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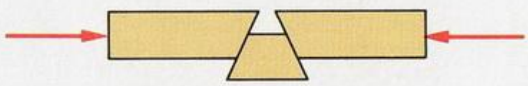
TENSION VERSUS COMPRESSION AND EULER BUCKLING

Sometimes supporting a load in tension is more problematic and more risky than supporting the same load in compression; but this doesn't have to be always true. With unreliable materials and with primitive joints, for example, tension can cause catastrophic results. However, in modern technology as well as in nature, a tension structure is often the lightest, cheapest, and safest solution. Some compressive constructions, despite of being simple and fairly reliable, such as masonry, they are heavy and consume a lot of labour. Nature does not offer many examples of compressive structures, but anthills are a rare exception. Such a design is quite unsuitable that have characteristics of living things.

The way in which materials fail under tension is much different than the way they fail under compression.

A thin rod, rope, or membrane under tension will eventually tear. Its strength is not affected by its length under static loading.



A short strut is likely to fail by local crushing—that is, by the actual fracture of the material.

A longer rod, strut, or membrane will not fail by fracturing, but by bending or buckling. This is likely to occur at a small load.

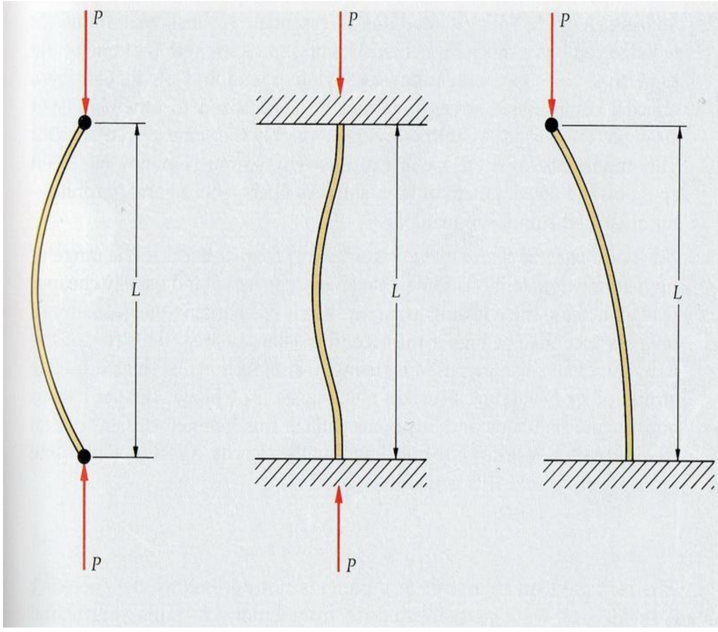
Failure in tension usually occurs by a separation of the molecules at the weakest cross section of the member. With a uniform member, the length does not make any difference to the strength in tension under static loading.

The behaviour of materials that fail under compression depends on their length. A low wall of bricks, a short material strut, or a short bone may remain straight and stable under the load until the material finally fails because of a local crushing mechanism. A longer strut or a higher wall is liable to buckle even under a light load.

The Swiss mathematician Leonard Euler (1707-1783) derived a formula for calculating the load at which a long rod will buckle when it is subjected to a compressive force along its length.

Euler interest in the problem arose rather because he had just invented the calculus of variations, and he was looking for a problem to try it out on. Someone suggested that he might attempt to calculate the height of a vertical rod that would buckle under its own weight. Euler succeeded in calculating the length of the rod, and his first results were incorporated in a book published in 1744.

Although Euler's calculations are correct as far as they go, his work antedated the concepts of the stress and strain by almost a century, and he did not have access to Young's modulus. For these reasons his work would have had limited practical application at the time.

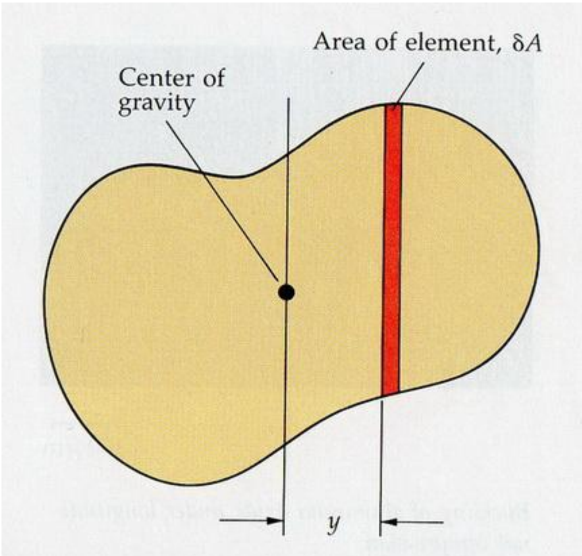


Various Euler conditions. A strut or panel that is free to hinge (left) has a buckling load P equal to $\pi^2(EI/L^2)$, where E is Young's modulus, I is the second moment of area (see page 00), and L is the length of the strut or panel. If both ends are fixed in position and direction, the buckling load is equal to $4\pi^2(EI/L^2)$. With one end encastré and the other pin-jointed and free to move sideways, P is $\pi^2(EI/4L^2)$.

In the modern technology, Euler says that a long road or column will buckle at an axial load P according to equation:

$$P = k\pi^2 \frac{EI}{L^2}$$

Where E is Young's modulus, I is the second moment of area of the cross section of the rod (sometimes wrongly called the moment of inertia), L is the length of the rod, k is a constant that depends on the end conditions-that is, on the extent to which the ends of the rod are held stationary or are free to rotate. When both ends are pin-joined (i.e., allowed to rotate), the value of k is 1. When both ends are clamped (i.e., fixed in direction and position), the value of k is 4. When one end is fixed and the other free, k is equal to ¼.



The second moment of area, I, sometimes called the moment of inertia, is expressed as $\int \delta A(Y^2)$.

Buckling load for a rod depends on:

- The unsupported length of the rod:

The length of the rod is highly effective in buckling. Longer rods may buckle with smaller forces. If the rod is not supported (having 2 free ends), all the length would be considered as unsupported length. This unsupported length change due to various fixing conditions.

-Stifness:

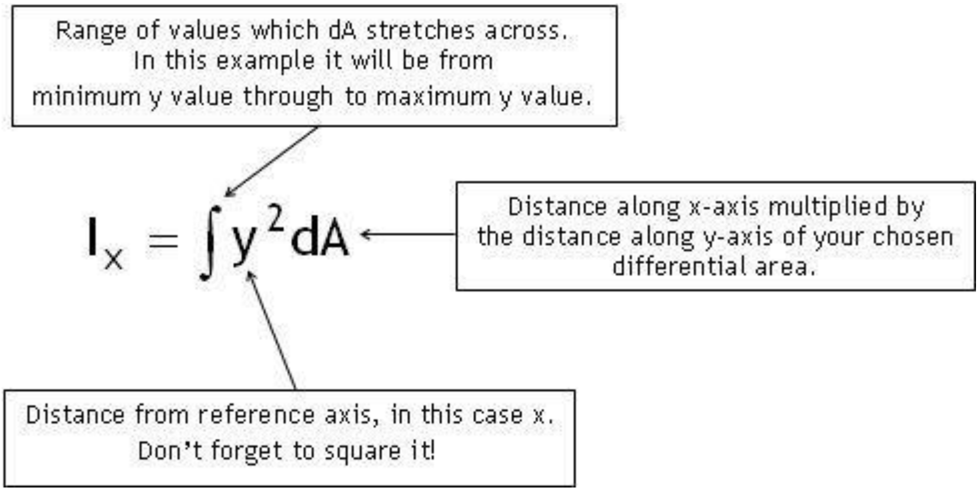
The stiffness, *k*, of a body is a measure of the resistance offered by an elastic body to deformation (bending, stretching or compression).

$$k = \frac{P}{\delta}$$

P is a steady force applied on the body
δ is the displacement produced by the force (for instance, the deflection of a beam, or the change in length of a stretched spring)

The stiffness is directly proportional to Modulus of Elasticity which is an important factor in buckling load. To prevent buckling developments of high-stiff fibers, boron, graphite is used for some advanced composite materials

-Second moment of Area of the cross-section:



It's easier to change the I value of material rather than changing the material itself. Increasing the second moment of area is possible by changing the dimensions of given rod or by changing the shape. Using hollow sections such as tubes, struts would lead to high I value. (Note that minimum of Ix; Iy should be taken considering the material may buckle in both directions). In Nature hollow bamboos and leg bones use this strategy to prevent buckling.

The buckling load of a strut or panel is not necessarily the breaking load. In fact, as we can deduce from Euler's formula the actual compressive stress at which long struts will buckle may be very small. So, when the compressive load is taken of , the strut may recover undamaged. It may simply spring back like a bow.

Buckling is a useful safety mechanism in the real world. For example many small plants such as grass buckles so easily (before high compressive load can be applied) and then recovers. If there were no buckling all the grasses that human and other animals step would have deformed heavily and probably die.

Long and thin grasses are easily buckled.

There are some cases in which buckling are used for safety mechanism for animals. Hedgehogs are one of these examples. Hedgehogs can easily climb trees in search of food. However, it is hard for them to climb down. So, they curl themselves like a ball and fall to ground. Their close spines buckle in an Eulerian manner on the impact and absorb the energy of the fall and mitigate the shock of the impact.

The thin spines of a hedgehog would buckle on an impact protecting the hedgehogs.

REFERENCES:

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