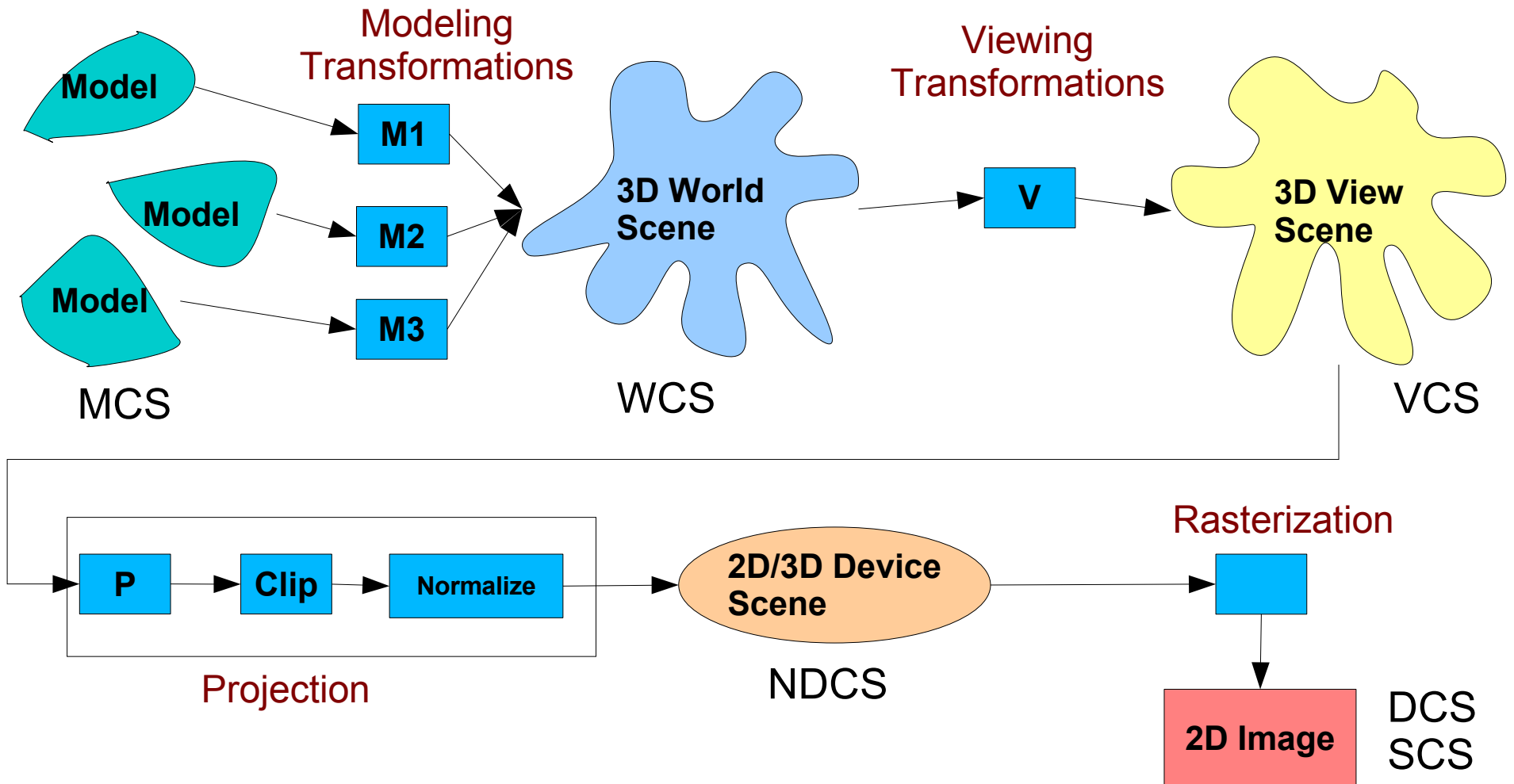

2 DIMENSIONAL VIEWING

Ceng 477 Computer Engineering
Lecture notes
METU

Viewing Pipeline Revisited



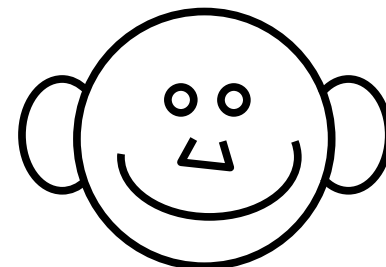
- Model coordinates to World coordinates:
Modelling transformations

Model coordinates:

1 circle (head),
2 circles (eyes),
1 line group (nose),
1 arc (mouth),
2 arcs (ears).
With their relative
coordinates and sizes

World coordinates:

All shapes with their
absolute coordinates and sizes.
circle(0,0,2)
circle(-.6,.8,.3) circle(.6,.8,.3)
lines[(-.4,0),(-.5,-.3),(.5,.3),(.4,0)]
arc(-.6,0,.6,0,1.8,180,360)
arc(-2.2,.2,-2.2,-.2,.8,45,315)
arc(2.2,.2,2.2,-.2,.8,225,135)

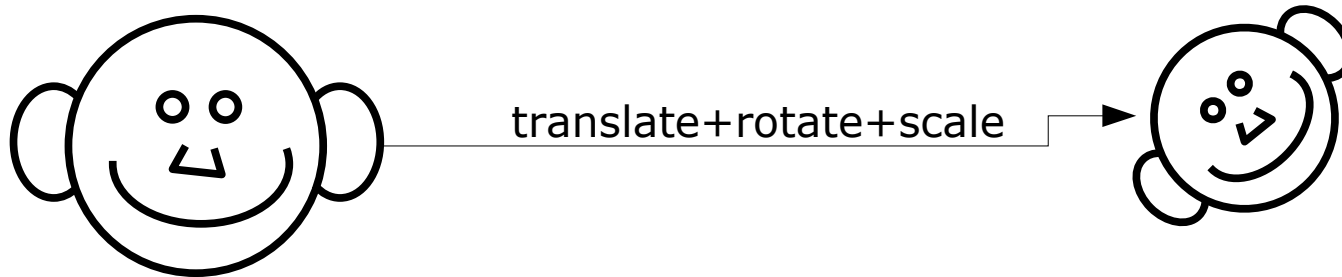


-
- World coordinates to Viewing coordinates:
Viewing transformations

World coordinates

Viewing coordinates:

Viewers position and view angle. i.e. rotated/translated



-
- Projection: 3D to 2D. Clipping depends on viewing frame/volume. Normalization: device independent coordinates (ie. cm.)

Viewing coordinates:

Device Independent Coordinates:

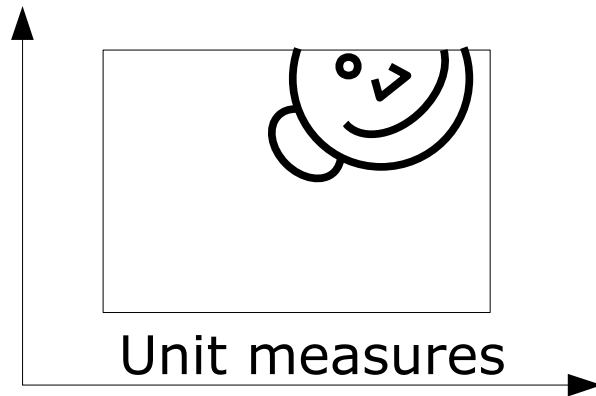
Invisible shapes deleted, others reduced to visible parts.

3 arcs, 1 circle, 1 line group

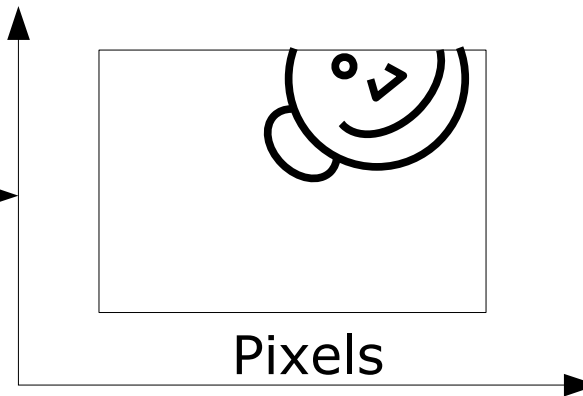


-
- Device Independent Coordinates to Device Coordinates. Rasterization

Device Independent Coordinates



Screen Coordinates or Device Coordinates



Basic Geometric Transformations

- Given the shape, transform all the points of the shape? Transform the points and/or vectors describing it.
- For example:
Polygon: corner points
Circle, Ellipse: center point(s), point at angle 0
- Some transformations preserves some of the attributes like sizes, angles, ratios of the shape.

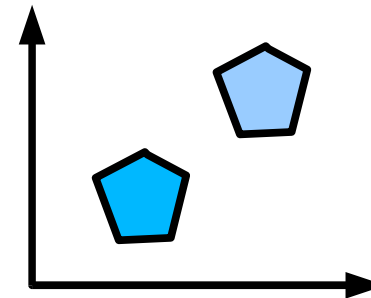
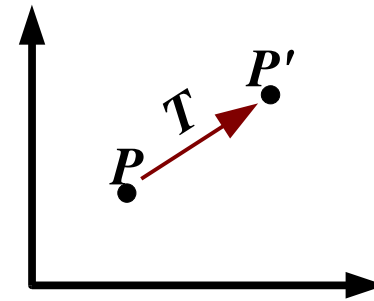
Translation

- Simply move the object to a relative position.

$$x' = x + t_x \quad y' = y + t_y$$

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$P' = P + T$$



Rotation

- Reposition the object in a circular path around a point.

$$x' = r \cos(\phi + \theta) = r \cos \phi \cos \theta - r \sin \phi \sin \theta$$

$$y' = r \sin(\phi + \theta) = r \cos \phi \sin \theta + r \sin \phi \cos \theta$$

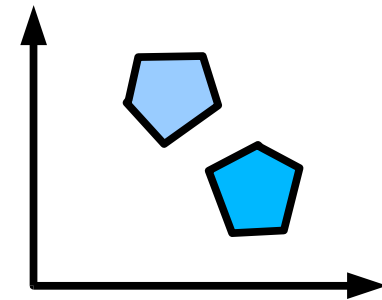
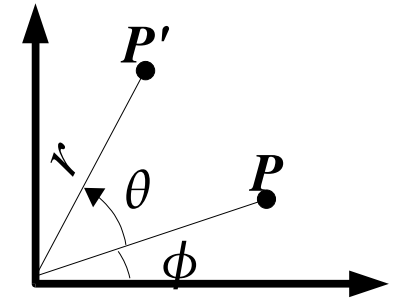
$$x = r \cos \phi \quad y = r \sin \phi$$

$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$P' = R \cdot P$$



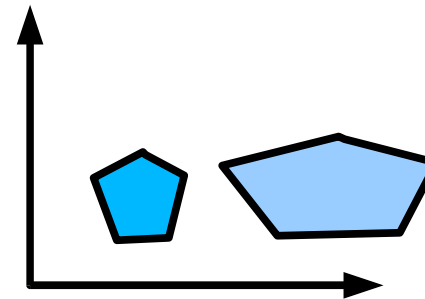
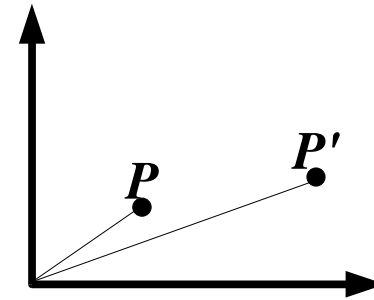
Scaling

- Change the sizes of the object.

$$x' = x s_x \quad y' = y s_y$$

$$S = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$P' = S \cdot P$$



Homogenous Coordinates

- All transformations can be represented by matrix operations.
- Translation is additive, rotation and scaling is multiplicative; making the operations complicated.
- Adding another dimension to transformations make translation also representable by multiplication.
Cartesian coordinates vs homogenous coordinates.

$$x = \frac{x_h}{h} \quad y = \frac{y_h}{h} \quad \mathbf{P} = \begin{bmatrix} x_h \\ y_h \\ h \end{bmatrix} = \begin{bmatrix} h \cdot x \\ h \cdot y \\ h \end{bmatrix}$$

-
- Many points in homogenous coordinates can represent same point in cartesian coordinates.
 - In homogenous coordinates, all transformations can be written as multiplications.

Transformations in Homogenous C.

- Translation
$$\mathbf{T}(t_x, t_y) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$
$$\mathbf{P}' = \mathbf{T}(t_x, t_y) \cdot \mathbf{P}$$

- Rotation
$$\mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\mathbf{P}' = \mathbf{R}(\theta) \cdot \mathbf{P}$$

- Scaling
$$\mathbf{S}(s_x, s_y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\mathbf{P}' = \mathbf{S}(s_x, s_y) \cdot \mathbf{P}$$

Composite Transformations

- Composition of similar type transformations

$$\mathbf{T}(t_{x1}, t_{y1}) \cdot \mathbf{T}(t_{x2}, t_{y2}) = \begin{bmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x2} + t_{x1} \\ 0 & 1 & t_{y2} + t_{y1} \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{T}(t_{x1} + t_{x2}, t_{y1} + t_{y2})$$

$$\mathbf{R}(\theta) \cdot \mathbf{R}(\phi) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} \cos \theta \cos \phi - \sin \theta \sin \phi & -\cos \theta \sin \phi - \sin \theta \cos \phi & 0 \\ \sin \theta \cos \phi + \cos \theta \sin \phi & -\sin \theta \sin \phi + \cos \theta \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta + \phi) & -\sin(\theta + \phi) & 0 \\ \sin(\theta + \phi) & \cos(\theta + \phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{R}(\theta + \phi)$$

$$\mathbf{S}(s_{x1}, s_{y1}) \cdot \mathbf{S}(s_{x2}, s_{y2}) = \begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{x1} s_{x2} & 0 & 0 \\ 0 & s_{y1} s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{S}(s_{x1} \cdot s_{x2}, s_{y1} \cdot s_{y2})$$

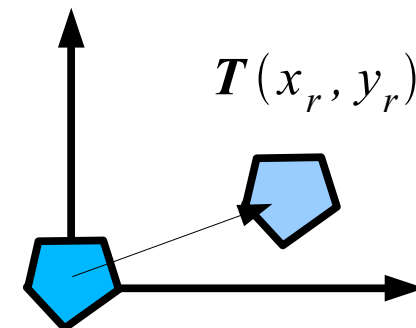
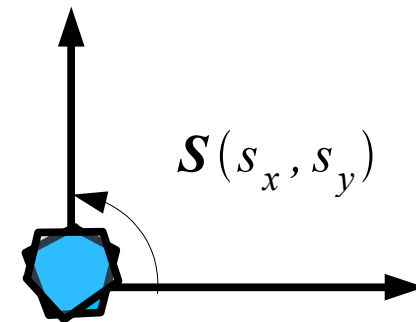
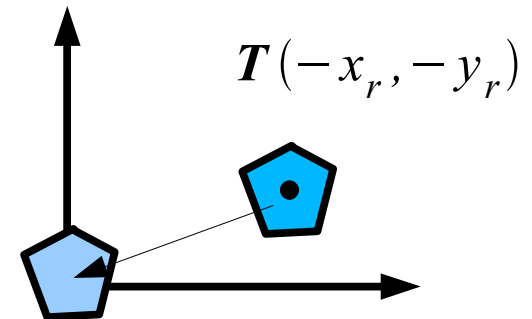
- Rotation on a pivot point

- Translate to origin
- Rotate
- Translate back

- $T(x_r, y_r) \cdot R(\theta) \cdot T(-x_r, -y_r) =$

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & x_r(1 - \cos \theta) + y_r \sin \theta \\ \sin \theta & \cos \theta & y_r(1 - \cos \theta) - x_r \sin \theta \\ 0 & 0 & 1 \end{bmatrix}$$



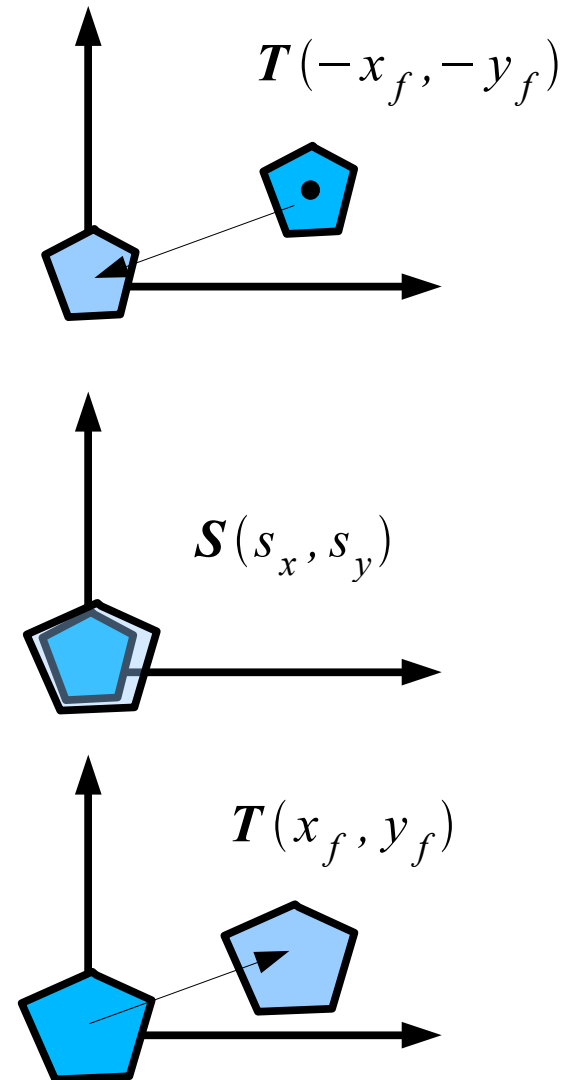
- Scaling on a fixed point

- Translate to origin
- Scale
- Translate back

- $T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f) =$

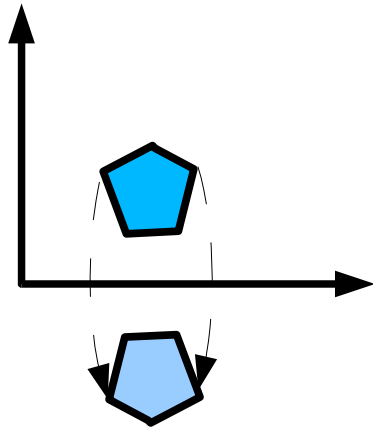
$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

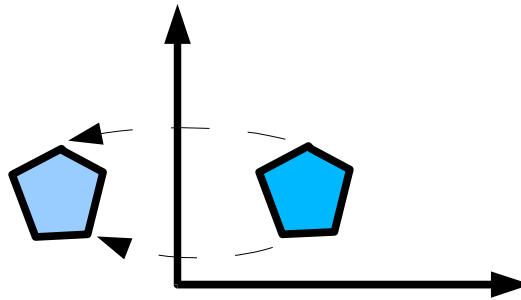


Other Transformations

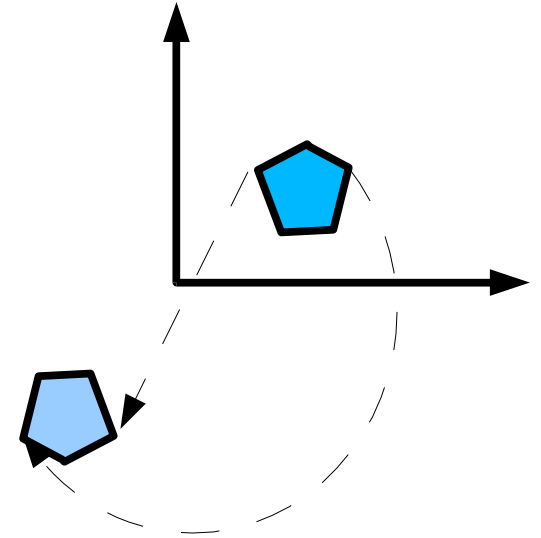
- Reflection



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

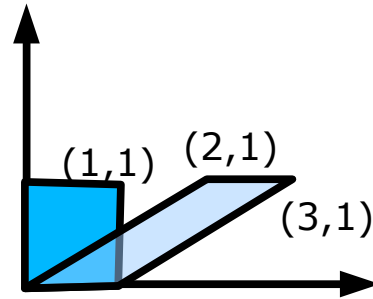
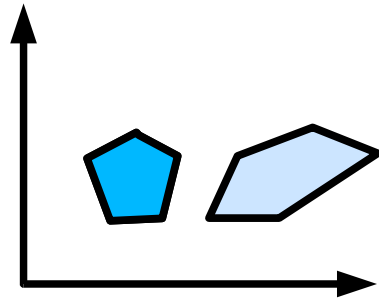


$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

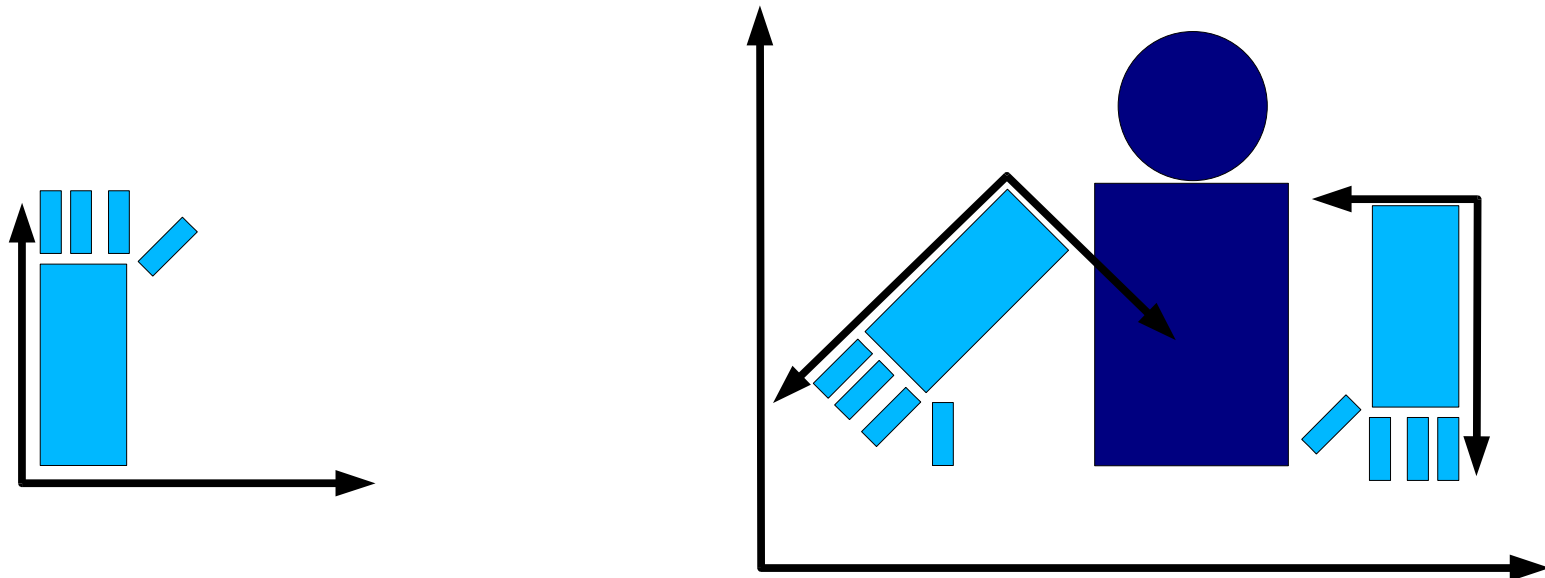
-
- Shear: Deform the shape like shifted slices.



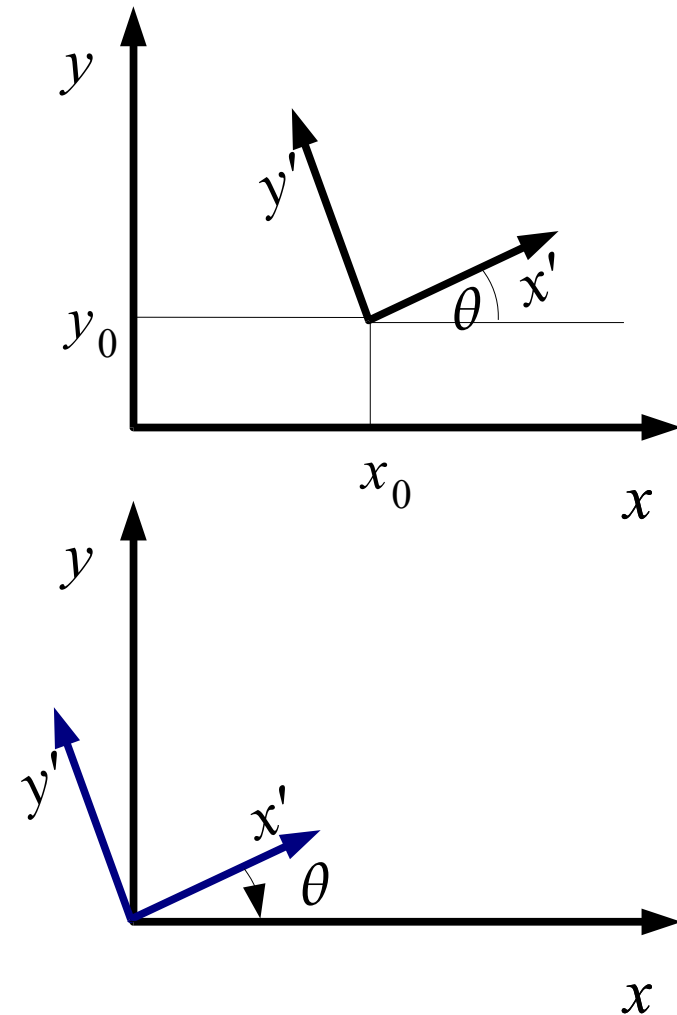
$$x' = x + sh_x \cdot y \quad y' = y$$
$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformations Between the Coordinate Systems

- Between different systems: Polar coordinates to cartesian coordinates
- Between two cartesian coordinate systems. For example relative coordinates or window to viewport transformation.



- How to transform from x, y to x', y' ?
- Superimpose x', y' to x, y
- Transformation:
 - Translate so that (x_0, y_0) moves to $(0, 0)$ of x, y
 - Rotate x' axis to x axis
- $R(-\theta) \cdot T(-x_0, -y_0)$



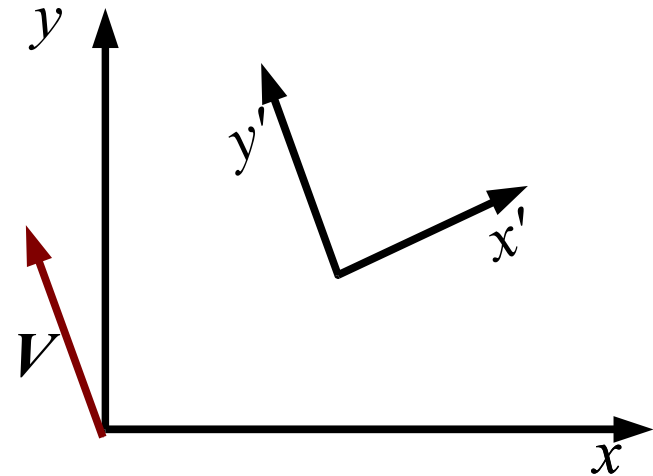
-
- Alternate method:
Specify a vector V for positive
 y' axis:

unit vector in the y' direction:

$$\vec{v} = \frac{\vec{V}}{|\vec{V}|} = (v_x, v_y)$$

unit vector in the x' direction, rotate \vec{v} 90

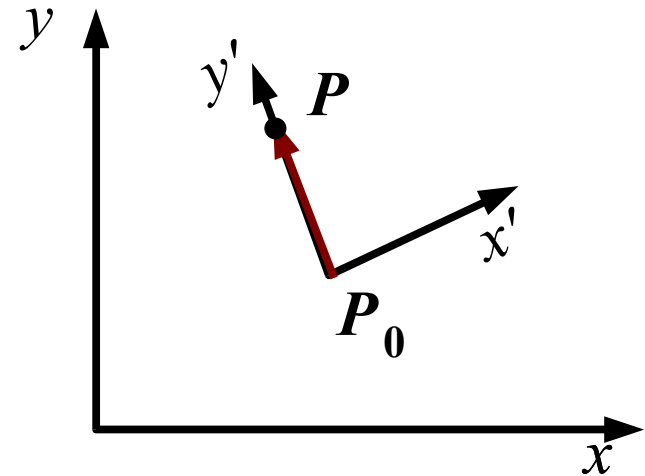
$$\vec{u} = (v_y, -v_x) = (u_x, u_y)$$



-
- Elements of any rotation matrix can be expressed as elements of a set of orthogonal unit vectors:

$$R = \begin{bmatrix} u_x & u_y & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} v_y & -v_x & 0 \\ v_x & v_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{V} = \frac{P - P_0}{|P - P_0|}$$



- Example:

$$P_0 = (2, 1) \quad P = (3.5, 3)$$

$$\vec{v} = \frac{P - P_0}{|P - P_0|} = \frac{(1.5, 2)}{\sqrt{1.5^2 + 2^2}} = \left(\frac{1.5}{2.5}, \frac{2}{2.5}\right) = (0.6, 0.8)$$

$$\vec{u} = (0.8, -0.6)$$

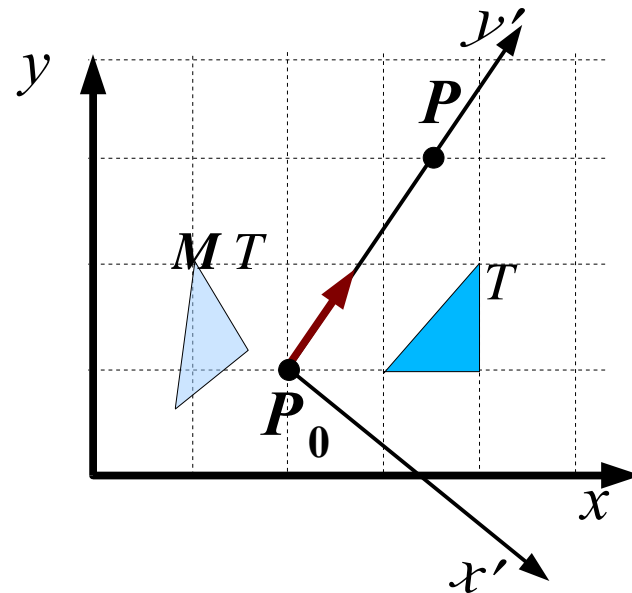
$$M = R(\vec{u}, \vec{v}) \cdot T(-2, -1) =$$

$$\begin{bmatrix} 0.8 & -0.6 & 0 \\ 0.6 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.8 & -0.6 & -1 \\ 0.6 & 0.8 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Let triangle T is defined as column vectors :

$$\begin{bmatrix} 3 & 4 & 4 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M \cdot T = \begin{bmatrix} 0.8 & 1 & 1.6 \\ 0.6 & 2 & 1.2 \\ 1 & 1 & 1 \end{bmatrix}$$



Affine Transformations

- Coordinations of the form:

$$x' = a_{xx}x + a_{xy}y + b_x$$

$$y' = a_{yx}x + a_{yy}y + b_y$$

- Translation, rotation, scaling, reflection, shear. Any affine transformation can be expressed as the combination of these.
- Rotation, translation, reflection:
preserve angles, lengths, parallel lines

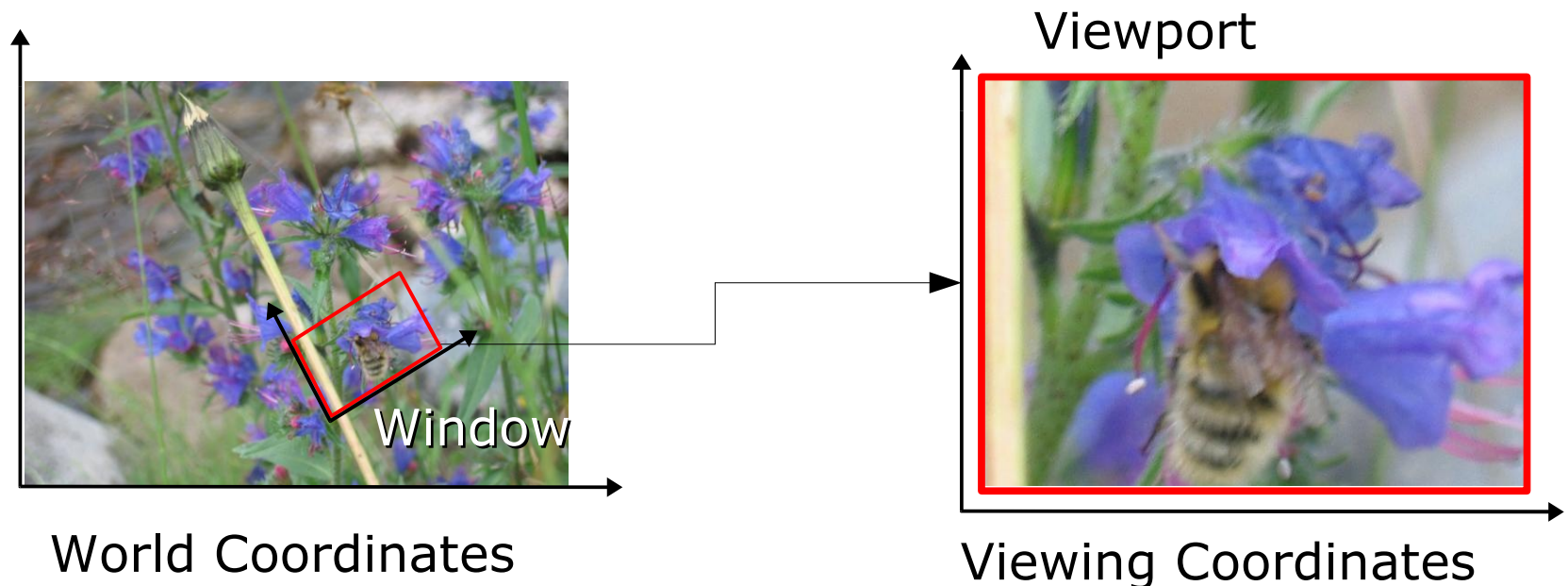
2D Viewing

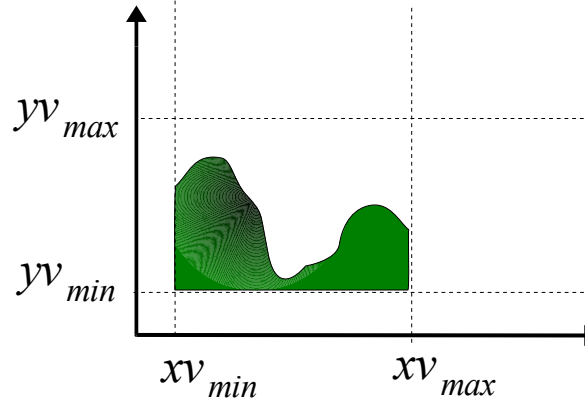
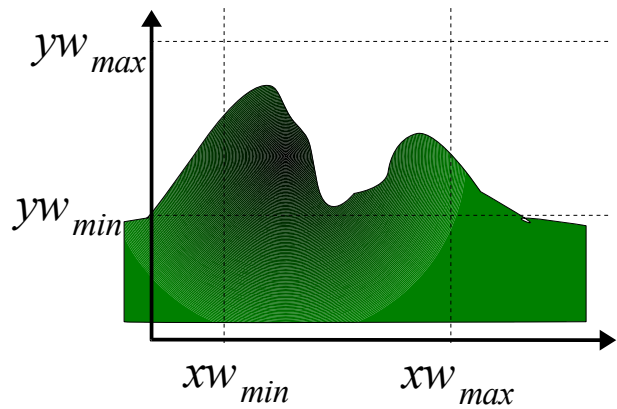
- World coordinates to Viewing coordinates

- Window to Viewport.

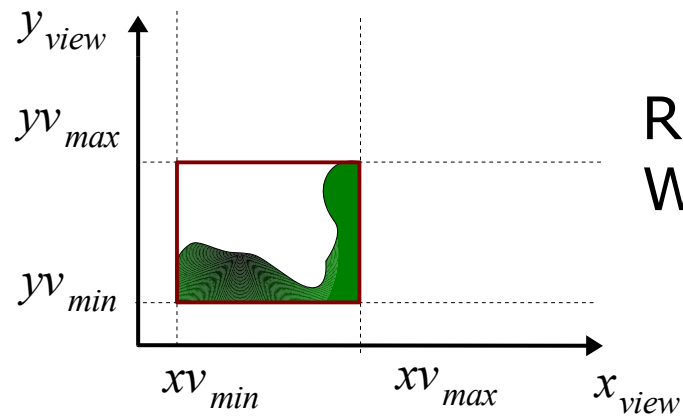
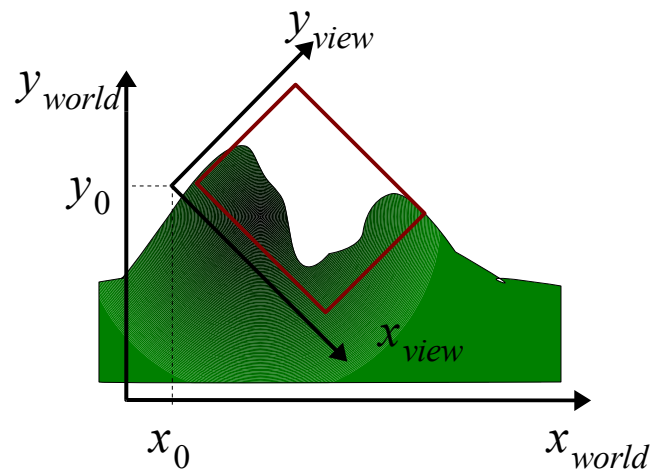
Window: A region of the scene selected for viewing

Viewport: A region on display device for mapping to window



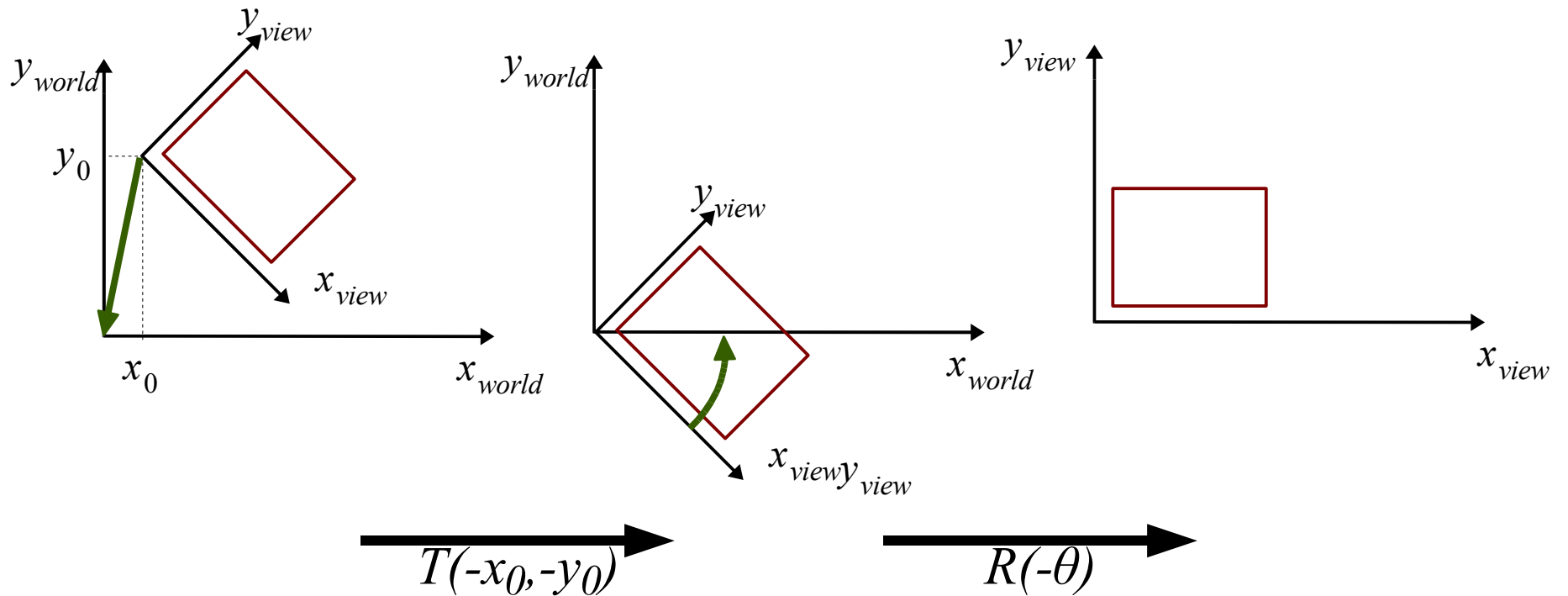


Rectangular Window



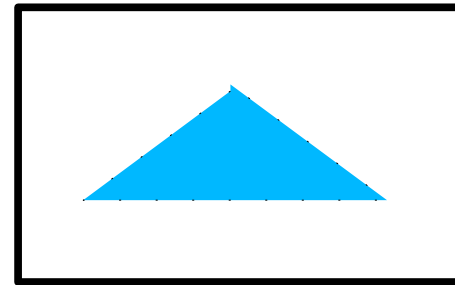
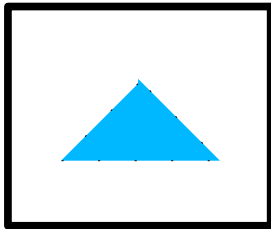
Rotated Window

- Window coordinates to viewing coordinates:



- $M_{WC,VC} = R \cdot T$

-
- Coordinate transformation: Different sizes and/or height width ratios?



- For any point:

$$\frac{x^v - x^v_{min}}{x^v_{max} - x^v_{min}} = \frac{x^w - x^w_{min}}{x^w_{max} - x^w_{min}}$$
$$\frac{y^v - y^v_{min}}{y^v_{max} - y^v_{min}} = \frac{y^w - y^w_{min}}{y^w_{max} - y^w_{min}}$$

should hold.

$$xv = xv_{min} + (xw - xw_{min}) \frac{(xv_{max} - xv_{min})}{xw_{max} - xw_{min}}$$
$$yv = yv_{min} + (yw - yw_{min}) \frac{(yv_{max} - yv_{min})}{yw_{max} - yw_{min}}$$

$$s_x = \frac{(xv_{max} - xv_{min})}{xw_{max} - xw_{min}}$$
$$s_y = \frac{(yv_{max} - yv_{min})}{yw_{max} - yw_{min}}$$

- Scaling over the fixed point:

$$\mathcal{S}(xw_{min}, yw_{min}, s_x, s_y)$$

- Translate window minimum to viewing minimum

$$\mathcal{T}(xv_{min} - xw_{min}, yv_{min} - yw_{min})$$

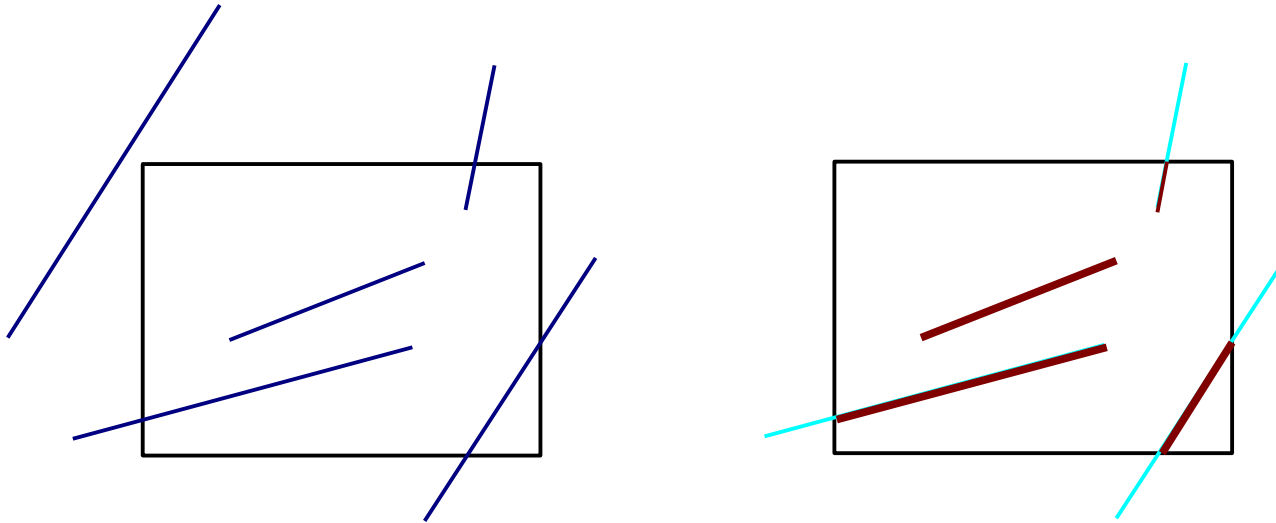
Clipping

- Clipping: identifying the parts of the objects that will be inside of the window.
- Scan converting all pixels and ignoring the pixels outside of the window is the easiest way. Is it a brilliant choice?
- Point clipping:

$$xw_{min} \leq x \leq xw_{max}$$

$$yw_{min} \leq y \leq yw_{max}$$

Line Clipping



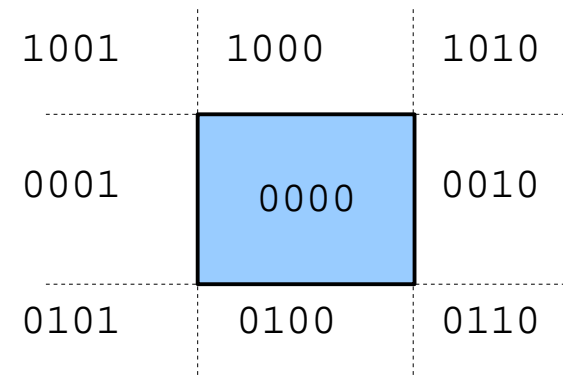
- For each line determine the intersection with boundaries. Parametric line equations:

$$\begin{aligned}x &= x_1 + u(x_2 - x_1), \\ y &= y_1 + u(y_2 - y_1), \quad 0 \leq u \leq 1\end{aligned}$$

Find u for all boundary lines. Expensive.

-
- Cohen-Shutherland Line Clipping:
Based on determination of completely invisible line segments.

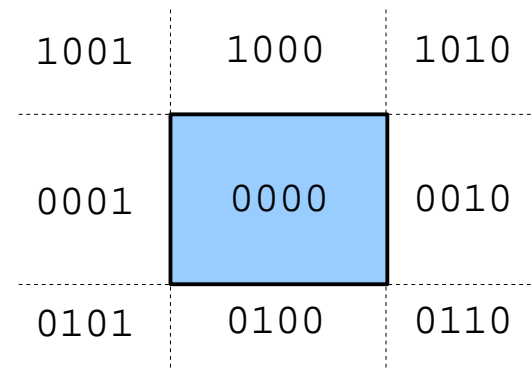
- Test bits for the point:
 - bit 1: left of the left border
 - bit 2: right of the right border
 - bit 3: below the bottom border
 - bit 4: above the top border



- When bits are defined for start and end point of the line segment, a single bitwise operation defines the visibility of the line segment. How?

-
- `#define bits(x,y) ((x<xmin)|(x>xmax)<<1|(y<ymin)<<2|(y>ymin)<<3)`

```
b1=bits(x1,y1) ; b2=bits(x2,y2);  
if (b1 == 0 && b2==0) {  
    /* both end points inside  
       trivial accept          */  
} else if (b1 & b2) {  
    /* line is completely outside  
       ignore it              */  
} else {  
    /* needs further calculation*/  
}
```



- For all borders find the transition from outside to inside
- Use the bits calculated before for outside tests

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

repeat

for $border = \{LEFT, RIGHT, BOTTOM, TOP\}$

if $\neg(bits(x_1, y_1) \vee bits(x_2, y_2))$

Both inside, terminate accept

else if $bits(x_1, y_1) \wedge bits(x_2, y_2)$

Same region, terminate reject

if $border = LEFT \vee border = RIGHT$

$$y_p = y_1 + m(x_{border} - x_1), x_p = x_{border}$$

else

$$x_p = x_1 + \frac{(y_{border} - y_1)}{m}, y_p = y_{border}$$

if $bordertest(x_1, y_1)$

$$x_1 = x_p; y_1 = y_p$$

else if $bordertest(x_2, y_2)$

$$x_2 = x_p; y_2 = y_p$$

- Example:**

$$\text{bits}(0,0)=0101 \quad \text{bits}(8,5)=0010$$

$$\text{bits}(0,0) \wedge \text{bits}(8,5)=0000$$

$$m = \frac{(5-0)}{(8-0)} = \frac{5}{8}$$

LEFT: $y_p = y_1 + \frac{5}{8}(1-0) = \frac{5}{8}$

$$\text{LEFT}(P_1) \rightarrow P_1' = (1, 5/8)$$

RIGHT: $y_p = y_1 + \frac{5}{8}(7-1) = \frac{5}{8} + \frac{5}{8}6 = \frac{35}{8}$

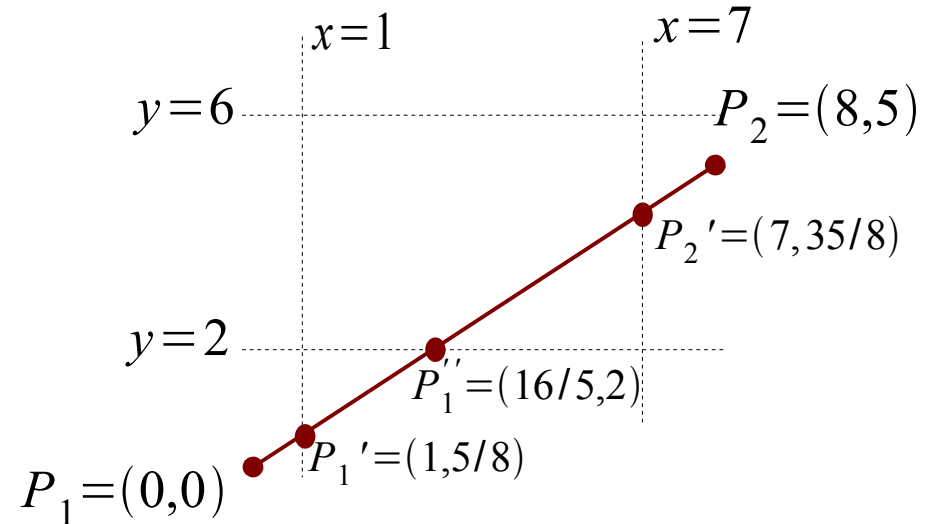
$$\text{RIGHT}(P_2) \rightarrow P_2' = (7, 35/8)$$

BOTTOM: $x_p = 1 + (2 - \frac{5}{8}) \frac{8}{5} = 1 + \frac{16}{5} - 1 = \frac{16}{5}$

$$\text{BOTTOM}(P_1') \rightarrow P_1'' = (16/5, 2)$$

P_1'' inside, P_2' inside, so terminate

$$L = (16/5, 2) \text{ to } (7, 35/8)$$



- Liang-Barsky Line clipping

- accelerate evaluation of parametric equations

$$xw_{min} \leq x_1 + u \Delta x \leq xw_{max}$$

$$yw_{min} \leq y_1 + u \Delta y \leq yw_{max}, \quad 0 \leq u \leq 1$$

rewrite these inequalities in the form:

$$u p_k \leq q_k, \quad k=1,2,3,4$$

$$p_1 = -\Delta x, \quad q_1 = x_1 - xw_{min}$$

$$p_2 = \Delta x, \quad q_2 = xw_{max} - x_1$$

$$p_3 = -\Delta y, \quad q_3 = y_1 - yw_{min}$$

$$p_4 = \Delta y, \quad q_4 = yw_{max} - y_1$$

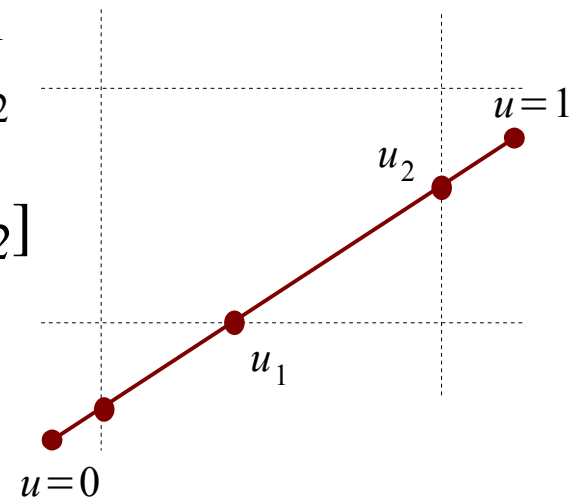
$$u = \frac{q_k}{p_k}$$

$$u p_k \leq q_k, \quad 0 \leq u \leq 1$$

if $p_k < 0$, outside to inside transition, candidate for u_1

if $p_k > 0$, inside to outside transition, candidate for u_2

- Problem: find the clipping interval $[u_1, u_2]$
- If $p_k < 0$ $u_1 = \max(u_1, p_k/q_k)$
- If $p_k > 0$ $u_2 = \min(u_2, p_k/q_k)$



$$u_1=0, u_2=1$$

for $k=1,2,3,4$

find p_k

$$\text{if } p_k < 0 \quad r_k = \frac{q_k}{p_k}$$

$$u_1 = \max(u_1, r_k)$$

$$\text{else if } p_k > 0 \quad r_k = \frac{q_k}{p_k}$$

$$u_2 = \min(u_2, r_k)$$

else parallel line, treat specially

if $u_1 > u_2$

line is outside, totally reject

$$x_2 = x_1 + u_2 \Delta x, y_2 = y_1 + u_2 \Delta y$$

$$x_1 = x_1 + u_1 \Delta x, y_1 = y_1 + u_1 \Delta y$$

- **Example:**

$$\Delta x = 8, \Delta y = 5$$

$$k=1$$

$$p_1 = -8, \quad q_1 = 0 - 1 = -1, \quad r_1 = \frac{1}{8}$$

$$u_1 = \max\left\{\frac{1}{8}, 0\right\} = \frac{1}{8}$$

$$k=2$$

$$p_2 = 8, \quad q_2 = 7 - 0 = 7, \quad r_2 = \frac{7}{8}$$

$$u_2 = \min\left\{\frac{7}{8}, 1\right\} = \frac{7}{8}$$

$$k=3$$

$$p_3 = -5, \quad q_3 = 0 - 2 = -2, \quad r_3 = \frac{-2}{-5}$$

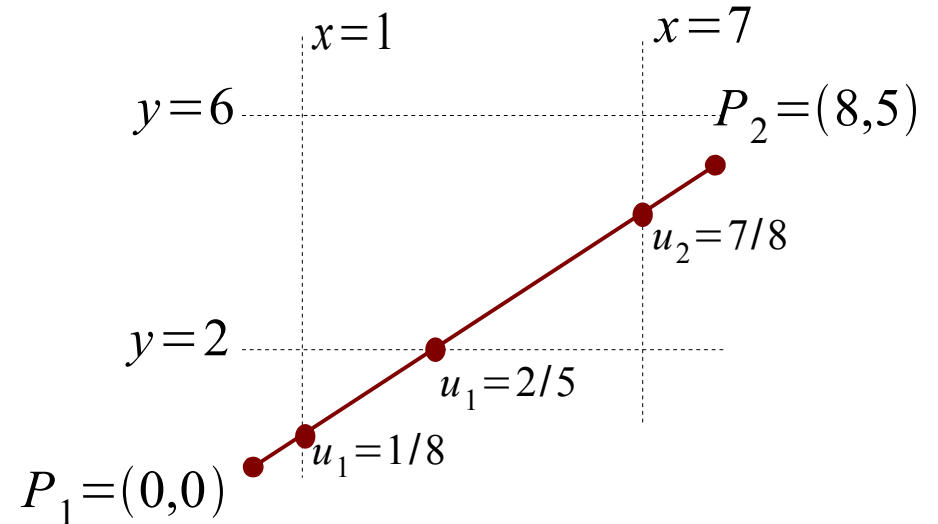
$$u_1 = \max\left\{\frac{1}{8}, \frac{2}{5}\right\} = \frac{2}{5}$$

$$P_1 = (0 + 2/5 \cdot 8, 0 + 2/5 \cdot 5) = (16/5, 2) \quad P_2 = (0 + 7/8 \cdot 8, 0 + 7/8 \cdot 5) = (7, 35/8)$$

$$k=4$$

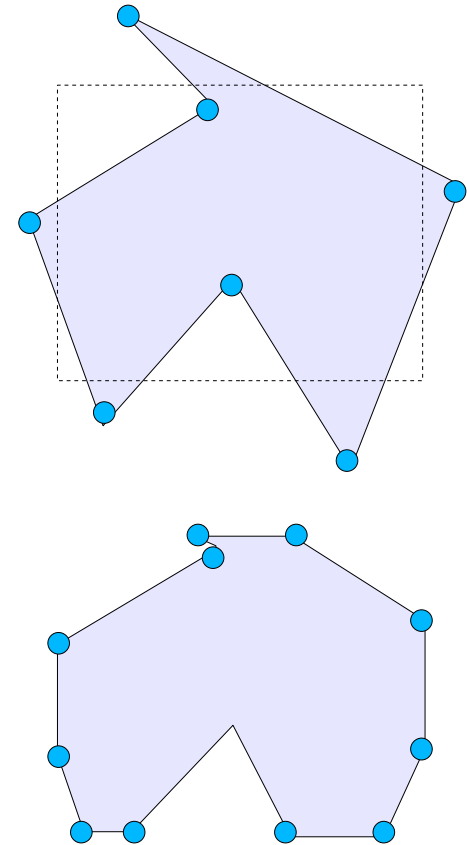
$$p_4 = 5, \quad q_4 = 6 - 0 = 6, \quad r_4 = \frac{6}{5}$$

$$u_2 = \min\left\{\frac{6}{5}, \frac{7}{8}\right\} = \frac{7}{8}$$



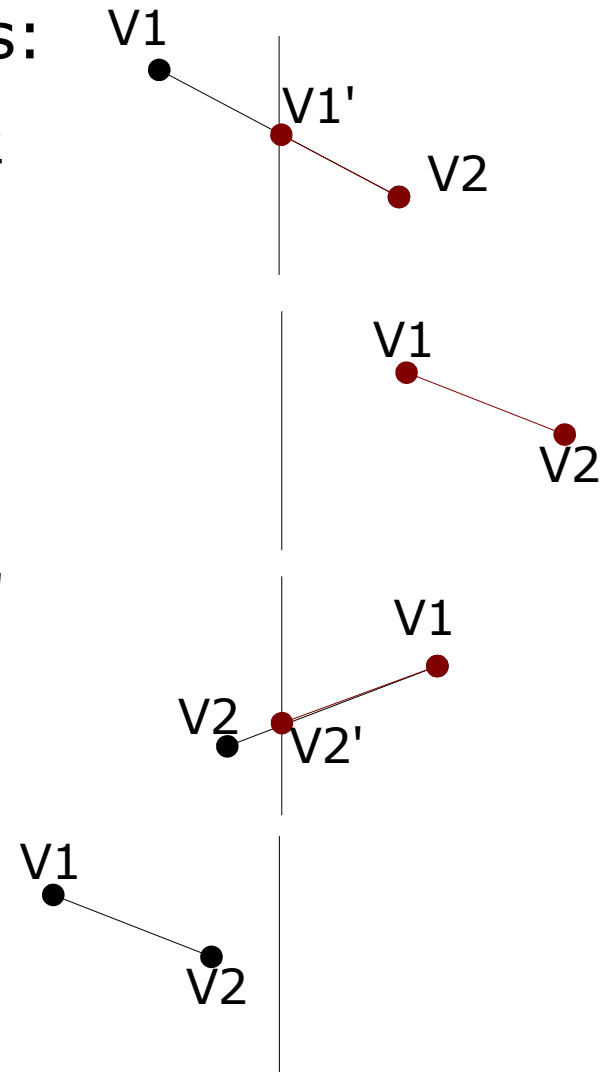
Polygon Clipping

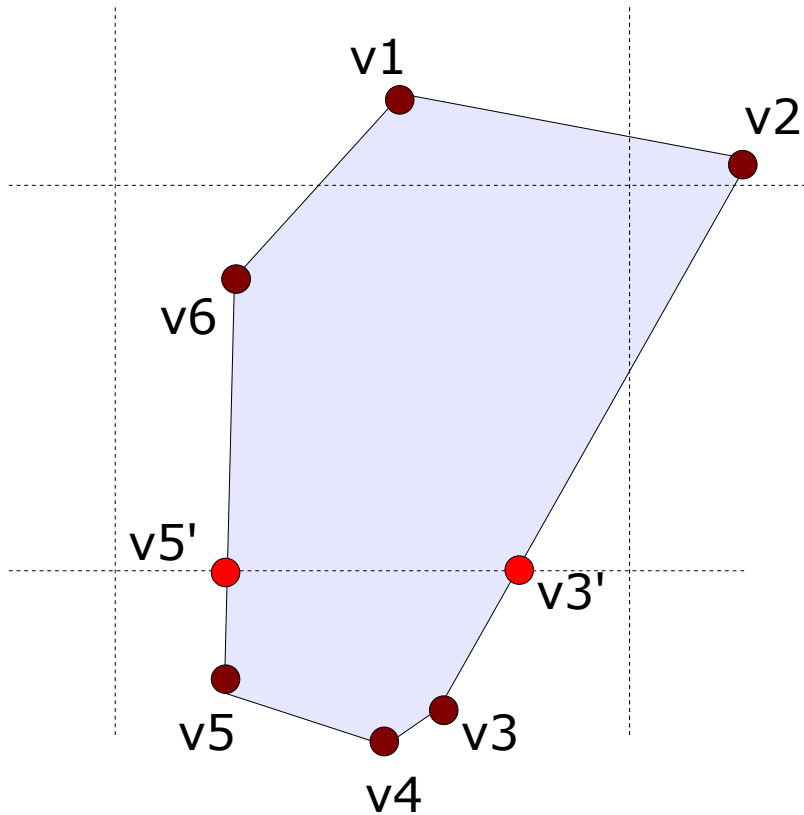
- Find the vertices of the new polygon(s) inside the window.
- Sutherland-Hodgeman Polygon Clipping:
Check each edge of the polygon against all window boundaries.
Modify the vertices based on transitions.



Shutherland-Hodgeman Polygon Clipping

- Traverse edges for borders; 4 cases:
 - V1 outside, V2 inside: take V1' and V2
 - V1 inside, V2 inside: take V1 and V2
 - V1 inside, V2 outside: take V1 and V2'
 - V1 outside, V2 outside: take none

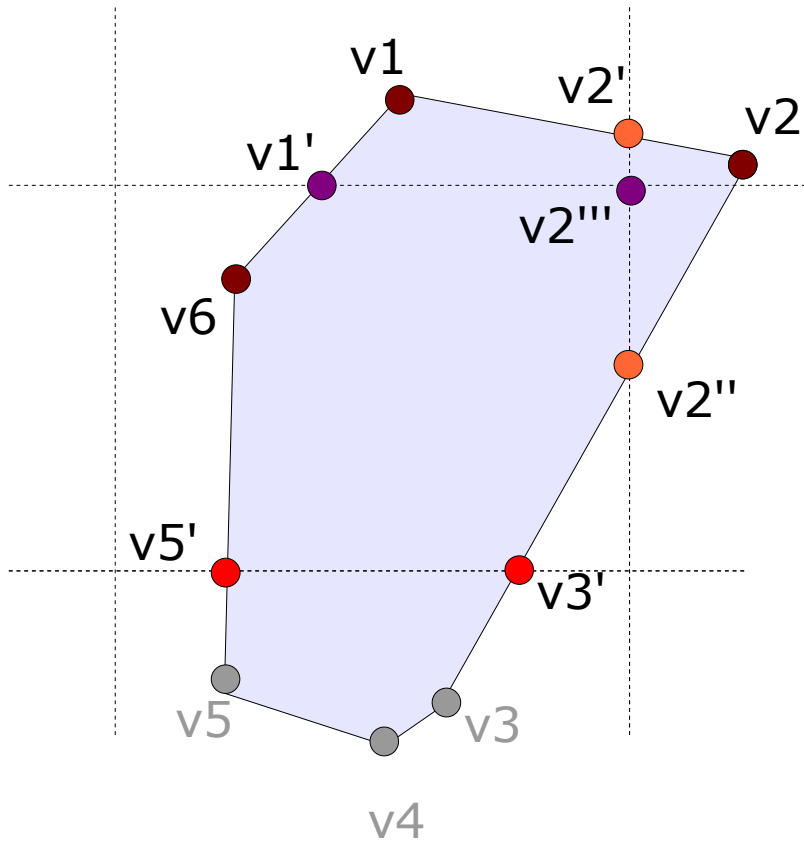




- Left border:

v1 v2	both inside	v1 v2
v2 v3	both inside	v2 v3
.....	" "
v1,v2,v3,v4v5,v6,v1		
- Bottom Border:

v1 v2	both inside	v1 v2
v2 v3	v2 i, v3 o	v2 v3'
v3 v4	both outside	none
v4 v5	both outside	none
v5 v6	v5 o, v6 i	v5' v6
v6 v1	both inside	v6 v1
v1,v2,v3',v5',v6,v1		



- $v1, v2, v3', v5', v6, v1$

- Right border:

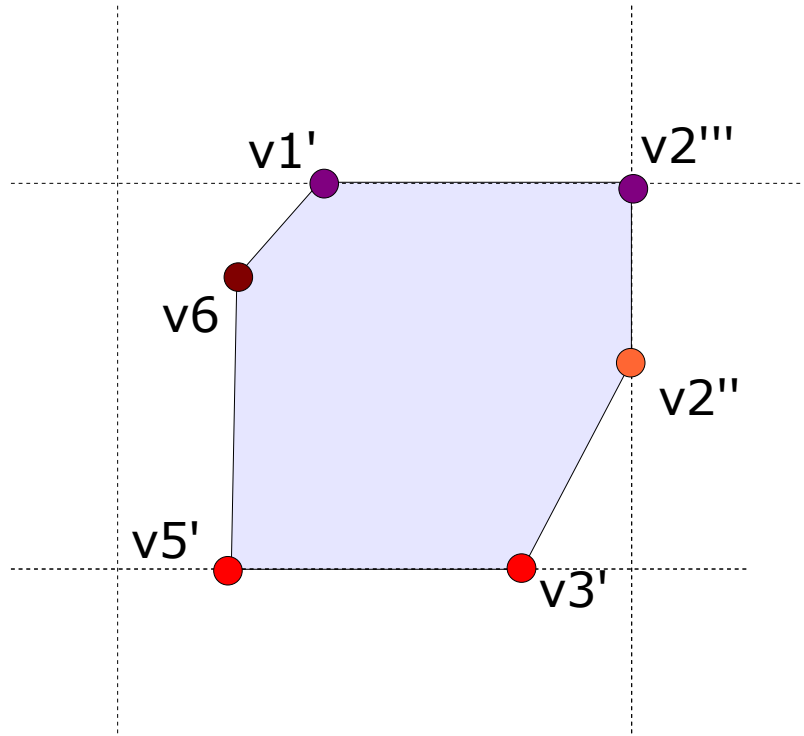
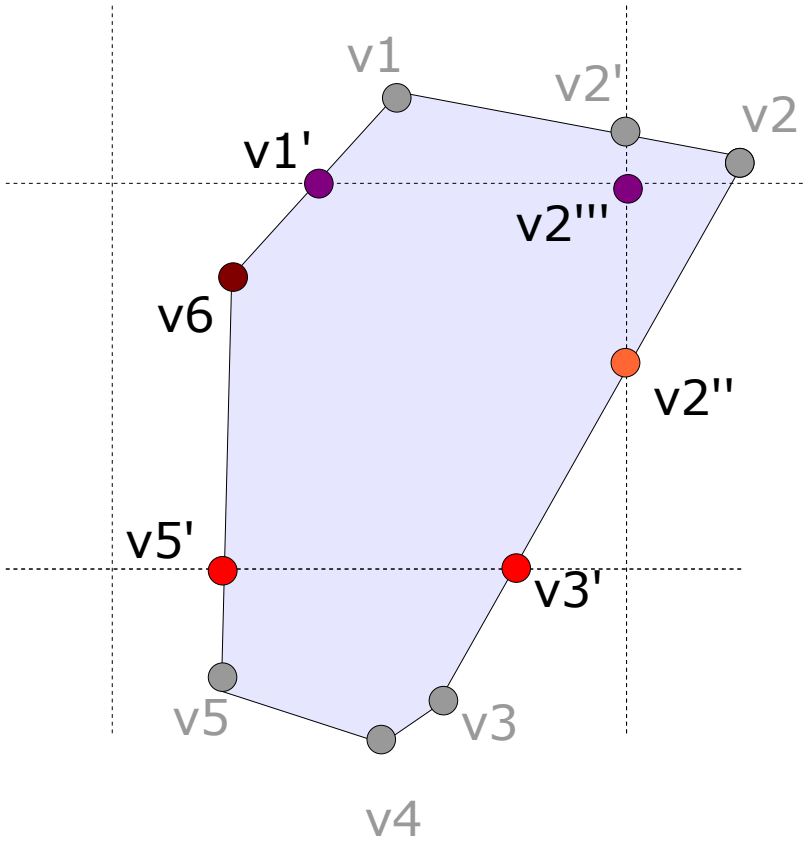
$v1 v2$	$v1 i, v2 o$	$v1 v2'$
$v2 v3'$	$v2 o, v3' i$	$v2'' v3'$
$v3' v5'$	both inside	$v3' v5'$
$v5' v6$	both inside	$v5' v6$
$v6 v1$	both inside	$v6 v1$

$v1, v2', v2'', v3', v5', v6, v1$

- Top Border:

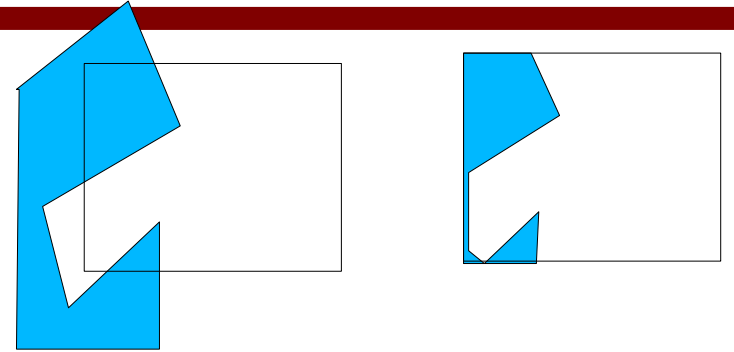
$v1 v2'$	both outside	none
$v2' v2''$	$v2' o, v2'' i$	$v2''' v2''$
$v2'' v3'$	both inside	$v2'' v3'$
$v3' v5'$	both inside	$v3' v5'$
$v5' v6$	both inside	$v5' v6$
$v6 v1$	$v6 i, v1 o$	$v6 v1'$

$v2''', v2'', v3', v5', v6, v1'$



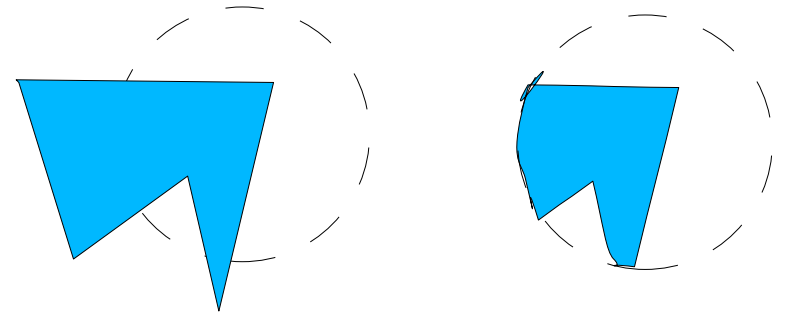
Other Issues in Clipping

- Problem in Shutherland-Hodges. Weiler-Atherton has a solution



- Clipping other shapes: Circle, Ellipse, Curves.

- Clipping a shape against another shape



- Clipping the exteriors.

