AE 361 APPLIED ELASTICITY LECTURE NOTES

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References: Ugural and Fenster

TOPICS:

1-STRESS TRANSFORMATIONS

2-MOHR'S CIRCLE

DESIGN PROJECT:Design options for the Course Project and the Project groups were determined. The design of a traffic light is the main option whereas the design of a wind turbine also offers an alternative.3 main concerns take part in the design,namely:

1-geometry and dimensions

2-loads

3-stress

First, a chosen traffic light post will be analyzed for safety factor. Second, the traffice sign will be redesigned with a different material for economy, strength and weight. Choosing another material due to cost and other parameters is a determining factor in this process. In the project, some conditions like fatigue, buckling, torsion, failures at some critical points will also be considered in design.

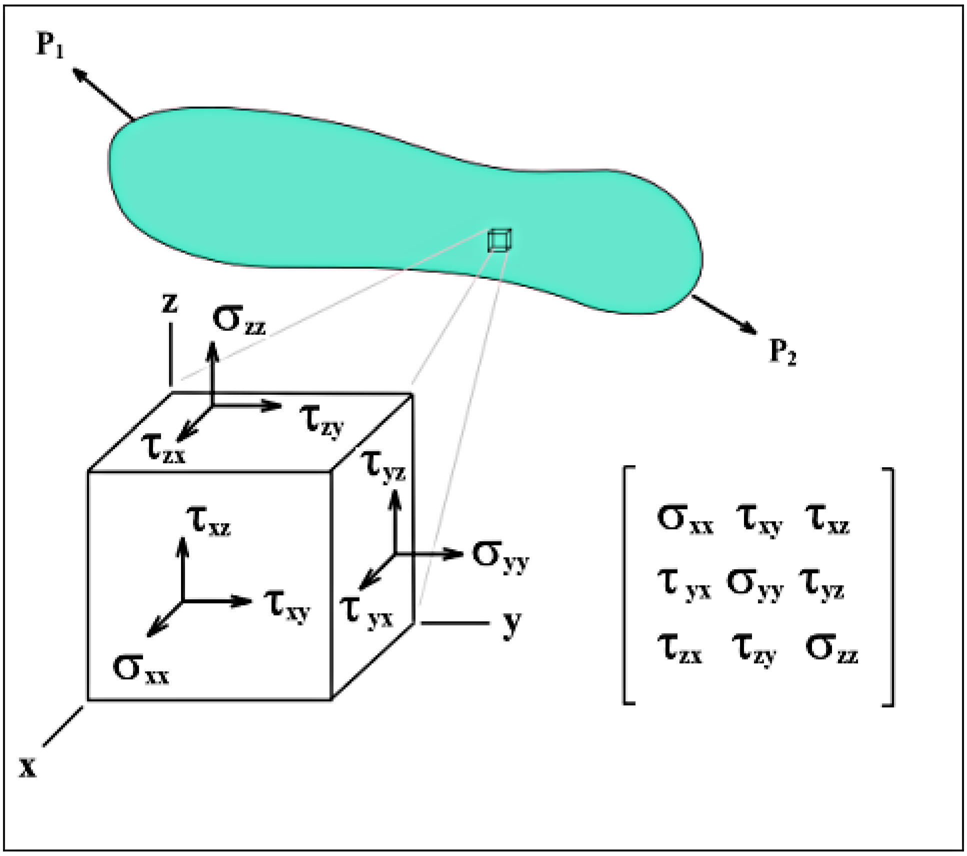
LAST TIME:

3-D Stress State Point at a Point:

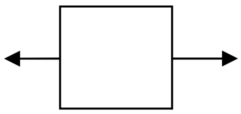
Normal Stresses	Shear Stresses
σ_{xx} σ_{yy} σ_{zz}	τ_{xy} τ_{xz} τ_{yz}

Stress Tensor: Stress Tensor components in Cartesian cordinates are as follows:

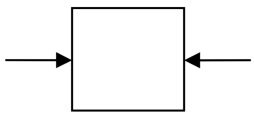
$$\underset{\sim}{\sigma} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$



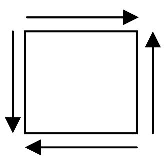
The directions according to applied loads on unit cube are as following:



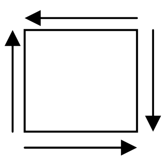
(+) tensile



(-) compressive



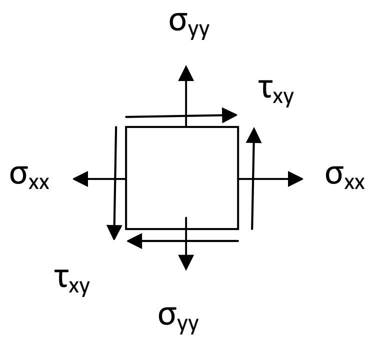
(+)

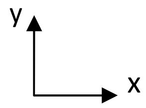


(-)

In 2-D Plane Stress Case: $\sigma_{zz} = 0$; $\tau_{xy} = 0$; $\tau_{xz} = 0$

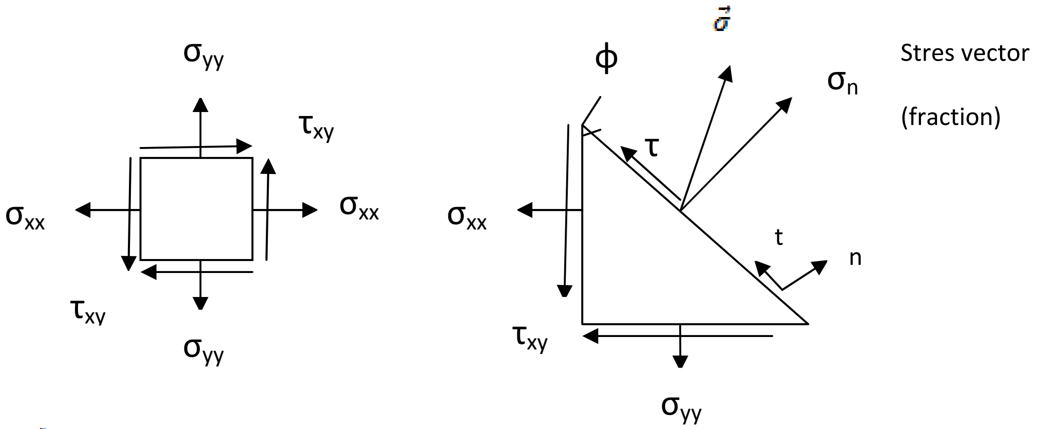
Then;





Stress Tensor= $\begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix}$

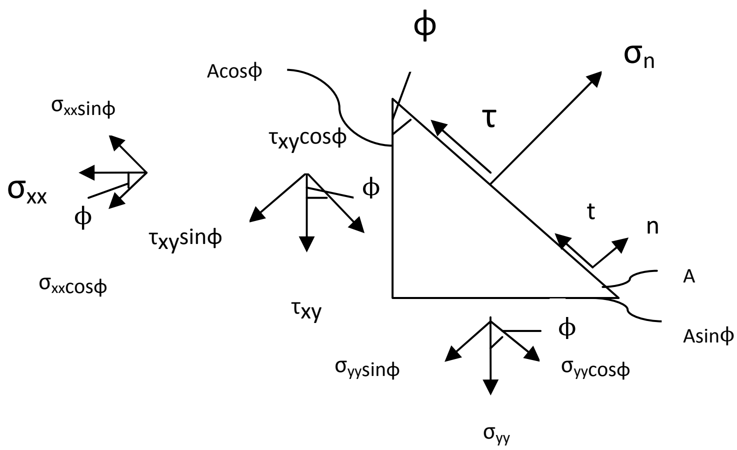
Given a 2-D stres state at a point.Find stres at an oblique angle:

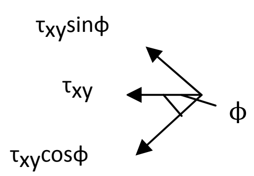


$$\sigma_n = \frac{\sigma x+\sigma y}{2} + \frac{\sigma x-\sigma y}{2}\cos(2\varphi) + \tau_{xy}\sin(2\varphi)$$

$$\tau = -\frac{\sigma x-\sigma y}{2}\sin(2\varphi) + \tau_{xy}\cos(2\varphi)$$

F.B.D. for the oblique Shape





From Newton's First Law : $\sum Fn = 0$

$$\sigma n\ A - \sigma YY sin\varphi\ A sin\varphi - \tau xy cos\varphi\ A sin\varphi - \sigma xx\ cos\varphi\ A cos\varphi - \tau xy sin\varphi\ A cos\varphi = 0$$

$$\sigma n = \sigma xx cos2\varphi + \sigma yy sin2\varphi + 2\ \tau xy\ sin\varphi\ cos\varphi$$

Rearrange equations considering following trigonometric relations;

$sin2\varphi = 2 sin\varphi cos\varphi$ $\qquad sin^2\varphi = \frac{1-cos2\varphi}{2}$

$cos2\varphi = 2cos^2\varphi - 1$ $\qquad cos^2\varphi = \frac{1+cos2\varphi}{2}$

$cos2\varphi = 1 - 2sin^2\varphi$

Then we have; $\qquad \sigma_n = \sigma_{xx}\frac{1+cos2\phi}{2} + \sigma_{yy}\frac{1-cos2\phi}{2} + \tau_{xy}\sin 2\phi$

$$\sigma_n = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2} cos2\varphi + \tau_{xy} sin2\varphi$$

Normal stress expression at an oblique angle

In order to find extrema of normal stress,we take the derivative of normal stress expression with respect to angle ϕ and equate it to zero;

$$\frac{\partial \sigma_n}{\partial \varphi} = 0$$

Then max and minimum stresses σ_1 and σ_2 (also called principal stresses):

$$\sigma_1, \sigma_2 = \frac{(\sigma_x+\sigma_y)}{2} \pm \sqrt{\left(\frac{(\sigma_x-\sigma_y)}{2}\right)^2 + \tau_{xy}{}^2}$$

The principal directions:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

Now we consider the tangential components and apply Newton'S 1st Law:

$$\sum Ft = 0$$

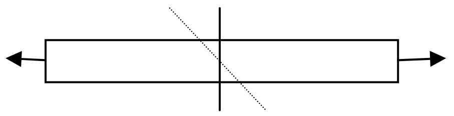
$$\tau\ A - \sigma YY cos\varphi\ A sin\varphi + \tau xy sin\varphi\ A sin\varphi + \sigma xx\ sin\varphi\ A cos\varphi - \tau xy cos\varphi\ A cos\varphi = 0$$

Rearrange and cancel areas:

$$\tau = \tau_{xy}((\cos\varphi)^2 - (\sin\varphi)^2) - \frac{(\sigma_{xx}-\sigma_{yy})}{2}\sin 2\varphi$$

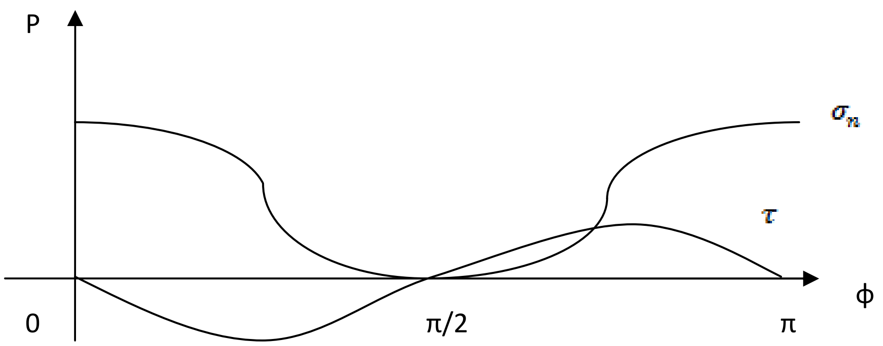
Shear stress expression at an oblique angle

EXAMPLE: Axial Loading

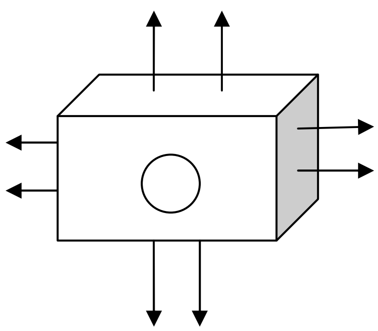


P= $\frac{P}{A}$

$$\sigma_n = \frac{P}{2} + \frac{P}{2}\cos 2\varphi \qquad \tau = -\frac{P}{2}\sin 2\varphi$$



- A material with a hole when pulled in many directions should be examined carefully along the significant points of the hole.



- We can also show that at $\phi=\phi_p$,the shear stresses associated with the principle directions are zero : $\tau = 0$.

- Also note that $\tau = \tau_p$ requires that φ is not equal to zero.

- Considering equation 2,in order to find extrema of shear stress,we take the derivative of shear stress expression with respect to angle ϕ and equate it to zero;

$$\frac{\partial \tau}{\partial \varphi} = 0 \qquad \text{Then ;} \qquad \tan 2\varphi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

- Extrema values of shear stresses can be shown as follows:

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

- Show that $\varphi_s - \varphi_p + \pi/4$
- Now consider stress equations at an oblique angle;

$$\sigma_n = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2}\cos(2\varphi) + \tau_{xy}\sin(2\varphi)$$

$$\tau = -\frac{\sigma x - \sigma y}{2}\sin(2\varphi) + \tau_{xy}\cos(2\varphi)$$

Square both,sum and drop cancelling terms and rearrange:

$$\left(\sigma - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right)^2 + \tau^2 = \left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)\left((\cos 2\varphi)^2 + (\sin 2\varphi)^2\right)$$

Trigonometrically we know $(\cos 2\varphi)^2 + (\sin 2\varphi)^2 = 1$.

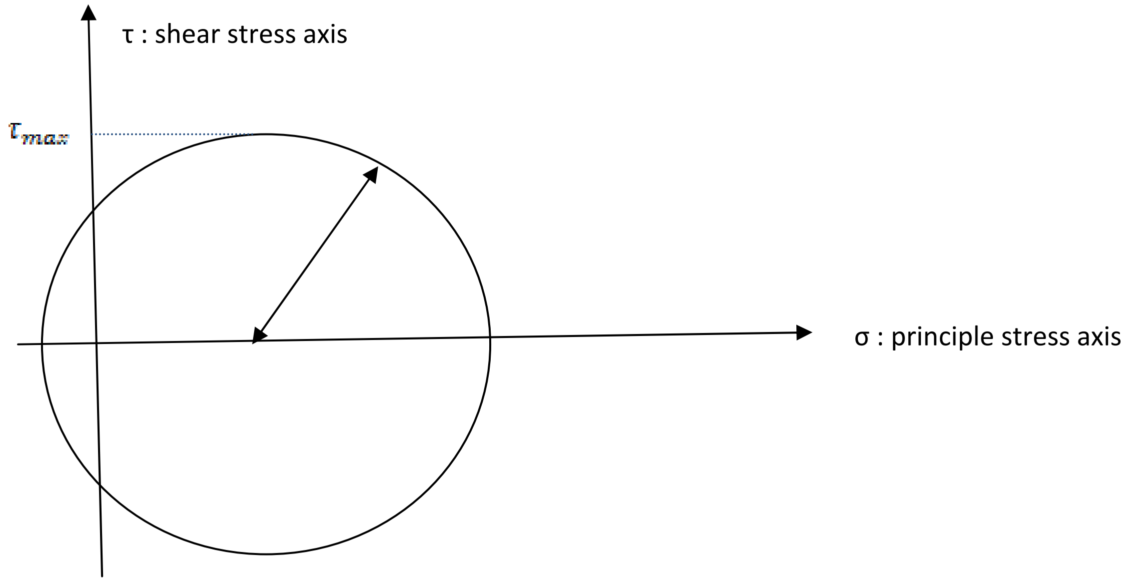
Then the equation appears to be:

$$\left(\sigma - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right)^2 + \tau^2 = \left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)$$

Now if we set $\sigma_A = \left(\frac{\sigma_x + \sigma_y}{2}\right)$; $\tau_A = 0$; $R = \sqrt{\left(\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right)}$;what the equation states becomes a circle of radius R,centered at (σ_A,0) in the σ-τ coordinates.This is called a **Mohr's Circle**. Mohr's circle helps illustrating principle stresses and transformations.This helps intrusion and visualization of these stresses for engineers.

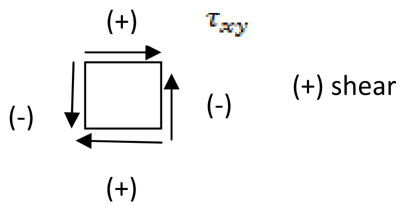
Constructing Mohr's Circle for Plane Stress:

Introduced by Otto Mohr in 1882,Mohr's Circle illustrates principle stresses and transformation via a graphical format:

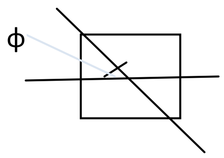


Note that Mohr's circle is symmetric along principle stress axis.

Notes: (1) In FBD (+) shear is defined on (+) surfaces .Any shear in clockwise direction in Mohr's circle is positive.

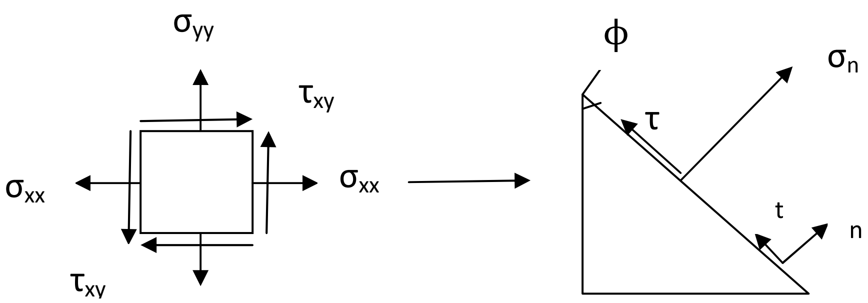


(2)The angle of rotation 2ϕ on Mohr's circle corresponds to ϕ on FBD.



PROCEDURE:

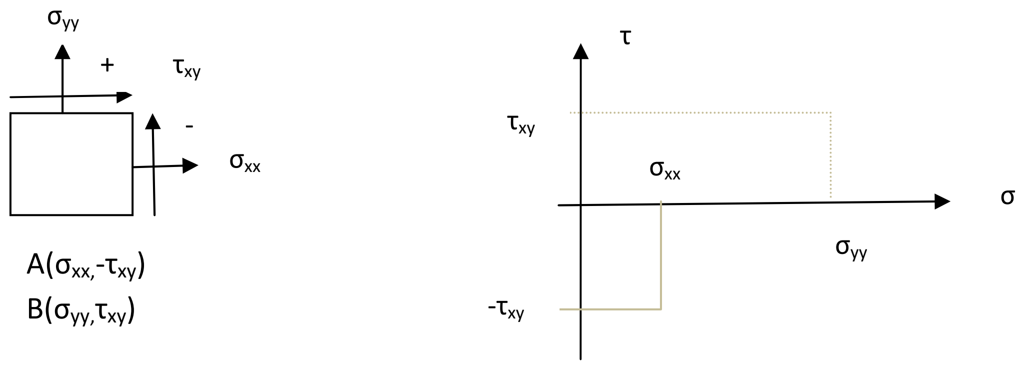
Given:



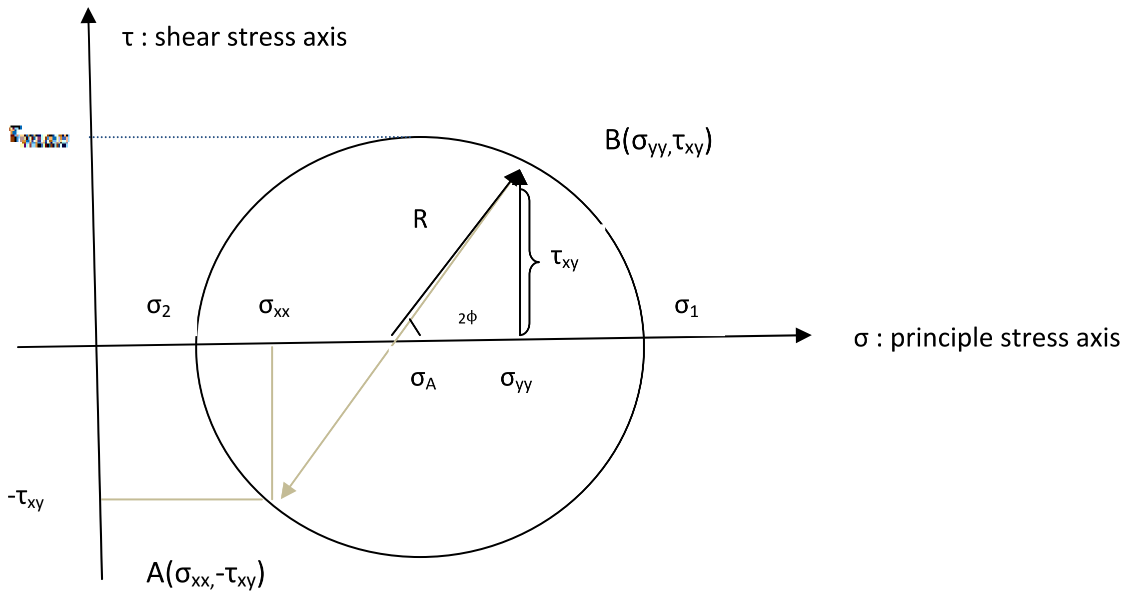
σ_{yy}

Find: τ , σ at an oblique angle ϕ.Find the principle stresses and planes.Then τ_{max} and its plane.

Step1:Draw current stres state on σ-τ plot.For two faces-x,y faces.



Step2:Determine the center and the radius of the circle by connecting the two points and draw a circle.



$$\sigma_A = \frac{\sigma_{xx}+\sigma_{yy}}{2}$$

$$R = \tau_{max} = \sqrt{{\tau_{xy}}^2 + (\sigma_{yy} - \sigma_A)^2} = \sqrt{{\tau_{xy}}^2 + \left(\sigma_{yy} - \frac{\sigma_{xx}+\sigma_{yy}}{2}\right)^2}$$

$$R = \sqrt{{\tau_{xy}}^2 + \left(\frac{\sigma_{yy}-\sigma_{xx}}{2}\right)^2}$$

Step3: Find principle stresses and principle plane.Note that they are defined as σ_1, σ_2:

$$\sigma_{1,2} = \sigma_A \mp R = \frac{\sigma_{xx}+\sigma_{yy}}{2} \mp \sqrt{{\tau_{xy}}^2 + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2}$$

$$\tan 2\varphi = \frac{\tau_{xy}}{\sigma_{yy}-\sigma_A} = \frac{2\tau_{xy}}{\sigma_{yy}-\sigma_{xx}}$$

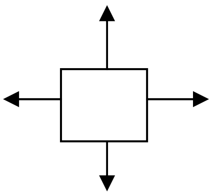
Step4: Rotate 2ϕ to find the new stres state.

Step5: Draw the FBD with the new stressses σ_{xx}' , σ_{yy}' , τ_{xy}' and the angle ϕ.

Example: $\sigma_{xx} = 2$; $\sigma_{yy} = 1$; $\tau_{xy} = 0$. Find principal stresses and their orientation.

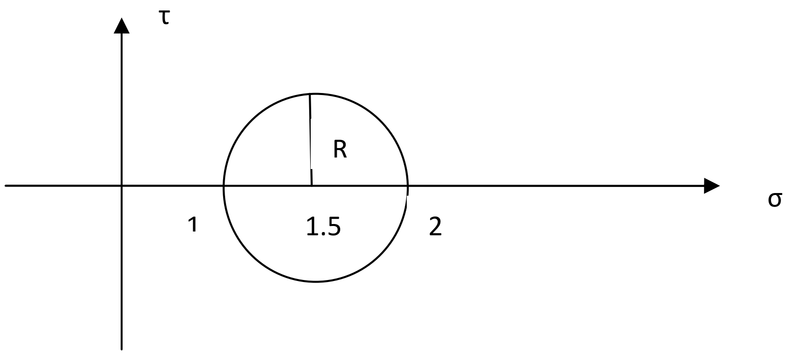
$\sigma_{yy} = 1$

Given:



$\sigma_{xx} = 2$

Then from the stress state we have the following points on Mohr's Circle: A(2,0) ; B(1,0)



$\sigma_A = \frac{\sigma_x+\sigma_y}{2}$=1,5

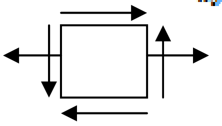
$R = \frac{\sigma_x-\sigma_y}{2}$=0,5

σ_1=2 ; σ_2=1

Example 2: $\sigma_{xx} = 2$; $\sigma_{yy} = 0$; $\tau_{xy} = 1$. Find principal stresses and their orientation.

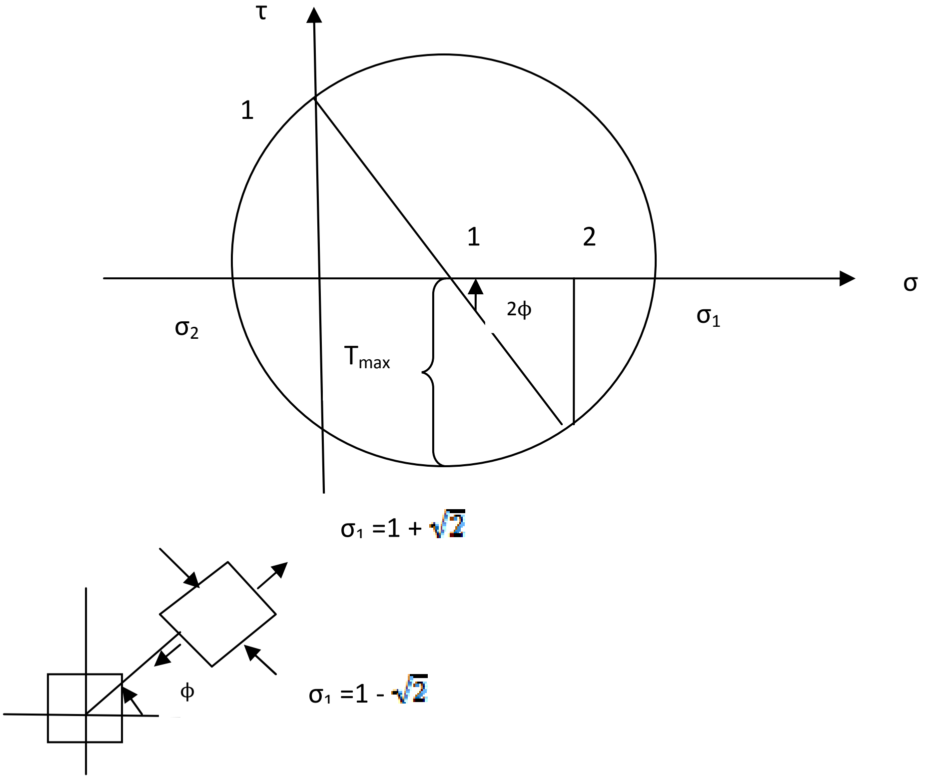
Given:

$\tau_{xy} = 1$



$\sigma_{xx} = 2$

Then from the stress state, we have the following points on Mohr's Circle: A(2,-1) ; B(0,1)



$\sigma_A = \frac{\sigma_x + \sigma_y}{2} = 1$; $2\phi = 45$

$\tau_{max} = R = \frac{\sigma_x - \sigma_y}{2} = \sqrt{2}$

$\sigma_1 = \sigma_A + R = 1 + \sqrt{2}$

$\sigma_2 = \sigma_A - R = 1 - \sqrt{2}$

GLOSSARY

Oblique: diagonal; inclined, sloping; indirect; of an angle which is not a right angle.

HISTORICAL BACKGROUND

Christian Otto Mohr (October 8, 1835 – October 2, 1918) was a German civil engineer, one of the most celebrated of the nineteenth century.

Mohr was born the son of a landowning family in Wesselburen in the Holstein region and at the age of 16 attended the Polytechnic School in Hanover.

Starting in 1855, his early working life was spent in railroad engineering for the Hanover and Oldenburg state railways, designing some famous bridges and making some of the earliest uses of steel trusses.

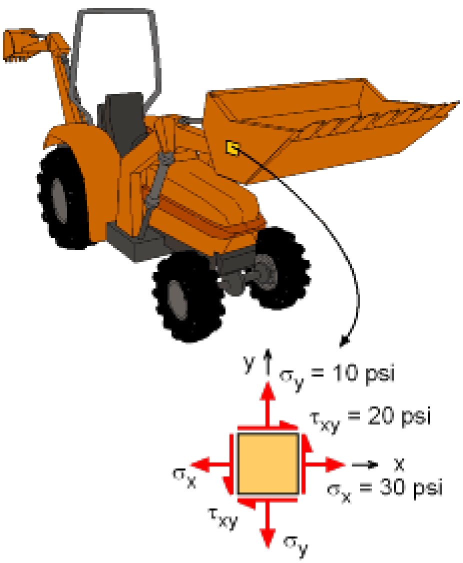
Even during his early railway years, Mohr had developed an interest in the theories of mechanics and the strength of materials. In 1867, he became professor of mechanics at Stuttgart Polytechnic, and in 1873 at Dresden Polytechnic in 1873. Mohr had a direct and unpretentious lecturing style that was popular with his students. In addition to a lone textbook, Mohr published many research papers on the theory of structures and strength of materials.

In 1874, Mohr formalized the idea of a statically determinate structure.

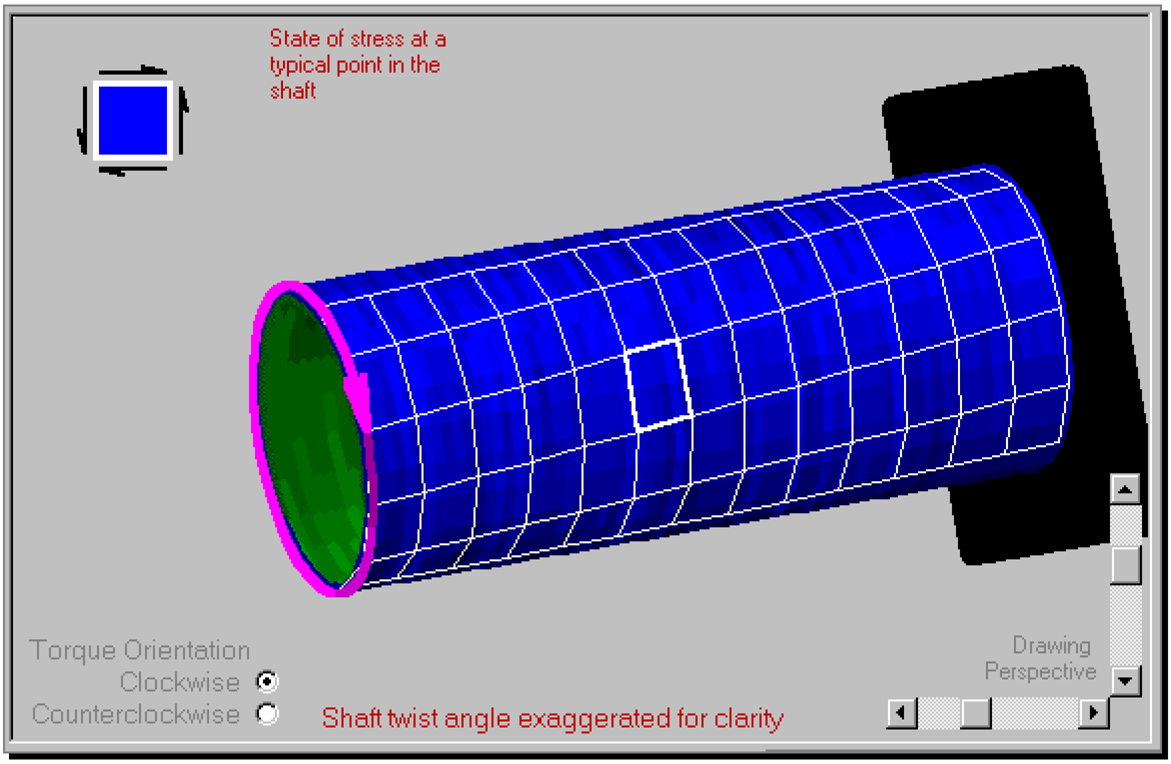
Mohr was an enthusiast for graphical tools and developed the method, for visually representing stress in three dimensions, previously proposed by Carl Culmann . In 1882, he famously developed the graphical method for analysing stress known as Mohr's circle and used it to propose an early theory of strength based on shear stress. He also developed the Williot-Mohr diagram for truss displacements and the Maxwell-Mohr method for analysing statically indeterminate structures, it can also be used to determine the displacement of truss nodes and forces acting on each member. The Maxwell-Mohr method is also referred to as the virtual force method for redundant trusses.

He retired in 1900, yet continued his scientific work in the town of Dresden until his death in 1918.

Some applications of Mohr's Circle



Stress State at the Bucket of a Tractor



Here,on the below link,you can find an online Mohr circle solver:

http://aa.nps.edu/~jones/online_tools/mohrs_circle/

References;

*Lecture Notes

*Mechanics of Materials by Beer,Johnston(2004).

*Wikipedia.

*Mukavemet, by Prof.Dr.EGOR P.POPOV,çeviri by Dr. Hilmi Demiray.