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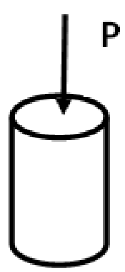
<u>ELASTICITY INSTABILITY & BUCKLING</u>

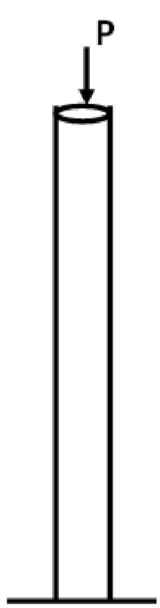
When compressive loads are applied to short thick members, failure occurs by compressive yielding or fracture.

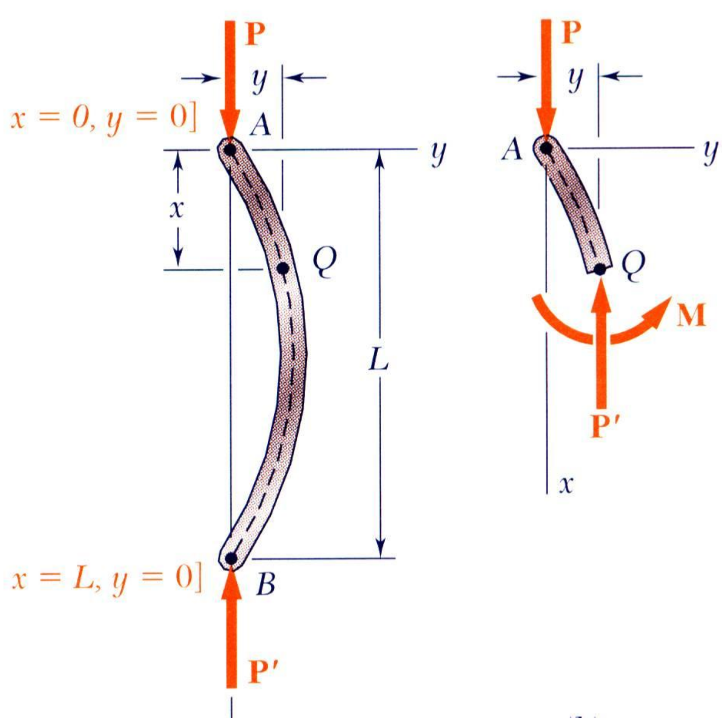


$$\frac{P}{A} = S_{yc}$$

When compressive loads are applied to long thin (slender) members, they sometimes induce sudden major changes in geometry such as bowing, wrinkling, bending or buckling.

Large deflections occur that destroy the equilibirium of the structure & produce unstable configuration that leads to collapse.





$M(x) = -P * y$

$\frac{d^2y}{dx^2} + \frac{P}{E*I} * y = 0$

$y = A * \sin(\sqrt{\frac{P}{E*I}} * x) + B * \cos(\sqrt{\frac{P}{E*I}} * x)$

<u>BC's</u>

at x=0 y(0)=0 $\Rightarrow$ B=0

at x=L y(L)=0 $\Rightarrow$ $0 = A \sin\left(\sqrt{\frac{P}{E*I}} * L\right)$

$\swarrow$ $\searrow$

A=0(trivial solution) or $\sin\left(\sqrt{\frac{P}{E*I}} * L\right) = 0$

$\sqrt{\frac{P}{E*I}} * L = n * \pi$ (n=1,2,3,......)

$P_{cr} = (\frac{n * \pi}{L})^2 * E * I$

Critical load is for n=1. (euler buckling load)

$P_{cr} = (\frac{\pi}{L})^2 * E * I$

Define:

K: radius of gyration

$I = A * k^2$

$\frac{P_{cr}}{A} = \frac{\pi^2 * E}{\left(\frac{L}{k}\right)^2}$

$\frac{L}{k} = slenderness\ ratio$

So the buckling is a function of the slenderness ratio.

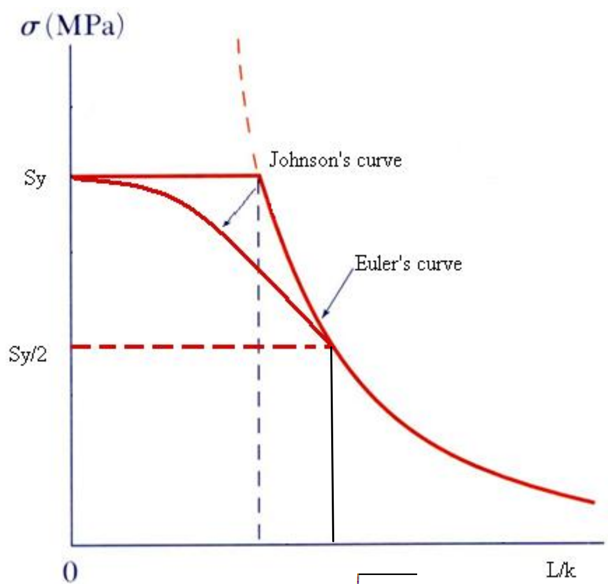
$$\frac{P_{cr}}{A} = load\ per\ unit\ area\ necessary\ to\ place\ the\ column\ in\ a\ condition\ of$$

$UNSTABLE\ EQUILIBRIUM$

(units ofstrength: strength of the column – not material)

$$y(x) = A * \sin(\sqrt{\frac{P}{E * I}} * x$$

- Euler Buckling Method
- Different End Conditions
- Intermediate length columns
- Eccentric Loading



$$\frac{L}{k} = \sqrt{\frac{\pi^2 * E}{S_y}}$$

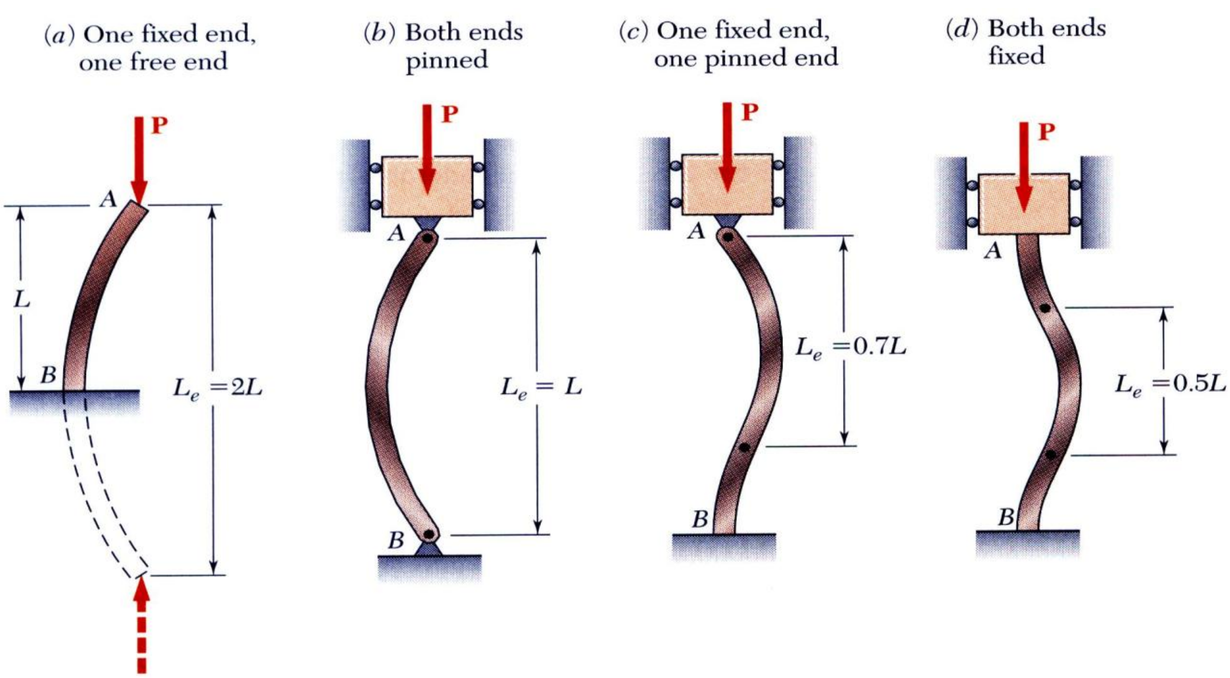
$$if\ \left(\frac{L}{k}\right)_{your\ problem} < \left(\frac{L}{k}\right)_1 = \sqrt{\frac{2*\pi^2*E*c}{S_y}}\ use\ Johnson's\ curve$$

Johnson's curve: $\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2*\pi}\right)^2 * \frac{1}{c*E} * \left(\frac{L}{k}\right)^2$

$$if\ \left(\frac{L}{k}\right)_{your problem} > \left(\frac{L}{k}\right)_1 = \sqrt{\frac{2*\pi^2*E*c}{S_y}}\ use\ Euler's\ curve$$

Euler's curve: $\frac{P_{cr}}{A} = \frac{\pi^2*E*c}{\left(\frac{L}{k}\right)^2}$ c: effect of end conditions

Different End Conditions:



$for\ case\ a$ $\frac{P_{cr}}{A} = \frac{1}{4} * \frac{\pi^2*E}{\left(\frac{L}{k}\right)^2}$

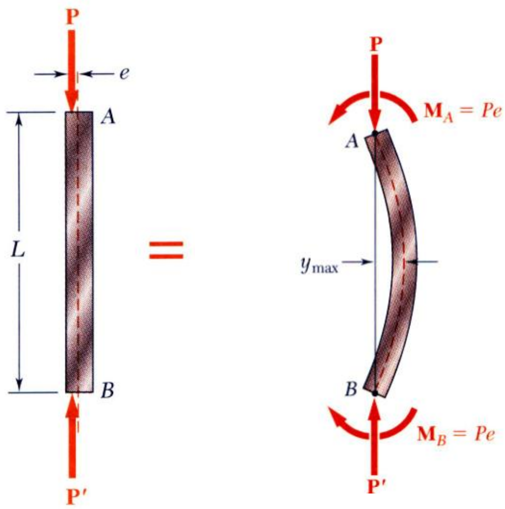
$for\ case\ b$ $\frac{P_{cr}}{A} = \frac{\pi^2*E}{\left(\frac{L}{k}\right)^2}$

$for\ case\ c$ $\frac{P_{cr}}{A} = \frac{2*\pi^2*E}{\left(\frac{L}{k}\right)^2}$

$for\ case\ d$ $\frac{P_{cr}}{A} = \frac{4*\pi^2*E}{\left(\frac{L}{k}\right)^2}$

Eccentric Loading:

- Eccentric loading is equivalent to a centric load and a couple.
- Bending occurs for any nonzero eccentricity. Question of buckling becomes whether the resulting deflection is excessive.



$$M = -P * (e + y)$$

$$\frac{d^2y}{dx^2} + \frac{P * y}{E * I} = \frac{-P * e}{E * I}$$

$$y = c_1 * \sin(\sqrt{\frac{P}{E * I}} * x) + c_2 * \cos\left(\sqrt{\frac{P}{E * I}} * x\right) - e$$

BC's:

at x=0 y=0 $\Longrightarrow$ $c_2 = e$

$$at\ x = \frac{L}{2}\ \frac{dy}{dx} = 0 \qquad c_1 = e * \tan\left(\frac{L}{2} * \sqrt{\frac{P}{E*I}}\right)$$

$$y = e * \left[\tan(\frac{L}{2} * \sqrt{\frac{P}{E * I}}) * \sin(\sqrt{\frac{P}{E * I}} * x) + \cos\left(\sqrt{\frac{P}{E * I}} * x\right) - 1\right]$$

$$Maximum\ deflection\ at\ x = \frac{L}{2}$$

$$y_{max} = e * \left[\sec\left(\frac{L}{2} * \sqrt{\frac{P}{E * I}}\right) - 1\right]$$

$$at\ x = \frac{L}{2}$$

$$M_{max} = |P * (e + y_{max})|$$

$$= P * e * \sec(\frac{L}{2} * \sqrt{\frac{P}{E * I}})$$

Maximum stress is:

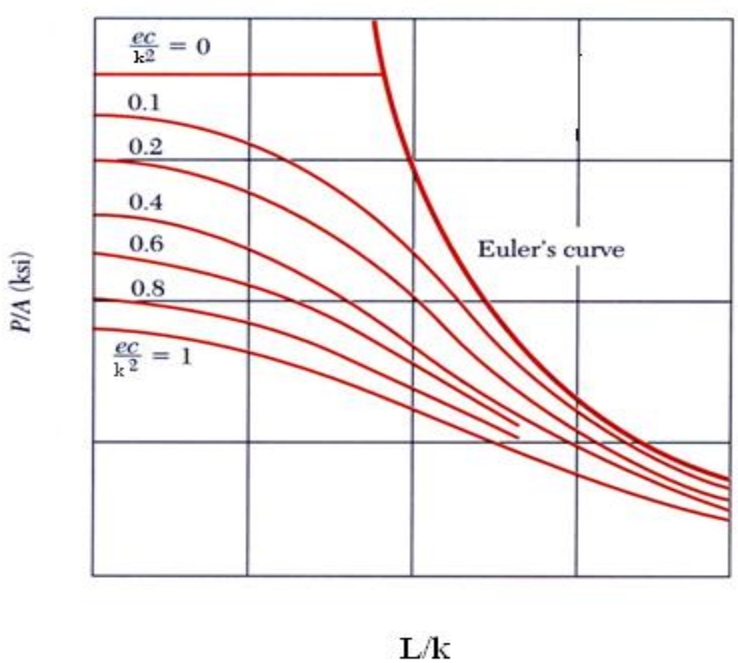
$$\sigma_{max} = \frac{P_{cr}}{A} + \frac{M_{max} * c}{I}$$

$$= \frac{P}{A} + \frac{P * e * c}{I} * \sec\left(\frac{L}{2} * \sqrt{\frac{P}{E * I}}\right)$$

$$\sigma_{max} = \frac{P}{A} * \left[1 + \frac{e * c}{k^2} * \sec(\frac{L}{2 * k} * \sqrt{\frac{P}{E * A}})\right]$$

$$\frac{e*c}{k^2} = eccentricity\ ratio$$

$$\frac{P_{cr}}{A} = \frac{S_{yc}}{1 + \left(\frac{e * c}{k^2}\right) * \sec\left(\frac{L}{2 * k} * \sec(\frac{L}{2 * k} * \sqrt{\frac{P_{cr}}{A * E}})\right)} \quad secant\ column\ formula$$



REFERENCES

- Classnotes
- Beer F.P.,Jonhnston E.R., DeWolf J.T "*Mechanics of Materials*" ,4.edition, Mc.Graw Hill