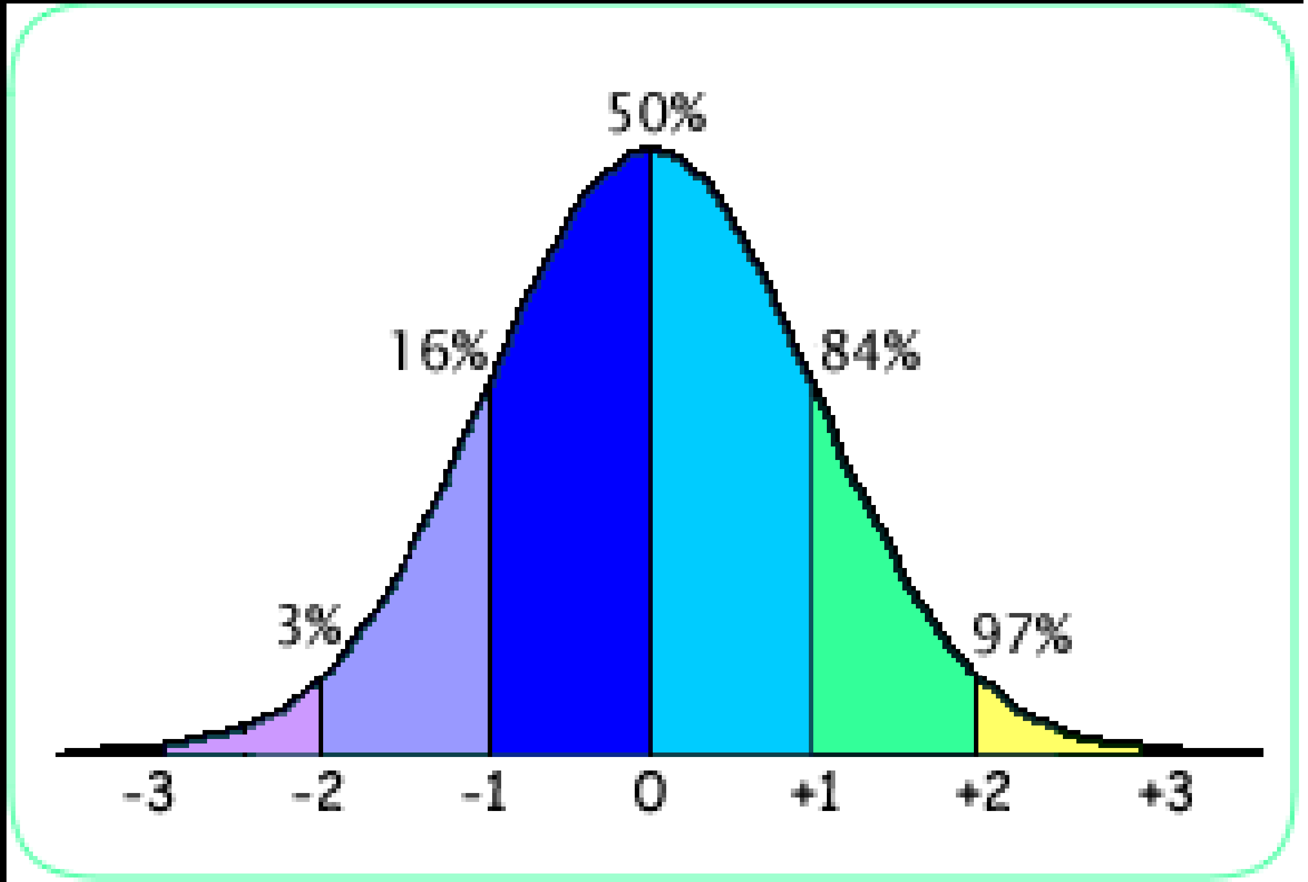
z-SCORES AND THE NORMAL CURVE MODEL



Understanding z-Scores

z-Scores

- A ***z*-score** is a *location* on the distribution. A *z*-score also automatically communicates the raw score's distance from the mean
- A *z*-score describes a raw score's location in terms of how far above or below the mean it is when measured in standard deviations

z-Score Formula

- The formula for computing a *z*-score for a raw score in a **sample** is:

$$Z = \frac{X - \bar{X}}{S}$$

z-Score Formula

- The formula for computing a *z*-score for a raw score in a **population** is:

$$z = \frac{X - \mu}{\sigma_X}$$

Example 1

- In a sample with a sample mean of 25 and a standard deviation of 5, calculate the z-score for the raw score 32.

Example 2

- In a sample with a sample mean of 50 and a standard deviation of 10, calculate the z-score for the raw score 44.

Example 3

- In a population with a mean of 100 and a standard deviation of 16, calculate the z-score for the raw score 132.

Computing a Raw Score

- When a *z*-score is known, this information can be used to calculate the original raw score. The formula for this is

$$X = (z)(S_x) + X^{bar}$$

Computing a Raw Score

- When a *z*-score is known, this information can be used to calculate the original raw score. The formula for this is

$$X = (z)(\sigma_X) + \mu$$

Example 4

- In a sample with a sample mean of 25 and a standard deviation of 5, calculate the raw score for $z = -0.43$

Example 5

- In a sample with a sample mean of 50 and a standard deviation of 10, calculate the raw score for $z = -1.30$

Example 6

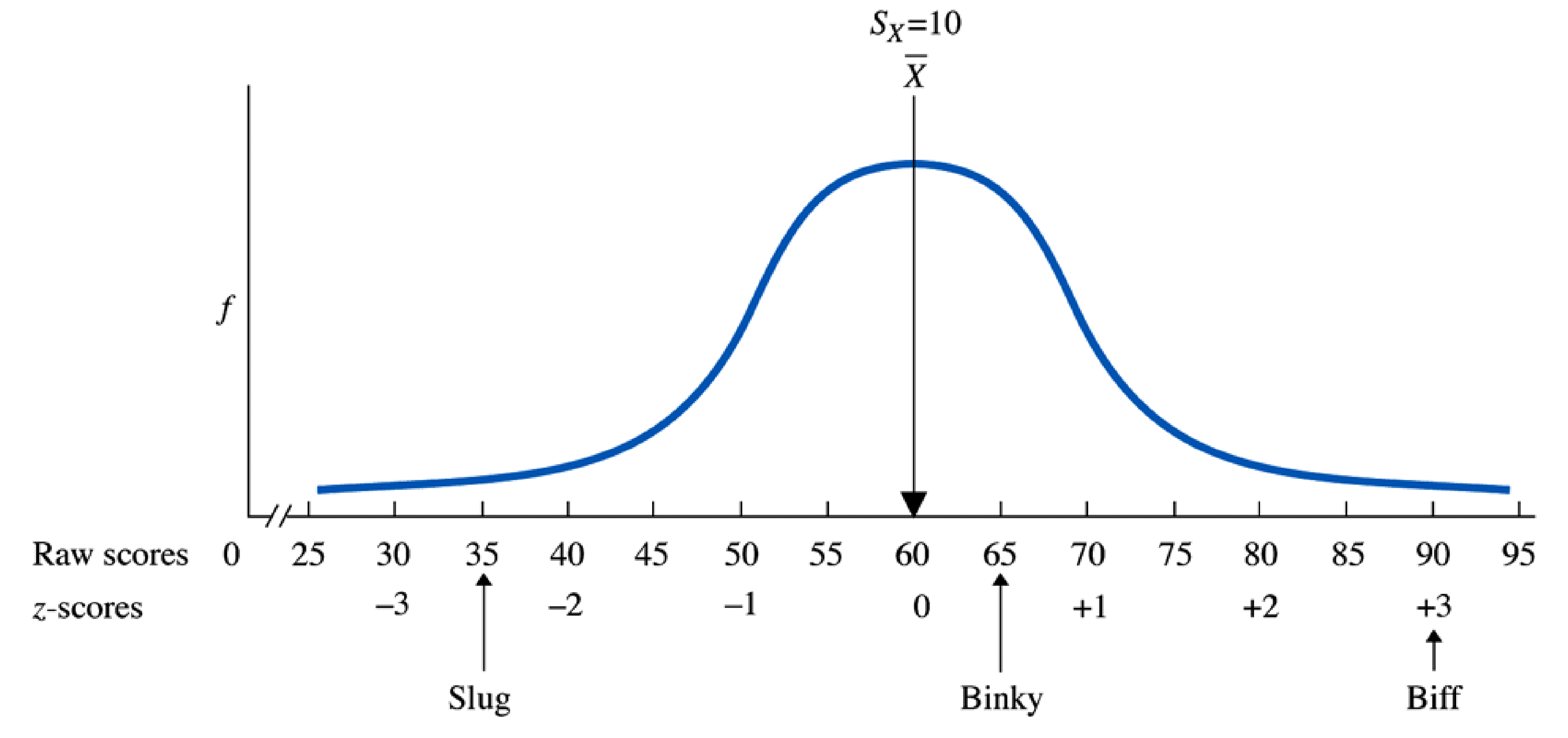
- In a population with a mean of 100 and a standard deviation of 16, calculate the raw score for $z = +1.40$

Interpreting z-Scores Using the z-Distribution

A *z*-Distribution

A *z*-distribution is the distribution produced by transforming all raw scores in the data into *z*-scores.

z-Distribution of Attractiveness Scores



Characteristics of the *z*-Distribution

1. A *z*-distribution always has the same shape as the raw score distribution
2. The mean of any *z*-distribution always equals 0
3. The standard deviation of any *z*-distribution always equals 1

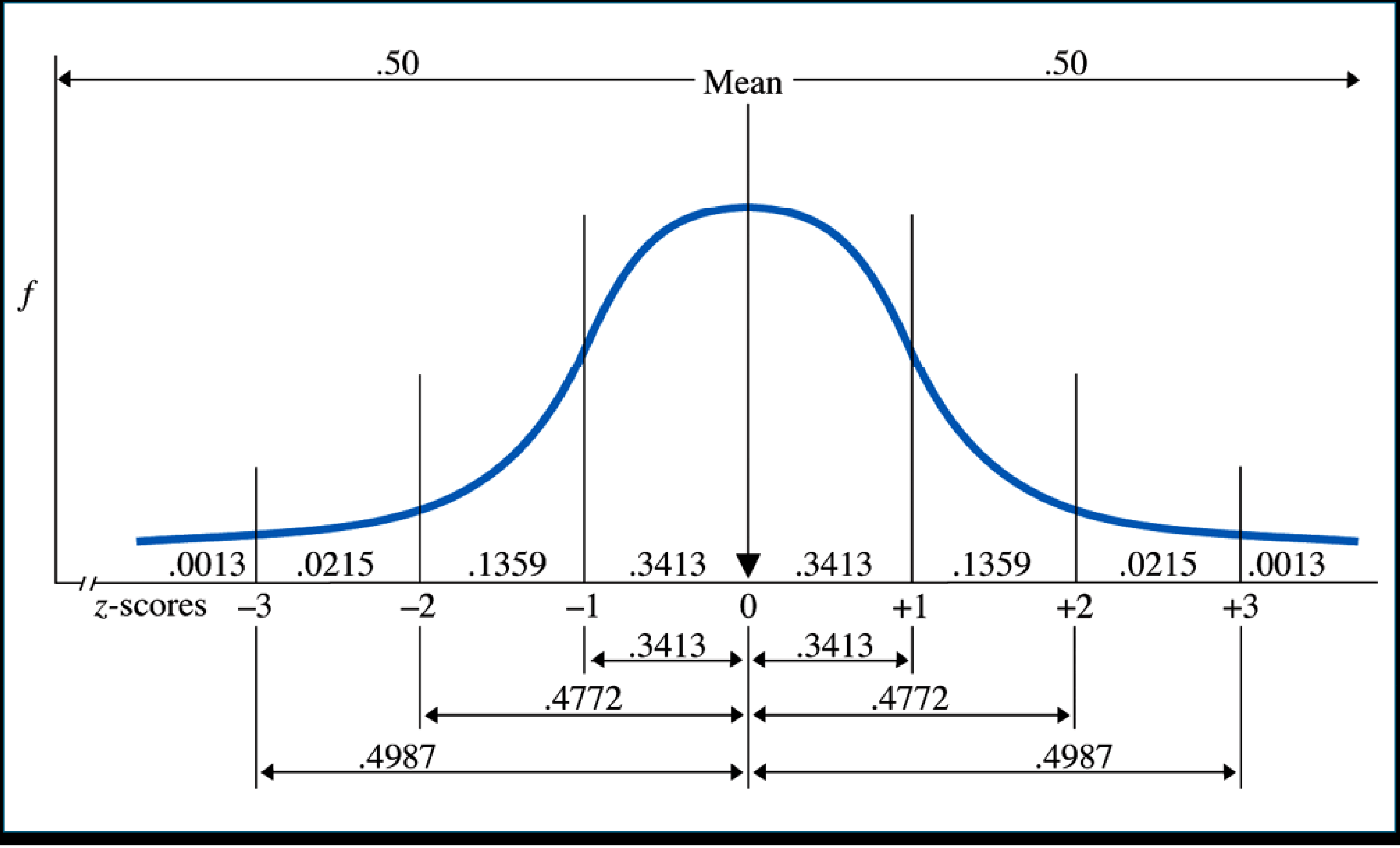
Relative Frequency

- Relative frequency can be computed using the proportion of the total area under the curve.
- The relative frequency of a particular *z*-score will be the same on all normal *z*-distributions.

The Standard Normal Curve

The **standard normal curve** is a perfect normal *z*-distribution that serves as our model of the *z*-distribution that would result from any approximately normal raw score distribution

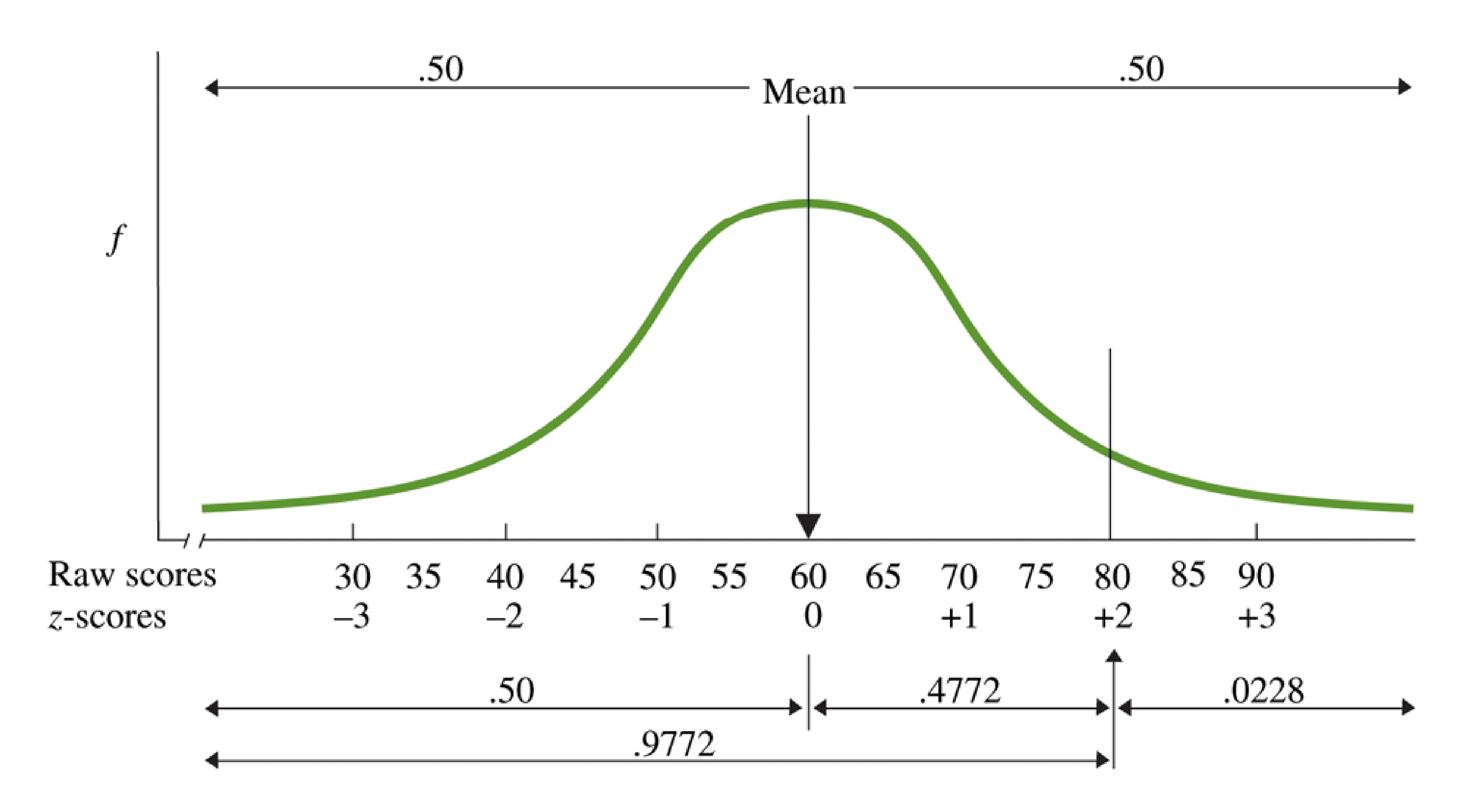
Standard Normal Curve



Uses of the Standard Normal Curve

- Calculate relative frequency of a score
- Calculate simple frequency of a score
- Calculate percentile of a score
- Calculate a raw score at a certain percentile

Proportions of the Standard Normal Curve



Example 7

- In a sample with a sample mean of 40 and a standard deviation of 4, find the relative frequency of the scores above 45.

Example 8

- In a sample with a sample mean of 40 and a standard deviation of 4, find the percentile of the score 41.5

Example 9

- In a sample with a sample mean of 65 and a standard deviation of 12, and sample size of 1000,
  - What is the relative frequency of scores below 59?
  - How many scores are between the mean and 70?
  - Which raw score signifies the top 3%?

Using z-Scores to Describe Sample Means

Sampling Distribution of Means

A distribution which shows all possible sample means that occur when an infinite number of samples of the same size *N* are randomly selected from one raw score population.

Central Limit Theorem

The **central limit theorem** tells us the sampling distribution of means

1. forms an approximately normal distribution,
2. has a μ equal to the μ of the underlying raw score population, and
3. has a standard deviation that is mathematically related to the standard deviation of the raw score population.

Standard Error of the Mean

The standard deviation of the sampling distribution of means is called the **standard error of the mean.** The formula for the true standard error of the mean is

$$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{N}}$$

z-Score Formula for a Sample Mean

The formula for computing a z-score for a sample mean is

$$z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}}$$

Example 10

If $\overline{X} = 13$, $N = 18$, $\mu = 12$, and $\sigma_{\overline{X}}$: 2.5, what is the *z*-score for this sample mean?

Example 11

- On a test the population mean is 100, population standard deviation is 16 and our sample size is 64. What proportion of sample means will be above 103?

Example 12

- In a sample with a sample mean of 150 and a standard deviation of 20, and sample size of 1000,
  - What is the proportion of scores below 100?
  - What is the proportion of scores above 170?
  - How many scores are between the mean and 160?
  - Which raw score signifies the top 8%?