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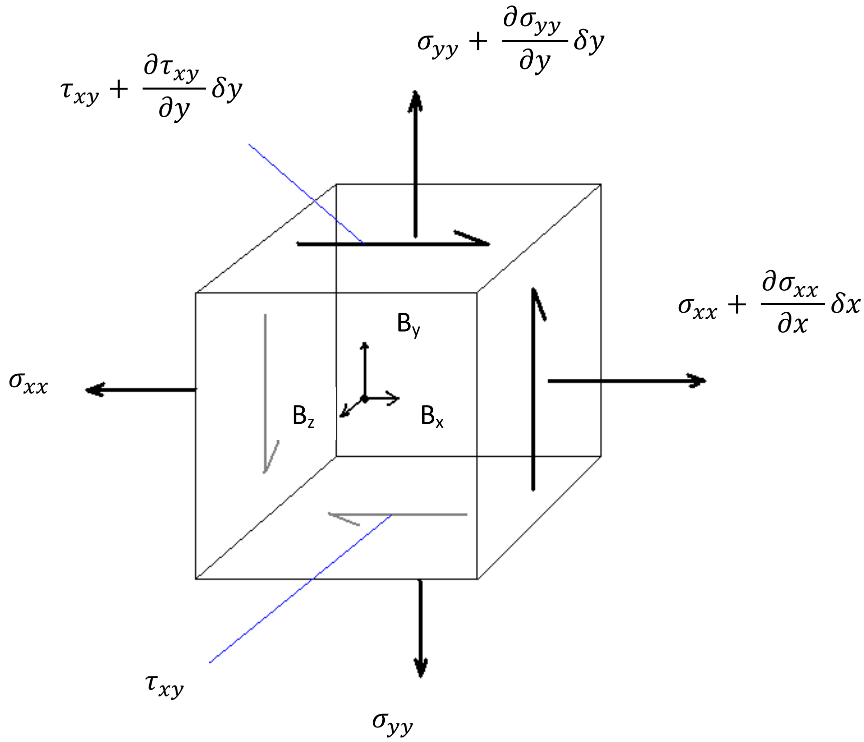
References: Juvinall(2006), Ugural and Fenster(2003)

Stress Equilibrium

Consider stress equilibrium of an infinitesimal volume element and assume that each component of stress is continuously differentiable function of x, y and z .Continuously differentiable function f of x

$$f(x+\Delta x) \approx f(x) + \frac{\partial f}{\partial x}\Delta x$$

The components of stresses on faces of an infinitesimal cube are in figure



Normal and shear stresses on cubic infinitesimal element

The subscript notation used for the nine stress components have the following meaning:

$\sigma_{\xi\eta}$: Stress on the ξ plane along η direction

Direction of stress component

Direction of the normal upon which the stress acts

In 2D, applying Newton's first law of motion, the equilibrium of forces in the x direction is

$$\sum F_x = 0$$

$$(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx)dydz - \sigma_{xx} dydz + (\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy)dxdz - \tau_{xy} dxdz + B_x dxdydz = 0$$

Which is equal to

$$\frac{\partial \sigma_{xx}}{\partial x} dxdydz + \frac{\partial \tau_{xy}}{\partial y} dxdydz + B_x dxdydz = 0$$

Cancelling out $dxdydz$ terms yields

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + B_x = 0$$

In 3D, the equation of equilibrium expands to, accordingly;

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + B_x = 0$$

Similarly $\sum F_y = 0$ and $\sum F_z = 0$ give

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + B_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + B_z = 0$$

In summary, the equations of equilibrium in 3D:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + B_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + B_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + B_z = 0$$

where B_x, B_y and B_z are body forces per unit volume.

In the case of two dimensional stresses, , the terms including "z" subscript dropping out, the above equations reduce to,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + B_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + B_y = 0$$

To remember the equations of equilibrium, following hint may be helpful;

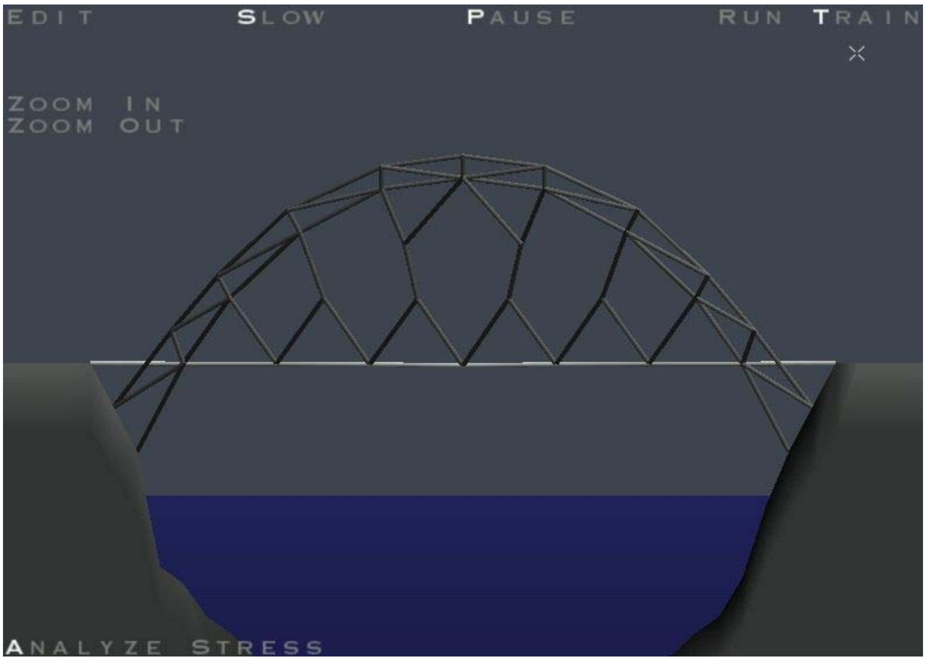
Hint for 3-D case: $$\sigma_{ij,j} = \frac{\partial \sigma_{ij}}{\partial j} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial j}$$

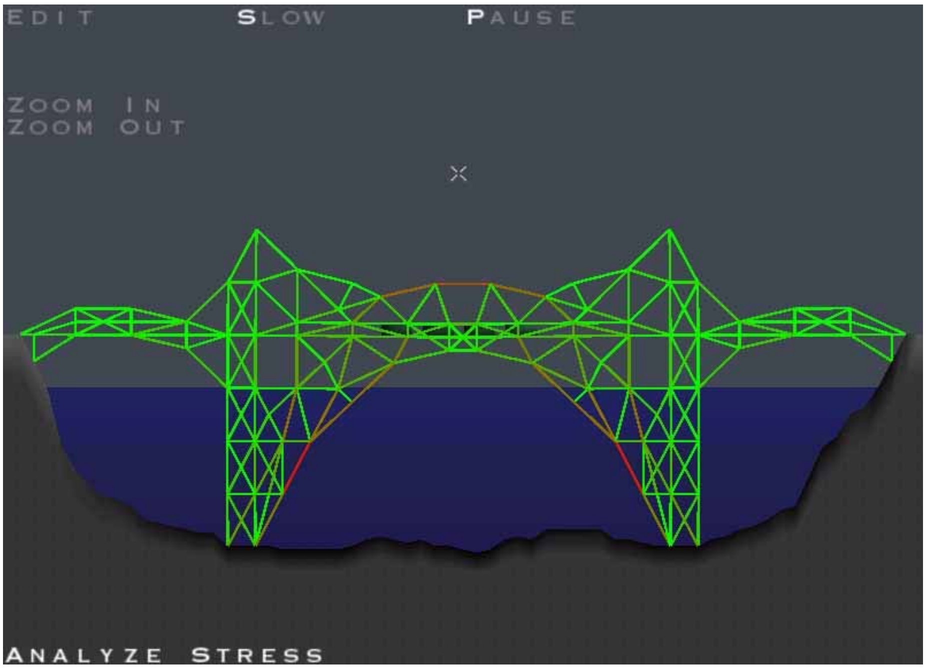
$$1 \equiv x \quad 2 \equiv y \quad 3 \equiv z$$

$$\sigma_{ij,j} + B_i = 0$$

APPLICATION

So what is *Bridge Builder*? The name says it all: you build bridges. The trick is that you need to build *structurally sound* bridges with limited resources. The goal of each level is to cross a span and then run a hefty train over your freshly-built trestle. Proper engineering is crucial: An understanding of triangles and support structures will go a long way.





Bridge Builder also does a great job communicating the status of your structure during the simulation/test mode. Bridge segments are color-coded by the amount of stress they're under. It's an elegant way to show you which areas of your bridge are robust and which are likely to fail.

Download Bridge Builder http://www.downloadtr.com/program.asp?id=3918