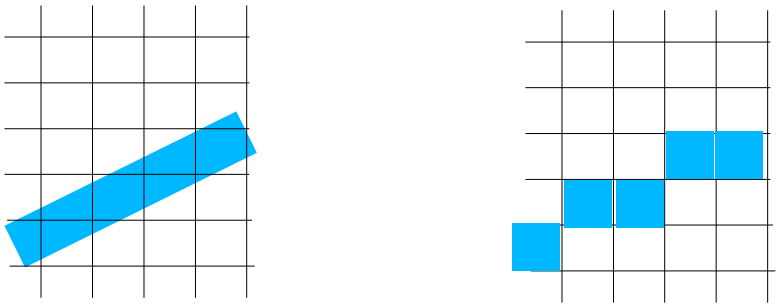
OUTPUT PRIMITIVES



CEng 477
Computer Graphics
METU, 2004

Output Primitives

- Graphic SW and HW provide subroutines to describe a scene in terms of basic geometric structures called output primitives.
- Output primitives are combined to form complex structures
- Simplest primitives
  - Point (pixel)
  - Line segment

Scan Conversion

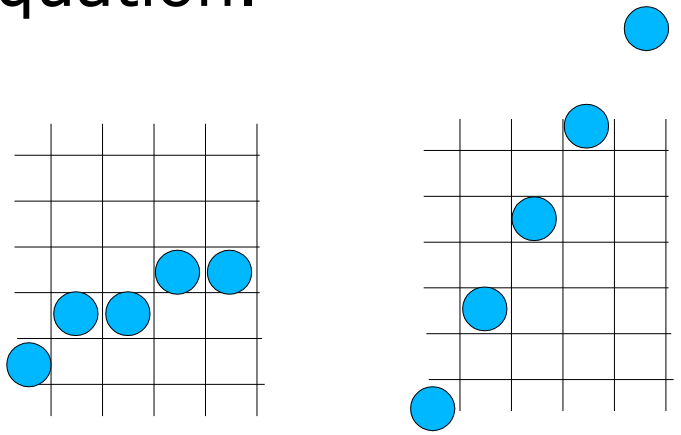
- Converting output primitives into frame buffer updates. Choose which pixels contain which intensity value.
- Constraints
  - Straight lines should appear as a straight line
  - primitives should start and end accurately
  - Primitives should have a consistent brightness along their length
  - They should be drawn rapidly

Line Drawing

- Simple approach:
  sample a line at discrete positions at one coordinate from start point to end point, calculate the other coordinate value from line equation.

$$y = m\,x + b \qquad x = \frac{1}{m}\,y + \frac{b}{m}$$

$$m = \frac{y_{start} - y_{end}}{x_{start} - x_{end}}$$



If $m > 1$, increment y and find x
If $m \le 1$, increment x and find y

Digital Differential Analyzer

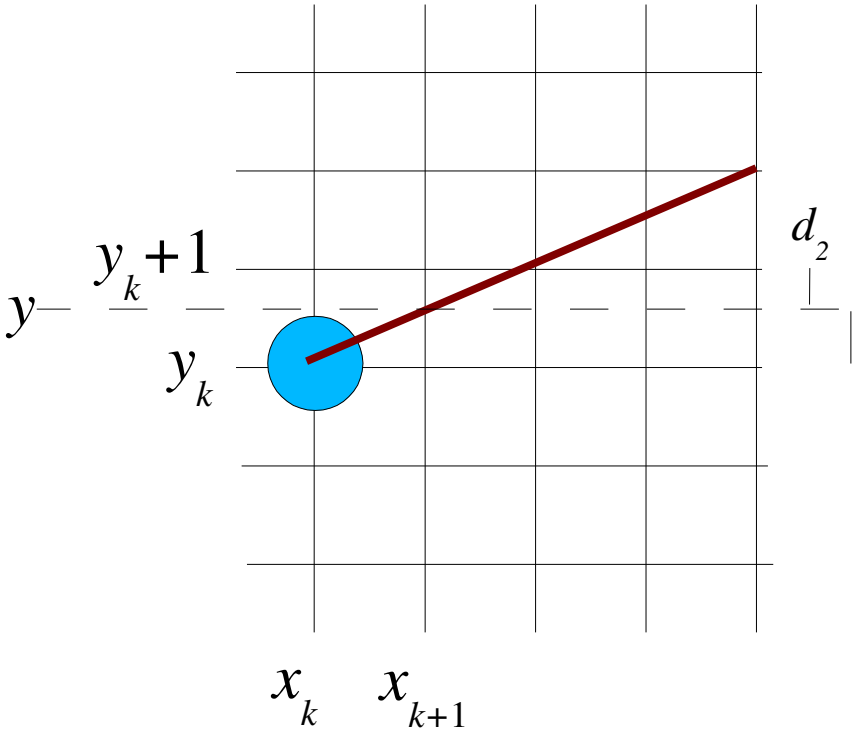
- Simple approach: too many floating point operations and repeated calculations
- Calculate y_{k+1} from y_k for a Δx value

$$\Delta y = m \Delta x \qquad y_{k+1} = y_k + m \qquad for \;\; \Delta x = 1, \;\; 0 < m < 1$$

$$\Delta x = \frac{\Delta y}{m} \qquad x_{k+1} = x_k + \frac{1}{m} \qquad for \;\; \Delta y = 1, \;\; m \geqslant 1$$

Bresenham's Line Algorithm

- DDA: Still floating point operations



$$\text{Assume } |m| \leq 1$$

$$\text{If already at } (x_k, y_k), \quad \text{choices:}$$

$$(x_k+1, y_k) \qquad \text{if } d_1 \leq d_2$$

$$(x_k+1, y_k+1) \qquad \text{if } d_1 \geq d_2$$

$$y = m(x_k+1)+b \Rightarrow \quad d_1 = y - y_k = m(x_k+1)+b-y_k$$

$$d_2 = (y_k+1) - y = y_k+1-m(x_k+1)-b$$

$$d_1 - d_2 = 2m(x_k+1) - 2y_k + 2b + 1$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_{end} - y_{start}}{x_{end} - x_{start}}$$

$$\text{define } p_k = \Delta x (d_1 - d_2) = 2\Delta y\, x_k - 2\Delta x\, y_k + c$$

$$c = 2\Delta y + \Delta x (2b-1) \quad \text{Independent from pixel position}$$

If $d_1 < d_2 \quad \Rightarrow \quad p_k < 0 \quad \Rightarrow$ Choose y_k

Else Choose $y_k + 1$

at step $k+1$:

$$p_{k+1} = 2\Delta y\, x_{k+1} - 2\Delta x\, y_{k+1} + c$$

$$p_{k+1} - p_k = 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k)$$

$x_{k+1} = x_k + 1 \Rightarrow$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)$$

$y_{k+1} - y_k = 0$ If y_k is choosen,

$y_{k+1} - y_k = 1$ If y_{k+1} is choosen

for x_0:

$$c = 2\Delta y + \Delta x (2b - 1) = 2\Delta y + \Delta x \left(2 y_0 - 2 x_0 \frac{\Delta y}{\Delta x}\right) - \Delta x$$

$$= 2\Delta y + 2\Delta x\, y_0 - 2\Delta y\, x_0 - \Delta x$$

$$p_0 = 2\Delta y\, x_0 - 2\Delta x\, y_0 + c = 2\Delta y\, x_0 - 2\Delta x\, y_0 + 2\Delta y + 2\Delta x\, y_0 - 2\Delta y\, x_0 - \Delta x$$

$$p_0 = 2\Delta y - \Delta x$$

$(x_{0,}, y_0)$ known

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

- Algorithm:

draw$(x_{0,}, y_0)$

$p_k \leftarrow 2\Delta y - \Delta x;\quad x_k \leftarrow x_0$

while $x_k < x_{end}$

  $x_{k+1} \leftarrow x_k + 1$

  if $p_k \le 0$ choose y_k

    $y_{k+1} \leftarrow y_k;\quad p_{k+1} \leftarrow p_k + 2\Delta y$

  else choose $y_k + 1$

    $y_{k+1} \leftarrow y_k + 1;\quad p_{k+1} \leftarrow p_k + 2\Delta y - 2\Delta x$

  draw(x_{k+1}, y_{k+1})

  $x_k \leftarrow x_k + 1$

  $p_k = p_{k+1}$

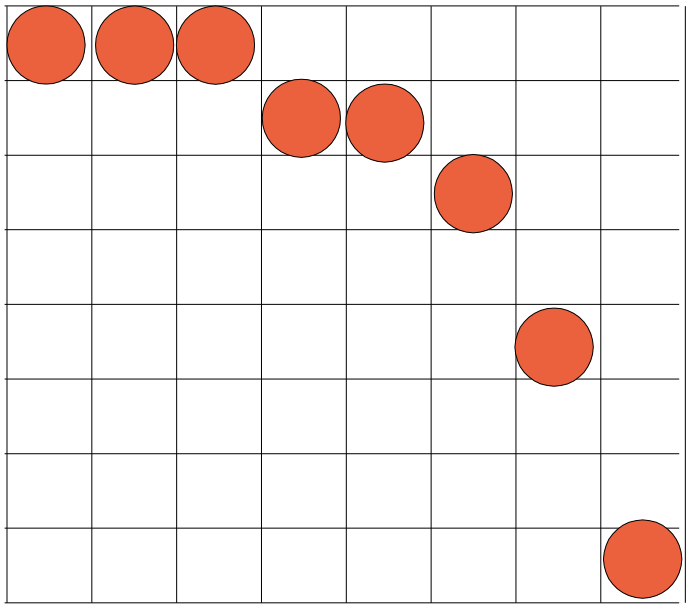
Circle Generation

$$(x-x_0)^2+(y-y_0)^2=r^2$$

unit steps in $x \Rightarrow y=y_0 \mp \sqrt{r^2-(x-x_0)^2}$

- Computationally complex
- Non uniform spacing
- Polar coordinates:

$$x=r\cos(\theta)+x_c$$
$$y=r\sin(\theta)+y_c$$



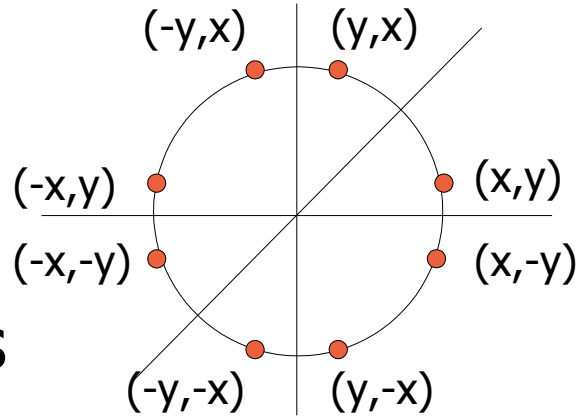
- Fixed angular step size to have equally spaced points

$$x_k = r\cos\theta \quad x_{k+1} = r\cos(\theta + d\theta)$$
$$y_k = r\sin\theta \quad y_{k+1} = r\sin(\theta + d\theta)$$

$$\begin{aligned} x_{k+1} &= r\cos\theta\cos d\theta - r\sin\theta\sin d\theta \\ &= x_k\cos d\theta - y_k\sin d\theta \\ y_{k+1} &= r\sin\theta\cos d\theta + r\cos\theta\sin d\theta \\ &= y_k\cos d\theta - y_k\sin d\theta \end{aligned}$$

fixed $d\theta$ so compute $\cos d\theta$ and $\sin d\theta$ initially

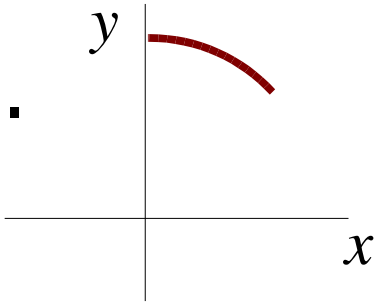
- Computation can be reduced by considering symmetry of circles:
- Still too complex, multiplications, trigonometric calculations



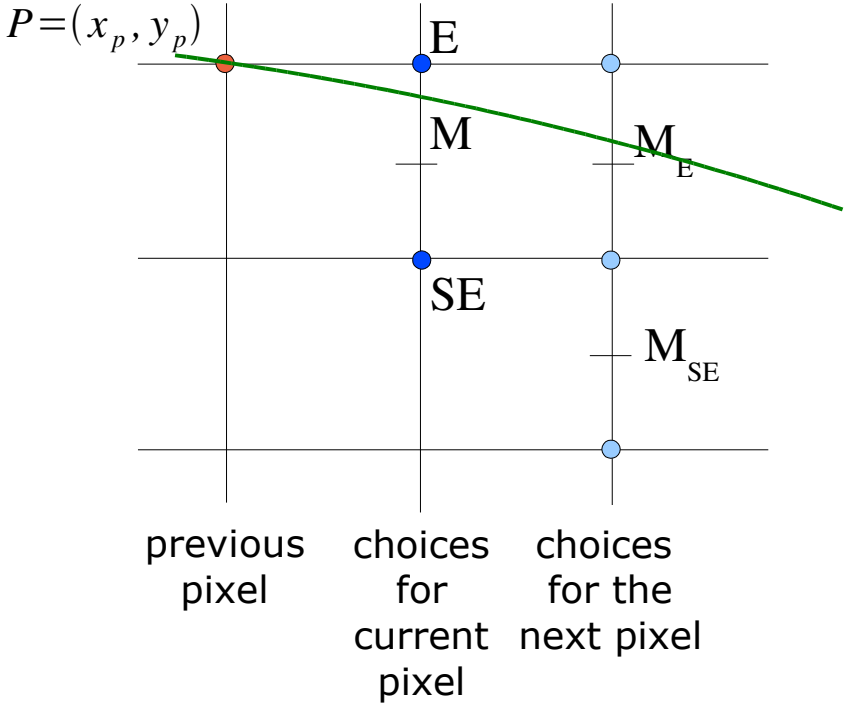
- Bresenham's circle generation algorithm involves simple integer operations
- Midpoint Circle Algorithm generates the same pixels

Midpoint Circle Algorithm

- Consider the second octant. Increment x , decide on y



- Assume that



select which of 2 pixels are closer to the circle by evaluating the function at E and SE.

$$f(x,y)=x^2+y^2-r^2=\begin{cases} 0 & \text{if on the circle} \\ >0 & \text{if outside the circle} \quad \text{choose SE} \\ <0 & \text{if inside the circle} \quad \text{choose E} \end{cases}$$

$$p_k=f(x_k+1,y_k-\frac{1}{2})=(x_k+1)^2+(y_k-\frac{1}{2})^2-r^2$$

$$p_{k+1}=f(x_{k+1}+1,y_{k+1}-\frac{1}{2})=(x_k+1+1)^2+(y_{k+1}-\frac{1}{2})^2-r^2$$

$$p_{k+1}-p_k=(x_k+1+1)^2+(y_{k+1}-\frac{1}{2})^2-r^2-(x_k+1)^2-(y_k-\frac{1}{2})^2+r^2$$

$$p_{k+1}=p_k+x_k^2+4x_k+4+y_{k+1}^2-y_{k+1}+\frac{1}{4}-x_k^2-2x_k-1-y_k^2+y_k-\frac{1}{4}$$

$$p_{k+1}=p_k+x_k^2+4x_k+4+y_{k+1}^2-y_{k+1}+\frac{1}{4}-x_k^2-2x_k-1-y_k^2+y_k-\frac{1}{4}$$

$$p_{k+1}=p_k+2x_k+3+(y_{k+1}^2-y_k^2)-(y_{k+1}-y_k)$$

If E is chosen $p_k<0$:

$y_{k+1}=y_k \Rightarrow p_{k+1}=p_k+2x_k+3$

If SE is chosen $p_k\geqslant 0$:

$$y_{k+1}=y_k-1 \Rightarrow p_{k+1}=p_k+2x_k+3-2y_k+1+1$$
$$=p_k+2x_k-2y_k+5$$

$$x_0=0;\quad y_0=r;\quad p_0=f(0,r-\frac{1}{2})=1+(r-\frac{1}{2})^2-r^2=\frac{5}{4}-r$$

All increments are integer, rounding $\frac{5}{4}$ will give 1 so,

$$p_0=1-r$$

- Algorithm:

drawoctants$(x_{0,} r)$

$p_k \leftarrow 1-r;\ \ x_k \leftarrow 0;\ \ y_k \leftarrow r$

while $x_k < y_k$

  if $p_k < 0$ choose E

    $y_{k+1} \leftarrow y_k;\quad p_{k+1} \leftarrow p_k + 2x_k + 3$

  else choose SE

    $y_{k+1} \leftarrow y_k - 1;\quad p_{k+1} \leftarrow p_k + 2x_k - 2y_k + 5$

  $x_{k+1} \leftarrow x_k + 1$

  drawoctants(x_{k+1}, y_{k+1})

  $x_k \leftarrow x_{k+1}$

  $y_k \leftarrow y_{k+1}$

  $p_k = p_{k+1}$

if $p_k<0$ choose E

$y_{k+1}\leftarrow y_k;\quad p_{k+1}\leftarrow p_k+2x_k+3$

else choose SE

$y_{k+1}\leftarrow y_k-1;\quad p_{k+1}\leftarrow p_k+2x_k-2y_k+5$

$x=0;\ y=10;\ r=10$ plot (0,10)

$p_k=1-10=-9$ choose E plot $(1,10)$

$p_k=-9+2+3=-4$ choose E plot $(2,10)$

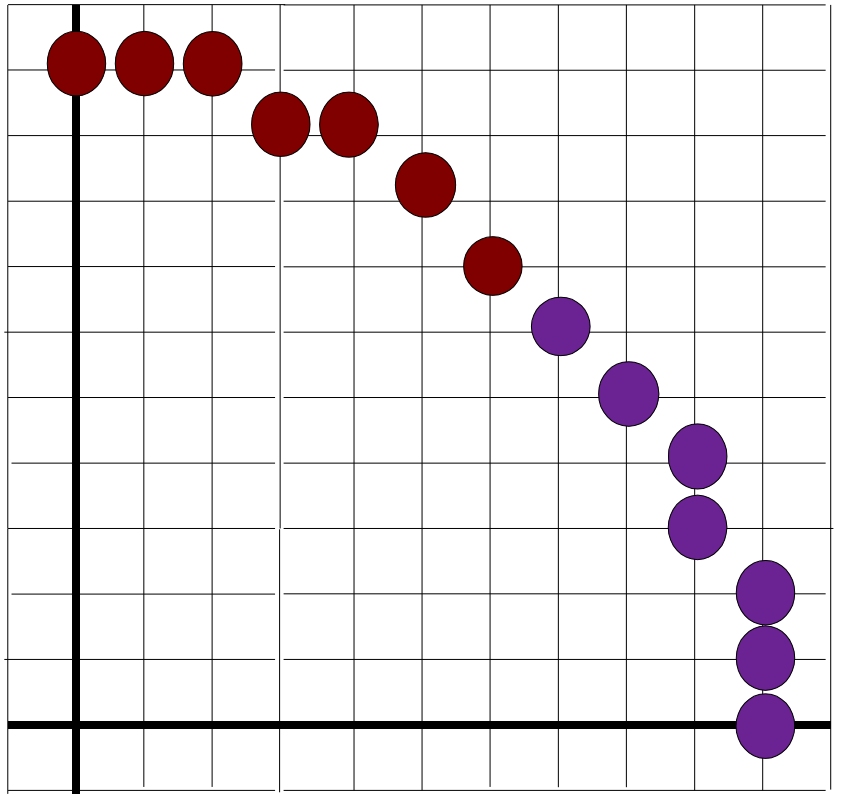
$p_k=-4+4+3=3$ choose SE plot $(3,9)$

$p_k=3+6-18+5=-4$ choose E plot $(4,9)$

$p_k=-4+8+3=7$ choose SE plot $(5,8)$

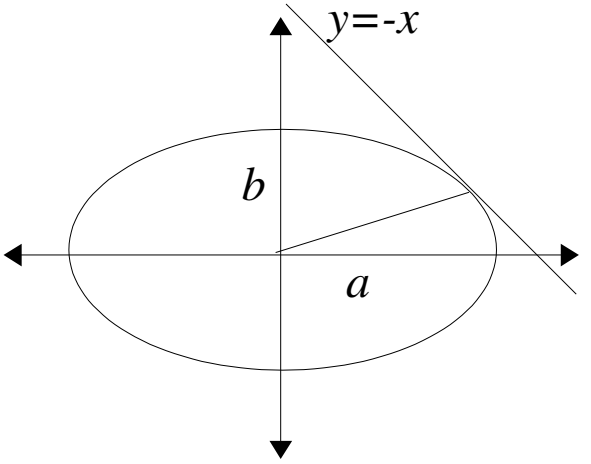
$p_k=7+10-16+5=6$ choose SE plot $(6,7)$

$p_k=6+12-14+5=9$ choose SE plot $(7,6)$



Ellipse Generation

- Similar to circle generation with mid-point. Inside test.
- Different formula for points up to the tangent $y=-x$, slope<1.
  $(0,b)$ to tangent: increment x find y
  tangent to $(a,0)$: decrement y find x
- Mid-point algorithm is applicable to other polynomial equations:
  - Parabola, Hyperbola



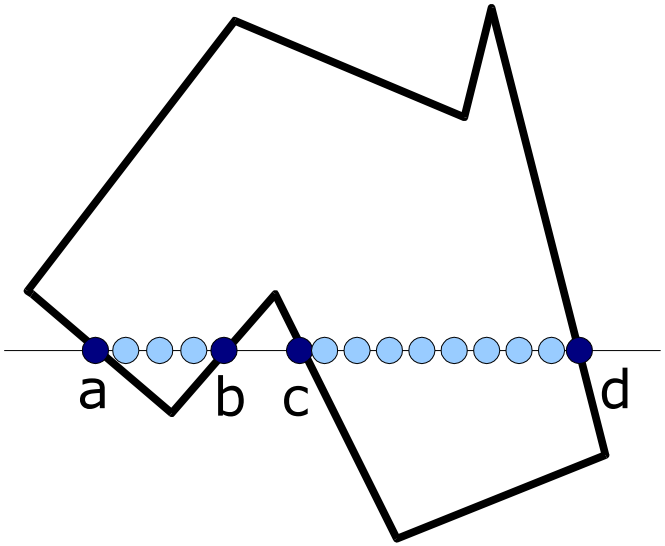
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Filled Area Primitives

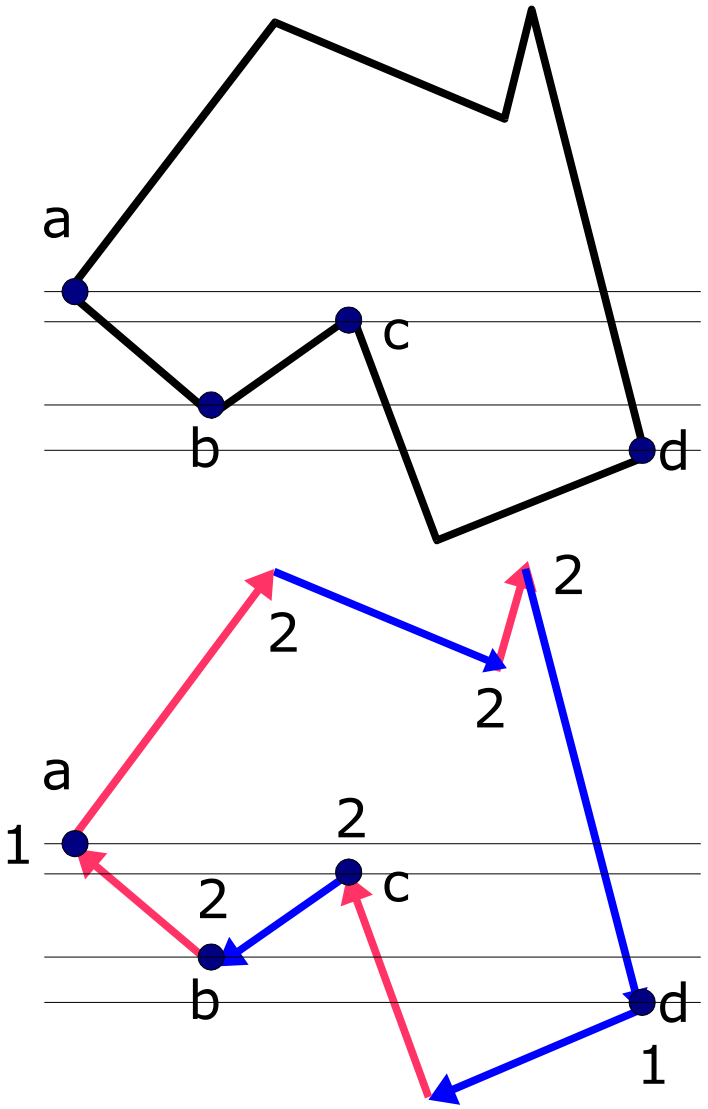
- Two basic approaches to area filling on raster systems:
  - Determine the overlap intervals for scan lines that cross the area (scan-line)
  - Start from an interior position and point outward from this point until the boundary condition reached (fill method)
- Scan-line: simple objects, polygons, circles,..
- Fill-method: complex objects, interactive fill.

Scan-line Polygon Fill

- For each scan-line:
  - Locate the intersection of the scan-line with the edges ($y=y_s$)
  - Sort the intersection points from left to right.
  - Draw the interiors intersection points pairwise. (a-b), (c-d)
- Problem with corners. Same point counted twice or not?



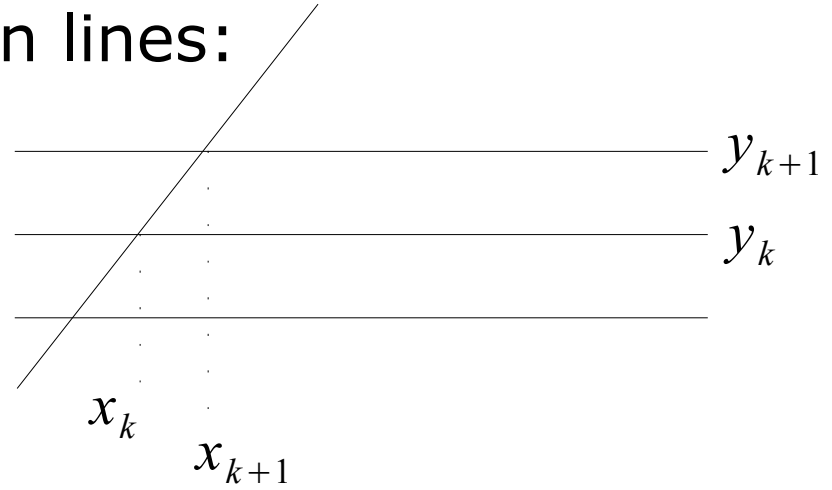
- *a*,*b*,*c* and *d* are intersected by 2 line segments each.
- Count *b*,*d* twice but *a* and *d* once. Why?
- Solution:
  Make a clockwise or counter-clockwise traversal on edges.
  Check if y is monotonically increasing or decreasing.
  If direction changes, double intersection, otherwise single intersection.



Scan-line Polygon Filling (coherence)

- **Coherence**: Properties of one part of a scene are related with the other in a way that can it be used to reduce processing of the other.
- Scan-lines adjacent to each other.
  Intersection of scan-lines are close to each other (like scan conversion of a line)
- Intersection points with scan lines:

$$x_{k+1} = round\left(x_k + \frac{1}{m}\right)$$



- Instead of floating point operations, use integer operations:

$$m = \frac{\Delta y}{\Delta x} \qquad x_{k+1} = x_k + \frac{\Delta x}{\Delta y}$$

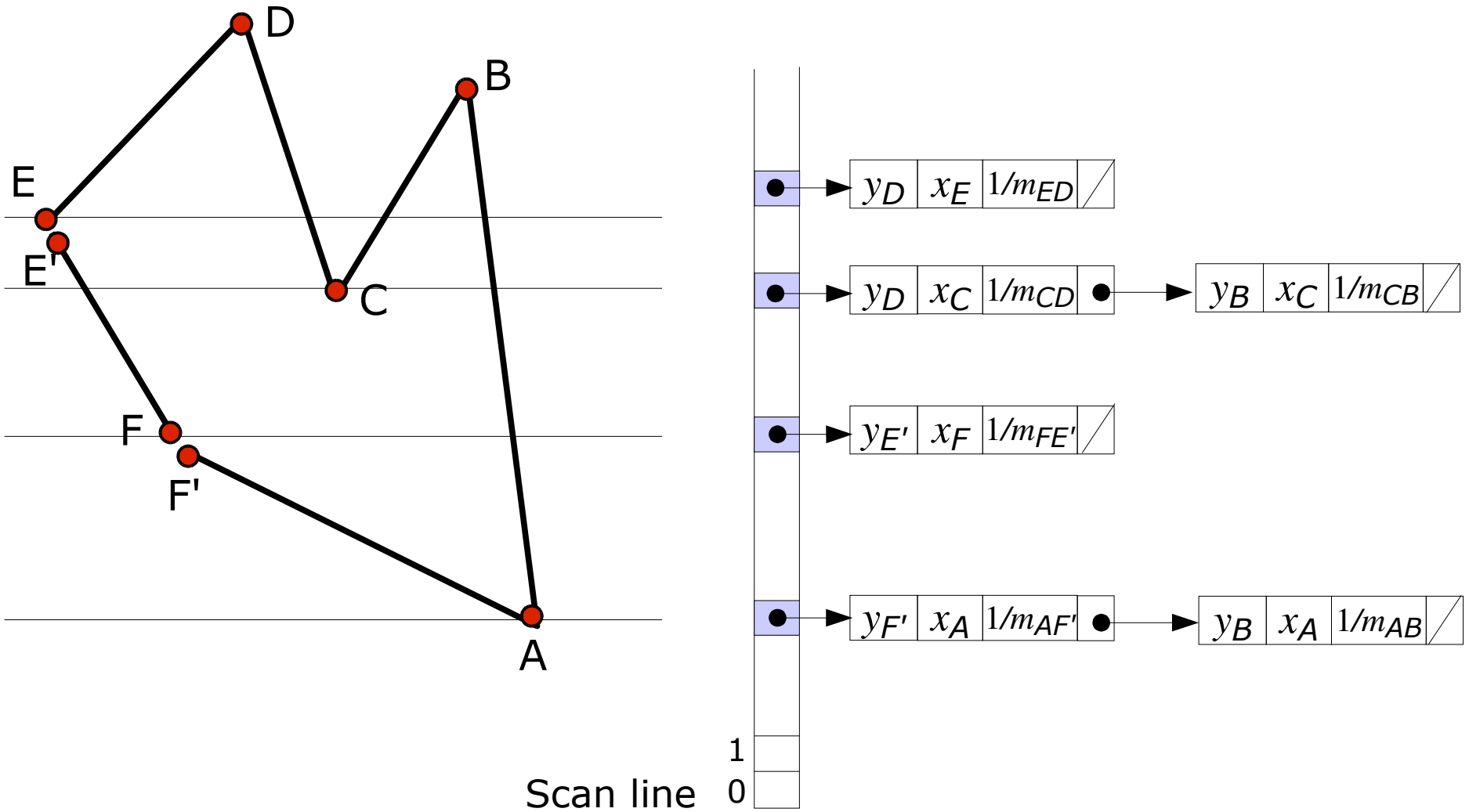
$counter \leftarrow 0$
for each scan-line
  $counter \leftarrow counter + \Delta x$
  while $counter \geqslant \Delta y$
    $x \leftarrow x + 1$
    $counter \leftarrow counter - \Delta y$

- Example:
  m = 8/5

scanline	counter	x
0	0	0
1	5	0
2	~~10~~ 2	1
3	7	1
4	~~12~~ 4	2
5	~~9~~ 1	3

Efficient Polygon Fill

- Make a (counter)clockwise traversal and shorten the single intersection vertices.
- Generate a sorted edge table on the scan-line axis. Each edge has an entry in smaller *y* valued corner point.
- Each entry keeps a linked list of all connected edges:
  - *x* value of the point
  - *y* value of the end-point
  - Slope of the edge


D
B
E
E'
C
F
F'
A
y_D x_E $1/m_{ED}$
y_D x_C $1/m_{CD}$
y_B x_C $1/m_{CB}$
$y_{E'}$ x_F $1/m_{FE'}$
$y_{F'}$ x_A $1/m_{AF'}$
y_B x_A $1/m_{AB}$
1
0
Scan line

- Start with the smallest scan-line
- Keep an active edge list:
  - Update the current x value of the edge based on m value
  - Add the lists in the current table entry based on their x value
  - Remove the completed edges
  - Draw the intermediate points of pairwise elements of the list.

- Example:
  A,B,C,D,E,F,A Polygon:
  (30,10),(24,32),(20,22),(16,34)
  (8,26),(12,16),(30,10)
  E'=(20,25), F'=(12,15)

  Edge Table:

Y	E1	E2
10	[15,30,-3]	[32,30,-3/11]
16	[25,12,-2/5]	
22	[34,20,-1/3]	[32,20,2/5]
26	[34,8,1]	

Y	S1	S1	S2	S2
10	30	30		
11	27	29.73		
12	24	29.45		
13	21	29.18		
14	18	28.91		
15	15	28.64		
16	12	28.36		
17	11.6	28.09		
18	11.2	27.82		
19	10.8	27.55		
20	10.4	27.27		
21	10	27		
22	9.6	20	20	26.73
23	9.2	19.67	20.4	26.45
24	8.8	19.33	20.8	26.18
25	8.4	19	21.2	25.91
26	8	18.67	21.6	25.64
27	9	18.33	22	25.36
28	10	18	22.4	25.09
29	11	17.67	22.8	24.82
30	12	17.33	23.2	24.55
31	13	17	23.6	24.27
32	14	16.67	24	24
33	15	16.33		
34	16	16		

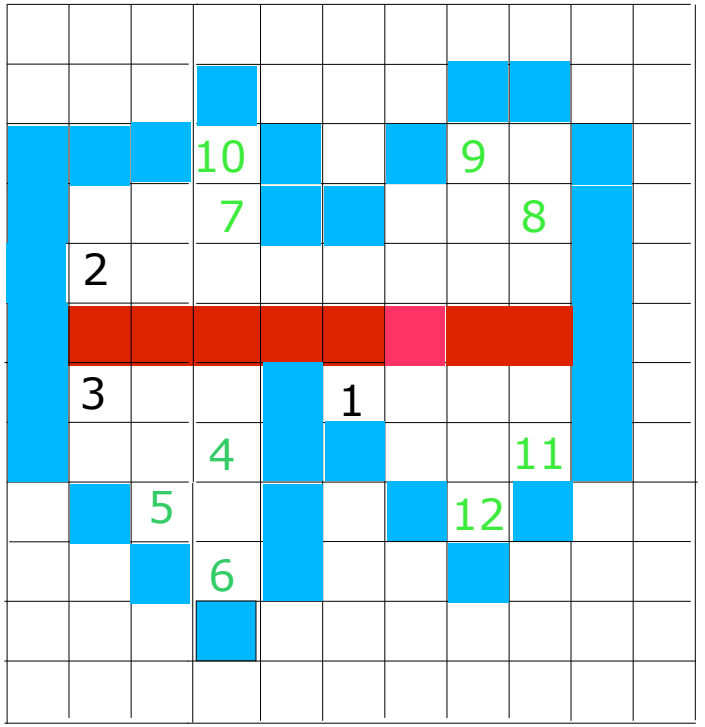
Boundary Fill Algorithm

- Start at a point inside a continuous arbitrary shaped region and paint the interior outward toward the boundary. Assumption: boundary color is a single color
- (*x*,*y*): start point; *b*:boundary color, fill: fill color

```
void boundaryFill4(x,y,fill,b) {
    cur = getpixel(x,y)
    if (cur != b) AND (cur != fill) {
        setpixel(x,y,fill);
        boundaryFill4(x+1,y,fill,b);
        boundaryFill4(x-1,y,fill,b);
        boundaryFill4(x,y+1,fill,b);
        boundaryFill4(x,y-1,fill,b);
    }
}
```

- 4 neighbors vs 8 neighbors: depends on definition of continuity.
  8 neighbor: diagonal boundaries will not stop
- Recursive, so slow. For large regions with millions of pixels, millions of function calls.
- Stack based improvement: keep neighbors in stack
- Number of elements in the stack can be reduced by filling the area as pixel spans and pushing only the pixels with pixel transitions.

- Check the neighbor pixels as filling the area line by line
- If pixel changes from null to boundary or null when scan-line finishes, push the pixel information on stack.
- After a scan-line finishes, pop a value from stack and continue processing.



Stack after scan line

3
2
1

Flood-Fill

- Similar to boundary fill. Algorithm continues while the neighbor pixels have the same color.
-

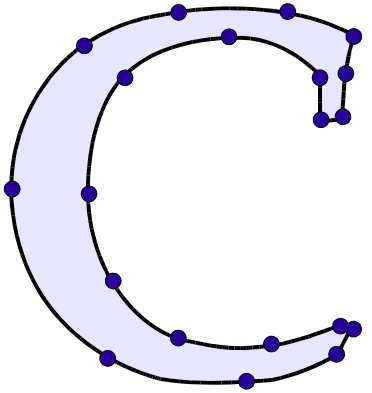
```
void FloodFill4(x,y,fill,oldcolor) {
   cur = getpixel(x,y)
   if (cur == oldcolor) {
      setpixel(x,y,fill);
      FloodFill4(x+1,y,fill,oldcolor);
      FloodFill4(x-1,y,fill,oldcolor);
      FloodFill4(x,y+1,fill,oldcolor);
      FloodFill4(x,y-1,fill,oldcolor);
   }
}
```

Character Generation

- Typesetting fonts:
  - Bitmap fonts: simple, not scalable.
  - Outline fonts: scalable, flexible, more complex process
-

```
0 0 0 0 0 0 0 0
0 0 0 1 1 1 0 0
0 0 1 1 0 1 1 0
0 1 1 0 0 0 1 1
0 1 1 0 0 0 1 1
0 1 1 1 1 1 1 1
0 1 1 1 1 1 1 1
0 1 1 0 0 0 1 1
0 1 1 0 0 0 1 1
0 1 1 0 0 0 1 1
0 1 1 0 0 0 1 1
0 0 0 0 0 0 0 0
```

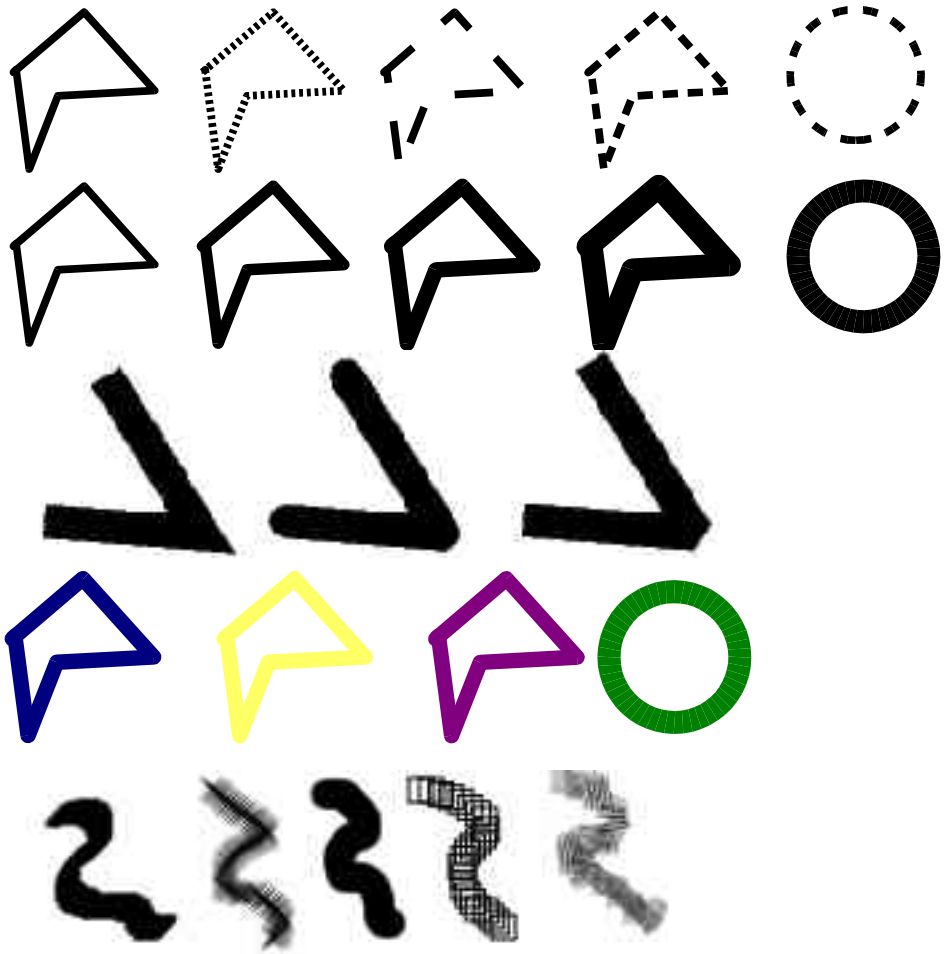
Pixelwise on/of information



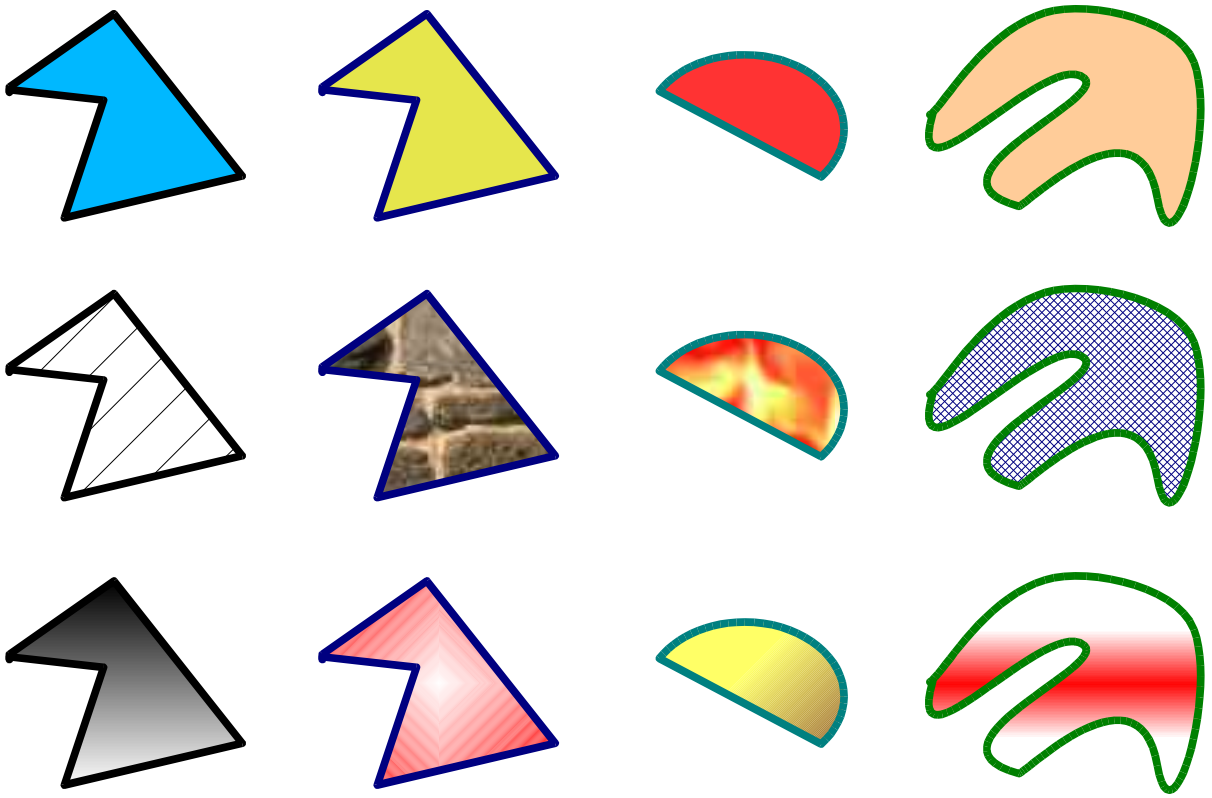
Points and tangents of the boundary

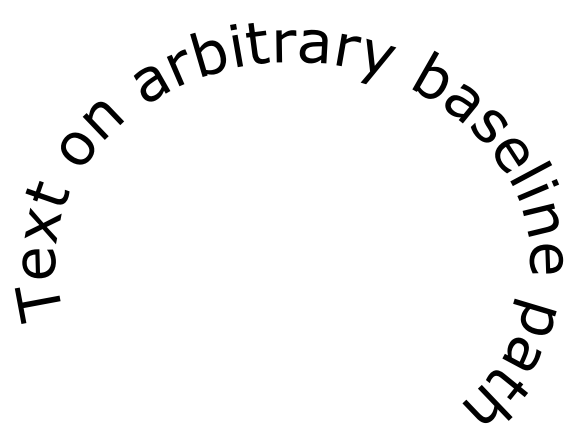
Attributes of Output Primitives

- Line attributes:
  - Line type
    dotted, dashed, ...
  - Line width
  - Line caps and join
    miter, round, bevel
  - Line color
  - Line brush



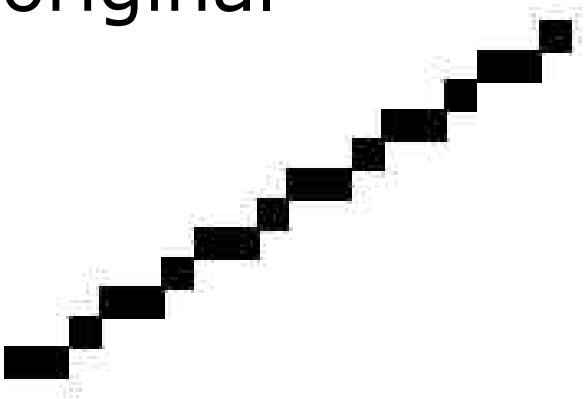
- Area Fill Attributes:
  - Solid
  - Pattern
  - Gradient



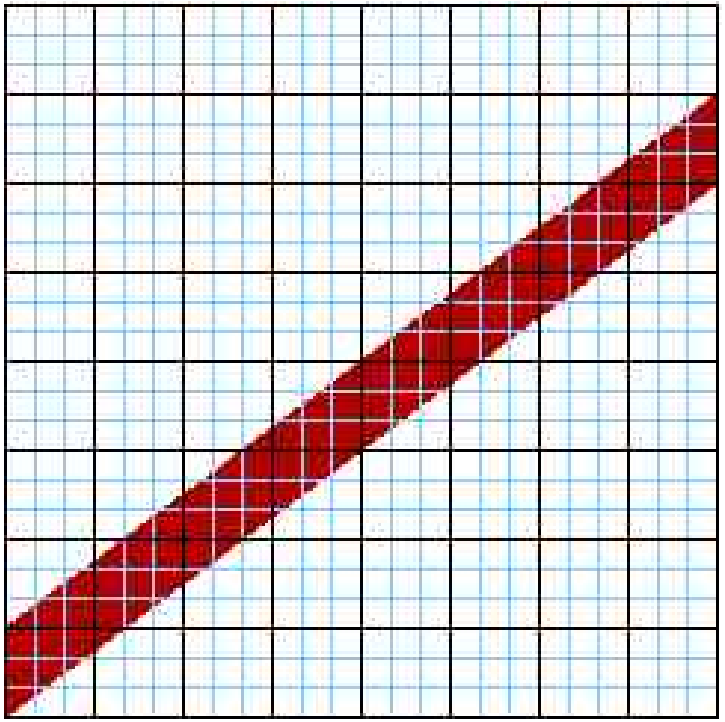
- Character Attributes
  - *Font*, **bold**, *italic*, underlined, outline, shadow
  - Spacing abcdef abcdef abcdef abcdef
  - Direction normal down rotated slanted
  - 

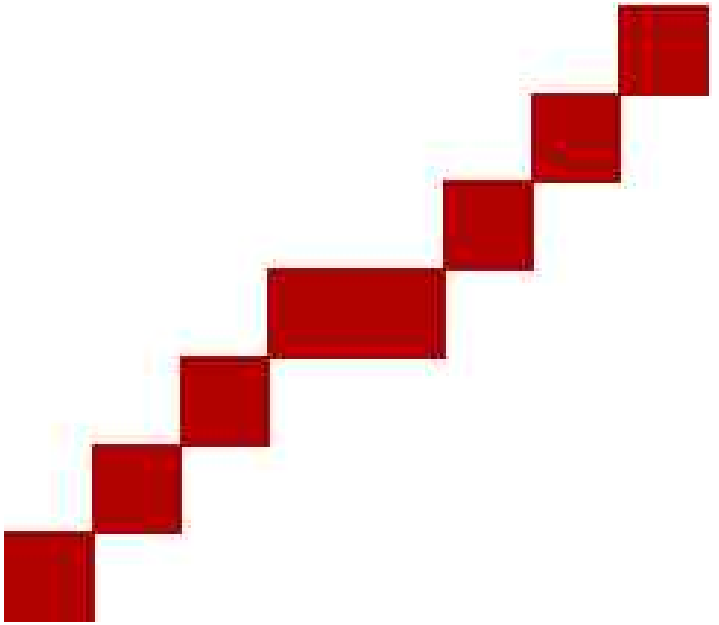
Antialiasing

- CG generated shapes:
  limited resolution sampling of original shapes with infinite resolution
- Loose information, Aliasing:
  Jagging, stairway effects.
- Use higher resolutions.
  Expensive, no limit.
- Another solution:
  Antialiasing: use intensity levels.

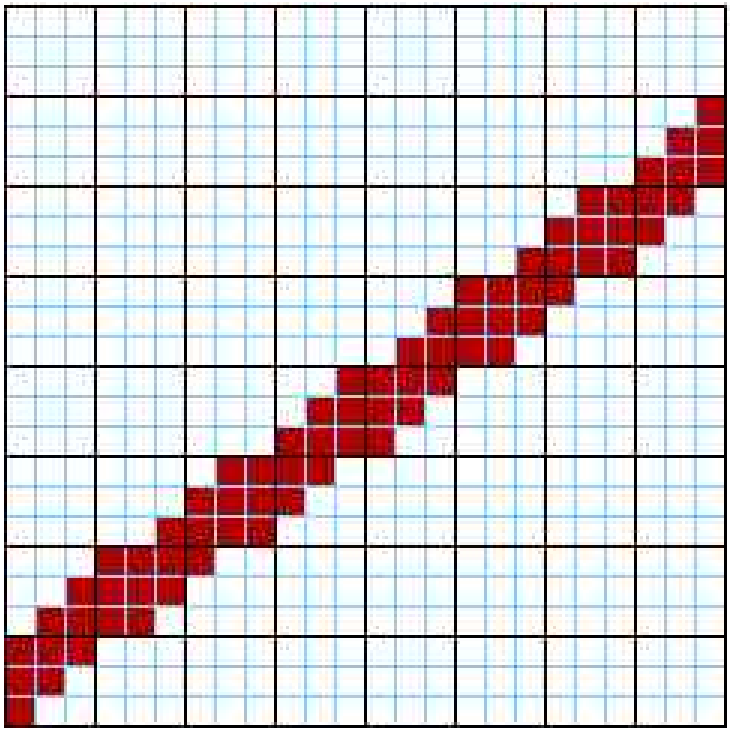


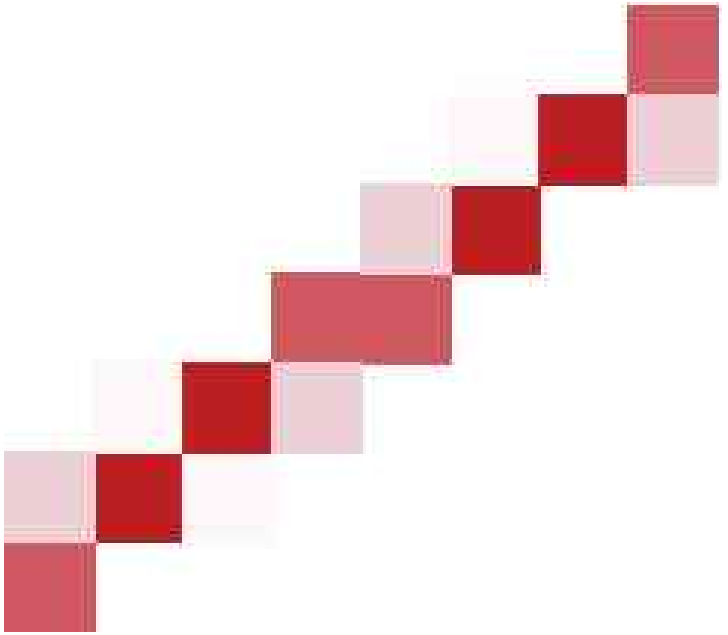
- Supersampling: sample at higher resolution.





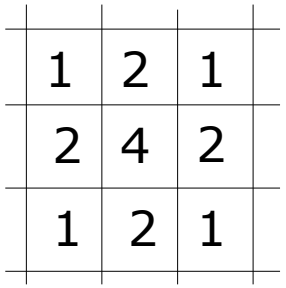
- Count the superpixels at each pixel. Give an intensity based on that value.



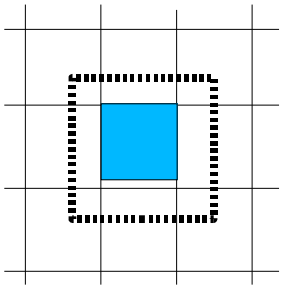


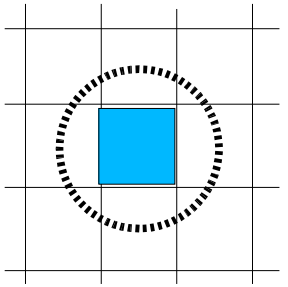
- Aliased vs. Antialiased.

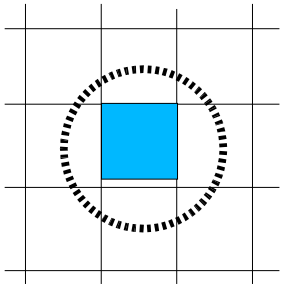
- Filtering techniques:
  Giving a larger weight to center pixel and lower weight to neighbor pixels. Weight mask or volume based filters.

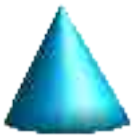


Pixel Weighting Mask









Box filter

Cone filter

Gaussian filter