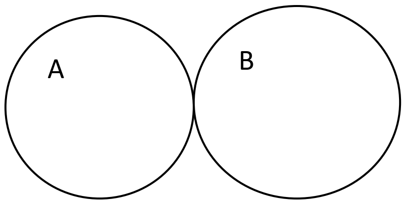
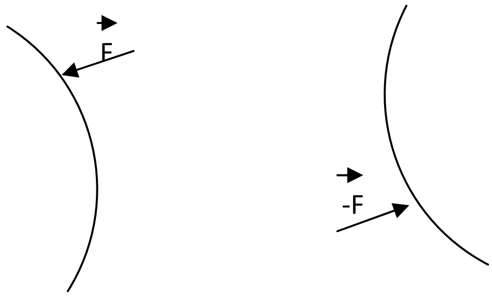
AEE 361 Lecture 3 Notes 07.10.2009

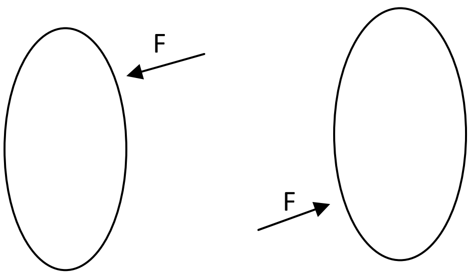
Prepared By: (01) M. Sinan HORASAN
(02) Cem Pekardan

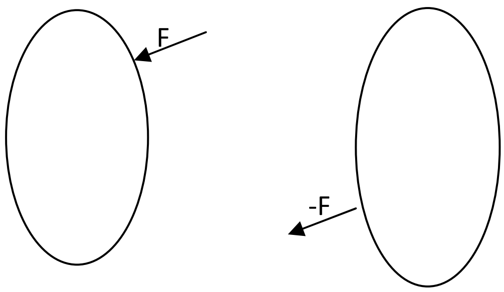
References: Ruina and Pratap, 2009; Bisplinghoff et al.; Ugural and Fenster

Correct FBD's

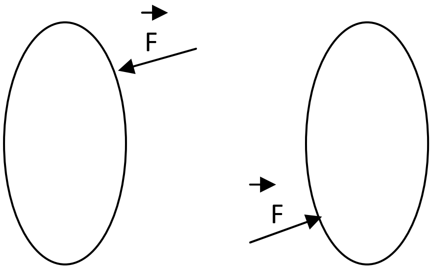


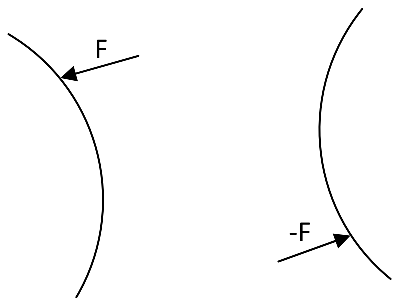


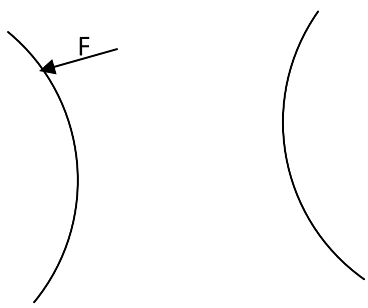


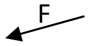


Incorrect FBDs









Concept of Stress at a point

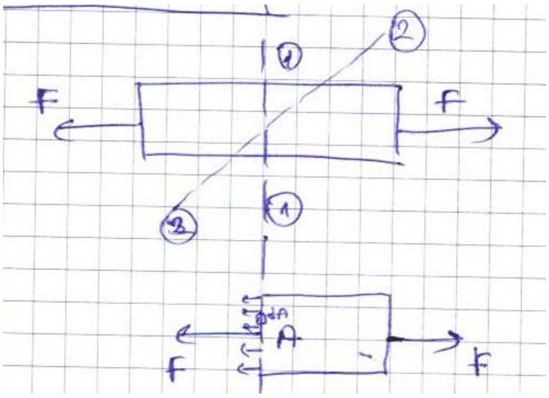


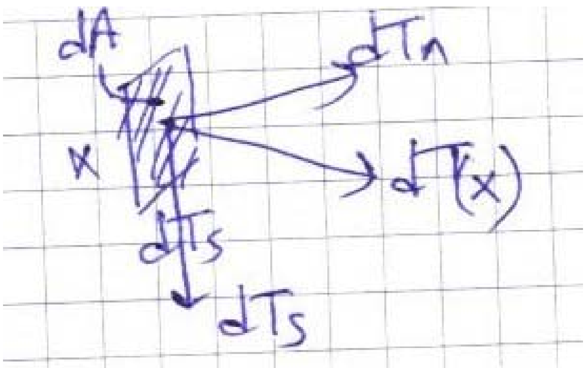
Figure 1

Stress at a point concept can be analyzed by cutting the body through an arbitrary section 1-1 in figure 1. Since the body is in equilibrium there has to be a force F independent of the where the body is cut. Moreover, stress distribution over the area becomes;

$$p = \frac{F}{A} \qquad\qquad p_x = \frac{dF}{dA}$$

If we permit our small area (delta area) to shrink towards zero, then from physical considerations, it is assumed that following limits will occur.

Normal Stress



$$\sigma = \lim_{dA \to 0} \frac{dT_n}{dA}$$

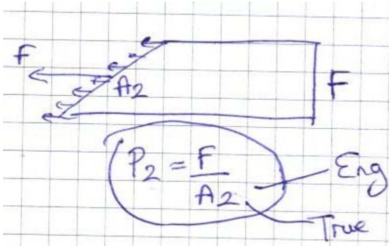
Shear Stress

$$\tau = \lim_{dA \to 0} \frac{dT_s}{dA}$$

The stress at a point

$$\overrightarrow{\sigma_A} = \lim_{dA \to 0} \frac{d\overrightarrow{T_s}}{dA}$$

For section 2-2,



$$p_2 = \frac{F}{A_2}$$

If the block is cut through the section 2-2, area changes and force distribution changes as we can see at the above formula.

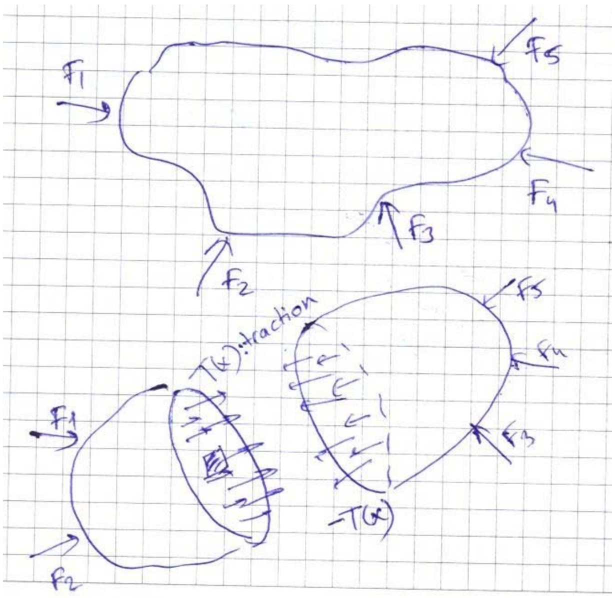


Figure 2

Newton's First Law

$$\sum F = 0$$

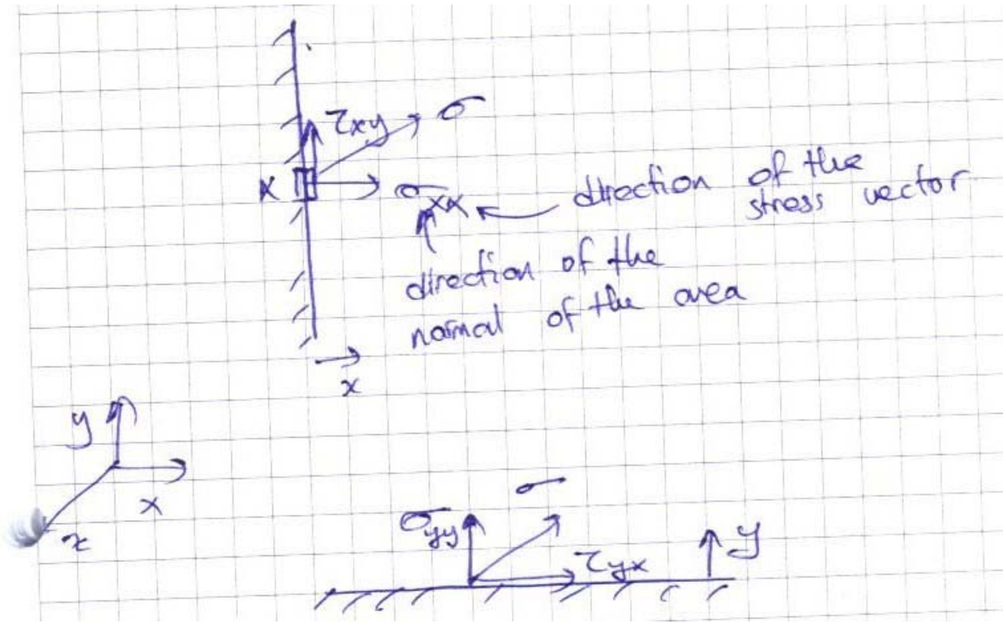
$$\sum M = 0$$

Newton's Third Law

For every action there is an equal an opposite reaction.

Stress at a point concept can also be understood by cutting the body through an arbitrary section as in figure 2. According to the Newton's first law, there has to be a force F in the opposite direction to maintain its equilibrium. Furthermore, this force should be distributed over the surface. This internal force distribution is represented with a function T(x).

The Stress Tensor at a point



x

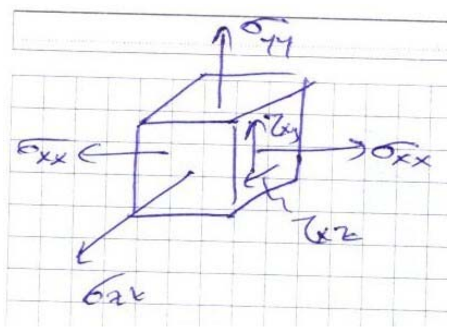
Sign Convention: By convention, first subscript of normal and shear stress symbols represents the outer normal of the area through which a stress acts. Second subscript represents the direction of the stress.

Therefore, we can say that if a stress' outer normal and direction is both positive and both negative, then this stress component is positive otherwise it is negative.

Stress Components	σ_{XX}	σ_{YY}	σ_{ZZ}
Stress Components	τ_{XY}	τ_{XZ}	τ_{YZ}
	τ_{YX}	τ_{ZX}	τ_{ZY}

Stress Tensor:

$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{pmatrix}$$



Since infinite number of planes is passing through a point in a body, in order to define the stresses at that point, we need to figure out three stress components passing through it. Assuming stresses to be uniformly distributed over the faces, stress components can be assembled in the above matrix form and each row represents the stresses acting on a certain plane.

Note: If a tensor is of zero order, it is a scalar. If it is a first order, tensor defines a vector. If a tensor is a second order, it is an array.

<u>In 2-D</u>

In 2-D or in a case with the stresses only at x and y planes is called plane stress. If a case presents only axial stresses, this situation is called <u>biaxial state</u>.

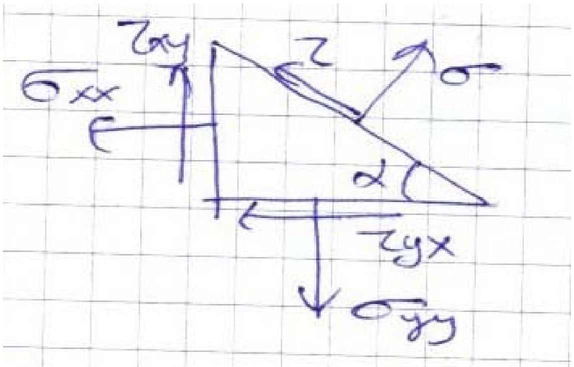
Plane Stress

$$\sigma_{zz} = 0, \tau_{xz} = \tau_{yz} = 0$$

We have 3 stress components

$$\sigma_{xx}, \sigma_{yy}, \tau_{xy}$$

Question: Find the stress at an oblique angle



Apply

$\sum F = 0$

$\sum M = 0$