Date: 09.12.2009, Wednesday

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NO FLAWS IN THE MATERIAL
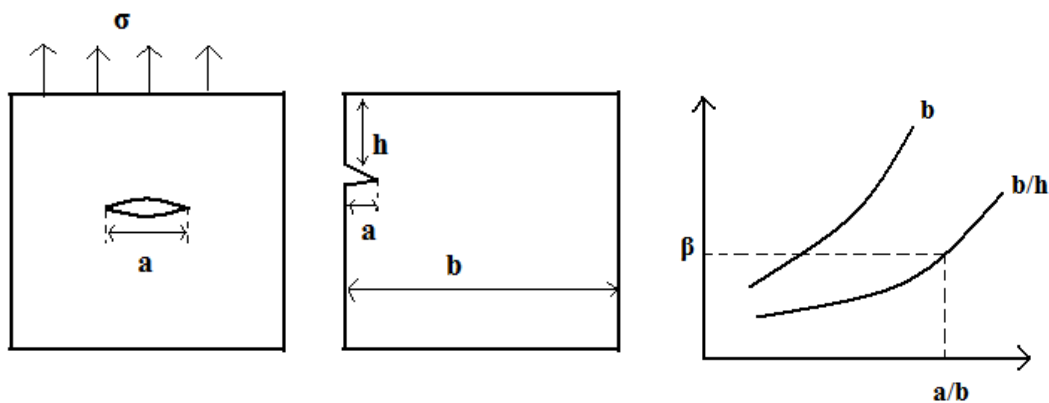
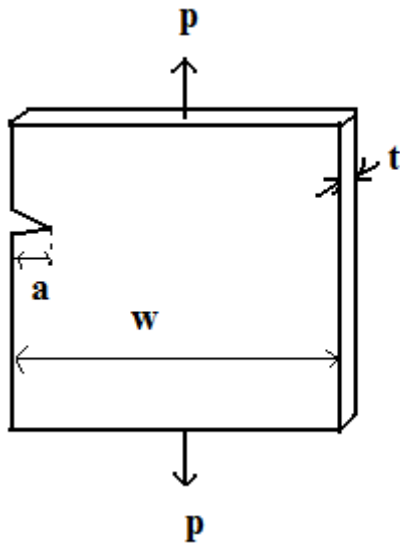**Ductile**

Brittle

2. Tresca (MSST)

3. Von Mises (MDET MOSST)

1. Max. Principle Stress Theory

4. WITH FLAWS

LEFM (Linear Elastic Fracture Mechanics)

FAILURE CRITERIA

		Elasticity(3-D)	Material(uniaxial exps)
1		σ_1	$\dfrac{S_y}{N}$
2		$\tau_{max} = \dfrac{\sigma_{max} - \sigma_{min}}{2}$	$\dfrac{S_y}{2N}$
3		$\sigma_{eff} = \dfrac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$	$\dfrac{S_y}{N}$
4		$K_I = \beta K_0 = \beta\sigma\sqrt{\pi a}$	K_{IC}

Example: (An edge crack on aircraft panel)

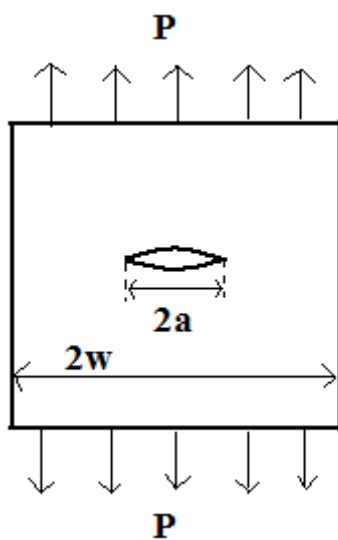w= 100mm

t= 16mm

Al 7075-1735 I

K_C= 31MPa$\sqrt{m}$

S_y= 392MPa

Estimate the maximum load P that can be applied without causing sudden fracture when on edge crack grows to length of a=20mm.

β ($\frac{a}{w}$ = 0.2)= 1.37

$K_I = 1.37\sigma_{all}\sqrt{\pi(0.02)}$ = 31 MPa$\sqrt{m}$ ➔ $\sigma_{all} = 90.27 MPa$

$P = \sigma_{all}wt$ ➔ $P = 144.4\ kN$

QUIZ

Ti-6Al-6V

$K_{IC} = 66MPa\sqrt{m}$

w= 60mm

P= 160kN

crack length= 18mm

N= 2.5

$S_y = 1149MPa$

Central transverse crack. Calculate the thickness t required to prevent sudden fracture.

β $(\frac{a}{w} = 0.15) = 1.02$

$$\frac{K_{IC}}{N} = K_I \quad \rightarrow \quad K_I = \frac{66}{2.5} = 26.4 \, MPa\sqrt{m}$$

$$K_I = \beta\sigma_{all}\sqrt{\pi a} \quad \rightarrow \quad 26.4 = 1.02 \times \sigma_{all} \times \sqrt{\pi(0.009)} \quad \rightarrow \quad \sigma_{all} = 153.924 \, MPa$$

$$\sigma_{all} < \frac{S_y}{N} \quad \text{is satisfied.}$$

$$P = \sigma_{all}2wt \quad \rightarrow \quad 160 = 153.924 \times 2(0.06) \times t \quad \rightarrow \quad t = 8.66m$$

Calculating stress intensity factors using finite element analysis

For solids with a complicated geometry, finite element methods (or boundary element methods) are the only way to calculate stress intensity factors. It is conceptually very straightforward to calculate stress intensities using finite elements – you just need to solve a routine linear elastic boundary value problem to determine the stress field in the solid, and then deduce the stress intensity factors by taking the limits given in Section 8.3.1.

Unfortunately this is easier said than done. The problem is that the stress and strain fields at a crack tip are *infinite*, and so standard finite element methods have problems calculating the stresses accurately. Two special procedures have been developed to help deal with this:
1. Special crack tip elements are available to approximate the singular strains at a crack tip;
2. Special techniques are available to calculate stress intensity factors from stresses *far* from the crack tip (where they should be accurate) instead of using the formal definition.

These methods can both give very accurate values for stress intensity factors – and can be used together to obtain the best results.

Crack tip elements

A very simple procedure can be used to approximate the strain singularity at a crack tip.
1. The solid near the crack tip must be meshed with quadratic elements (8 noded quadrilaterals or 6 noded triangles in 2D, or 20 noded bricks/10 noded tetrahedral in 3D).
2. The elements connected to the crack tip must be quadrilateral or brick elements
3. One side of each element connected to the crack tip is collapsed to make the three nodes on the side coincident, as shown in the figure to the right
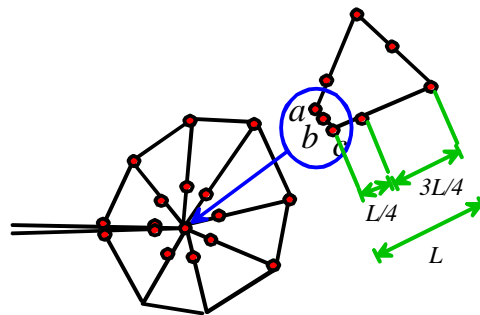
4. The mid-side nodes on the elements connected to the crack tip are shifted to ¼ point positions, as shown in the figure

5. If the coincident nodes *a,b,c* on each crack-tip element are constrained to move together, this procedure generates a $(r)^{-1/2}$ singularity in strain at the crack tip (good for linear elastic problems). If the nodes are permitted to move independently, a r^{-1} singularity in strain is produced (good for problems involving crack tip plasticity)

Calculating stress intensity factors using path independent integrals

Energy methods in fracture mechanics are discussed in detail in Section 9.4 Two crucial results emerge from this analysis

1. The 'energy release rate' for a mode I crack in a linear elastic solid with Young's modulus E and Poisson's ratio ν is related to the mode I stress intensity factor by

$$G = \frac{1-\nu^2}{E} K_I^2$$

2. The energy release rate for a crack can be calculated by evaluating the following line integral for *any* contour that starts on one crack face and ends on the other

$$G = \int_{\Gamma} (W\delta_{j1} - \sigma_{ij} \frac{\partial u_i}{\partial x_1}) m_j ds$$

where

$$W = \sigma_{ij}\varepsilon_{ij} / 2$$

is the strain energy density, σ_{ij} is the stress field, u_i is the displacement field, m_i is a unit vector normal to Γ, and the e_i basis vector is parallel to the direction of crack propagation.
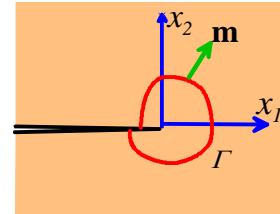
These results are ideally suited for FEM calculations. The path independent integral can be calculated for a contour far from the crack tip, where the stresses are accurate, and then the relationship between G and K_I can be used to deduce the stress intensity factors. Analogous, but rather more complex, procedures exist to extract all three components of stress intensity factor, as well as to compute stress intensity factors for 3D cracks, where the stress intensity factor is a function of position on the crack front.

References

- http://solidmechanics.org/text/Chapter9_3/Chapter9_3.htm

- Classnotes