- By dividing the numerator of the sample variance by *N* - 1, we have an unbiased estimator of the population variance.
- Definitional formula

$$s_X^2 = \frac{\Sigma(X - \overline{X})^2}{N - 1}$$

Estimated Population Variance

- Computational formula

$$s_X^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N-1}$$

Estimated Population Standard Deviation

- By dividing the numerator of the sample standard deviation by N - 1, we have an unbiased estimator of the population standard deviation.
- Definitional formula

$$s_X = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N - 1}}$$

Estimated Population Standard Deviation

- Computational formula

$$s_X = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N-1}}$$

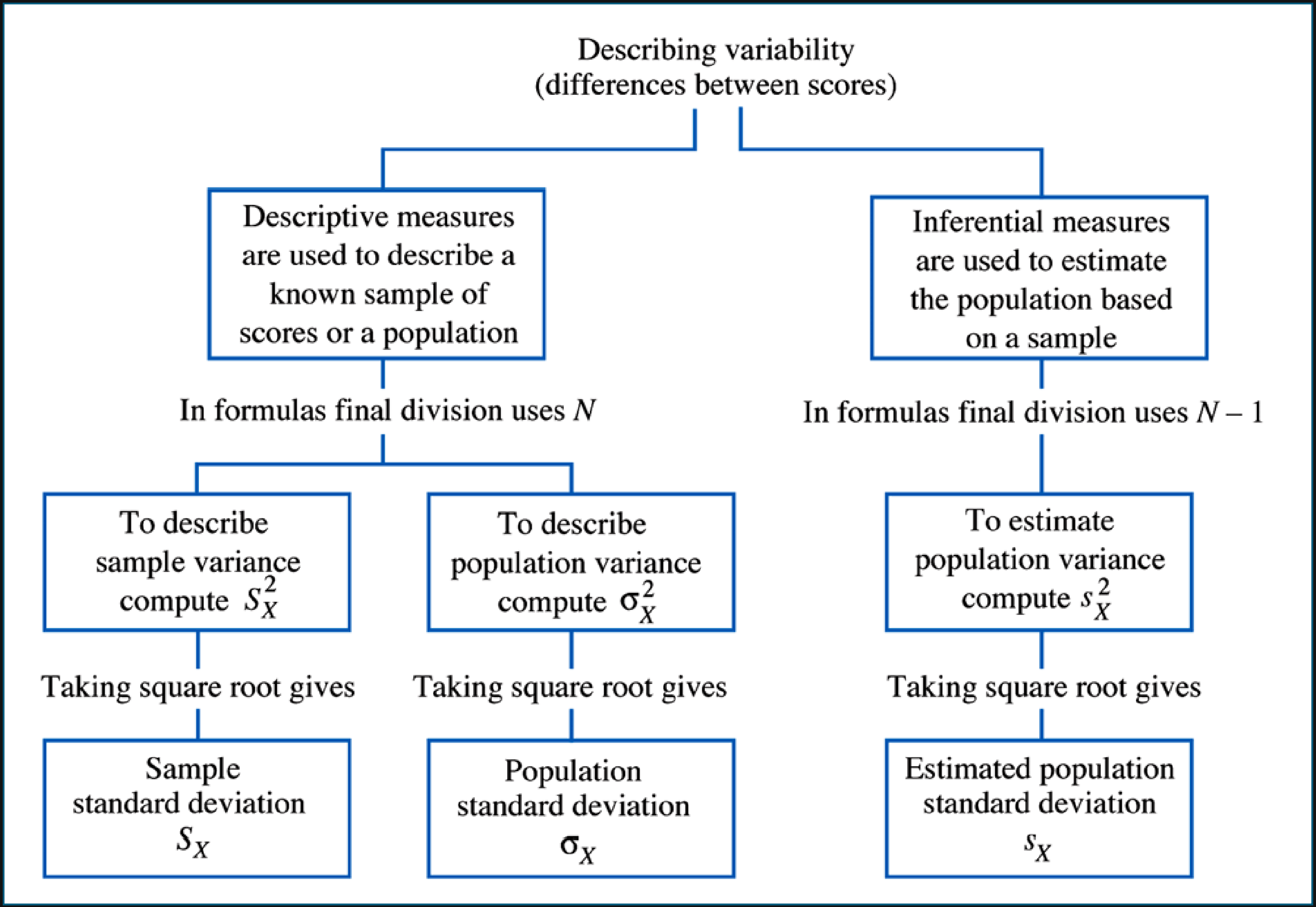
Unbiased Estimators

- s_X^2 is an unbiased estimator of σ^2
- s_X is an unbiased estimator of σ
- The quantity N - 1 is called the degrees of freedom

Uses of S_X^2, S_X, s_X^2, and s_X

- Use the sample variance S_X^2 and the sample standard deviation S_X to describe the variability of a sample.
- Use the estimated population variance s_X^2 and the estimated population s_X standard deviation for inferential purposes when you need to estimate the variability in the population.

Organizational Chart of Descriptive and Inferential



Proportion of Variance Accounted For

The *proportion of variance accounted for* is the proportion of error in our predictions when we use the overall mean to predict scores that is eliminated when we use the relationship with another variable to predict scores

Example 1

- Using the following data set, find
  - The range,
  - The sample variance and standard deviation,
  - The estimated population variance and standard deviation

14	14	13	15	11	15
13	10	12	13	14	13
14	15	17	14	14	15

Example 2

- For the following sample data, compute the range, variance and standard deviation

8 8 10 7 9 6 11 9 10 7

11 11 7 9 11 10 11 8 10 7

Example 3

- For the data set below, calculate the mean, deviation, sum of squares, variance and standard deviation by creating a table.
- 15 12 13 15 16 17 13 16 11 18

Example 4

- For the data set below, calculate the mean, deviation, sum of squares, variance and standard deviation by creating a table.
- 1 3 2 2 2 4 3 3 4 1

Example 5

- For the data set below, calculate the mean, deviation, sum of squares, variance and standard deviation by creating a table.
- 1 3 30 12 15 20 5 13 2 4


Viewer
THE END