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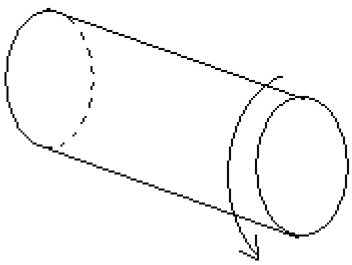
TORSION

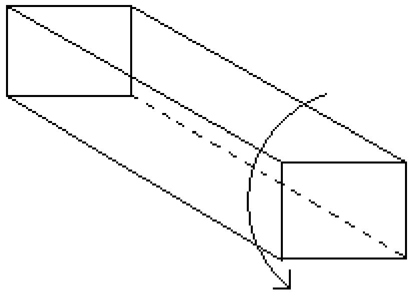
1- Mechanics of Materials Approach

2- Prandtl Stress Function Approach

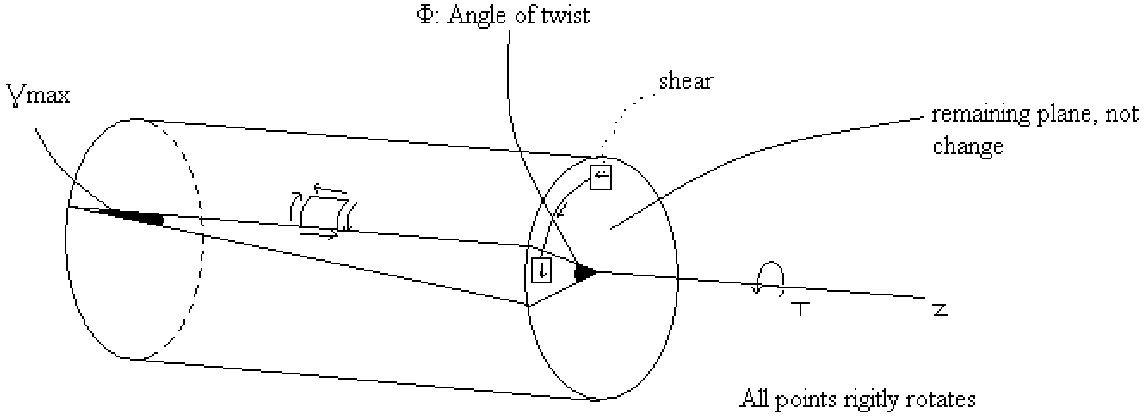
-more sufficient for axial cylinder problem

Sufficient insufficient sufficient





1-) Mechanics of Material Approach to Torsion of Circular Bars



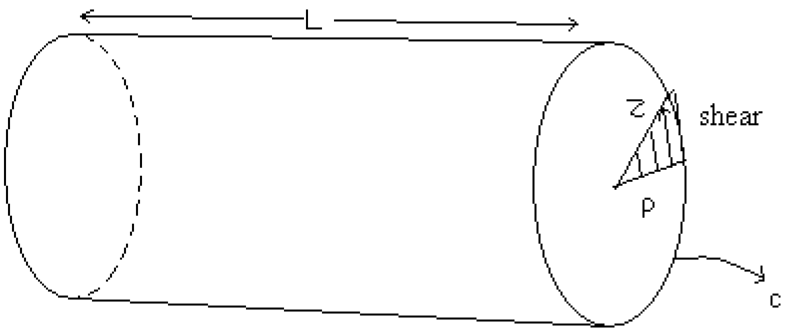
Assumptions:

1-) All plane sections perpendicular to torsion axis z remain perpendicular.

2-) Cross sections are undistorted in their individuals planes

-Shearing strain γ varies linearly from at the center to a maximum at the outer surface

3-) Material is homogenous



T=$\int \tau.\rho dA$

T=$\int_0^r \frac{\rho}{r}.\tau_{max}.\rho dA = \frac{\tau_{max}}{r}.\int \rho^2 \, dA$

Where $\int \rho^2 \, dA$ is polar moment of inertia

J=$\int \rho^2 \, dA$ and $\tau_{max} = \frac{T.r}{J}$ $\tau(\rho) = \frac{T.\rho}{J}$

Note: Polar moment of inertia for circle of radius r ; J=$\frac{\pi.r^4}{2}$

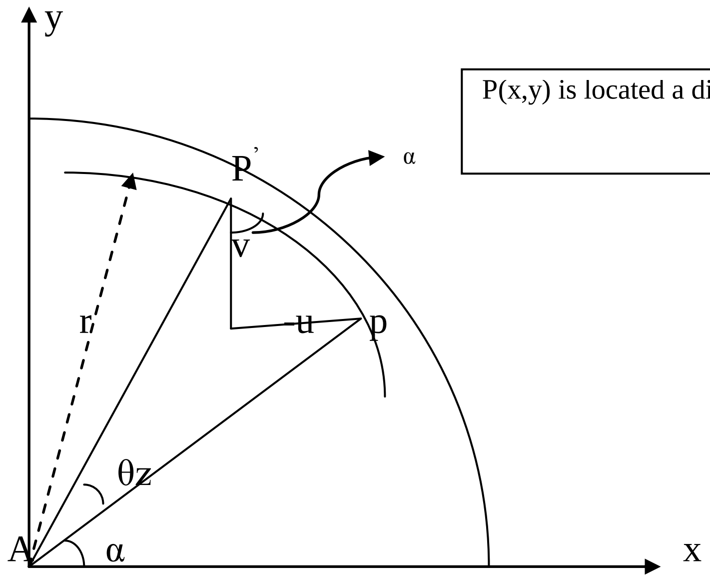
Angle of Twist Φ:

$\Phi.r = \gamma_{max}.L$

$\gamma_{max} = \frac{\tau_{max}}{G}$ (Hooke's law) so, $\gamma_{max} = \frac{T.r}{J.G}$

$\Phi.r = \frac{T.r}{J.G}.L$ $\Phi = \frac{T.L}{J.G}$ J.G : Torsional rigidity

2) theory of elasticity approach to torsion of bar



P(x,y) is located a distance r from the center

$u=-(r\theta z)\sin\alpha=-y\theta z$

$v=(r\theta z)\cos\alpha = x\,\theta z$

where the angular displacement of AP at a distance Z from the left end is θz

$\varepsilon_x=\gamma_{xy}=\varepsilon_y=\varepsilon_z=0$

$\gamma_{zy}=\frac{\partial w}{\partial x}-y\theta$, $\gamma_{zy}=\frac{\partial w}{\partial y}+x\theta$

$\sigma_x=\sigma_y=\sigma_z=\tau_{xy}=0$

$\tau_{zx}=G(\frac{\partial w}{\partial x}-y\theta)$………..1

$\tau_{zy}=G(\frac{\partial w}{\partial y}+x\theta)$…… …..2

assuming negligible body foces

$\frac{\partial \tau zx}{\partial z}=0$, $\frac{\partial \tau zy}{\partial z}=0$ $\frac{\partial \tau zx}{\partial x}+\frac{\partial \tau zy}{\partial y}=0$

Differentiating the equation 1 and 2 w.r.t. y and x respectively , we have

$\frac{\partial \tau zx}{\partial y}-\frac{\partial \tau zy}{\partial x}=H$

Where

$H=-2G\theta$

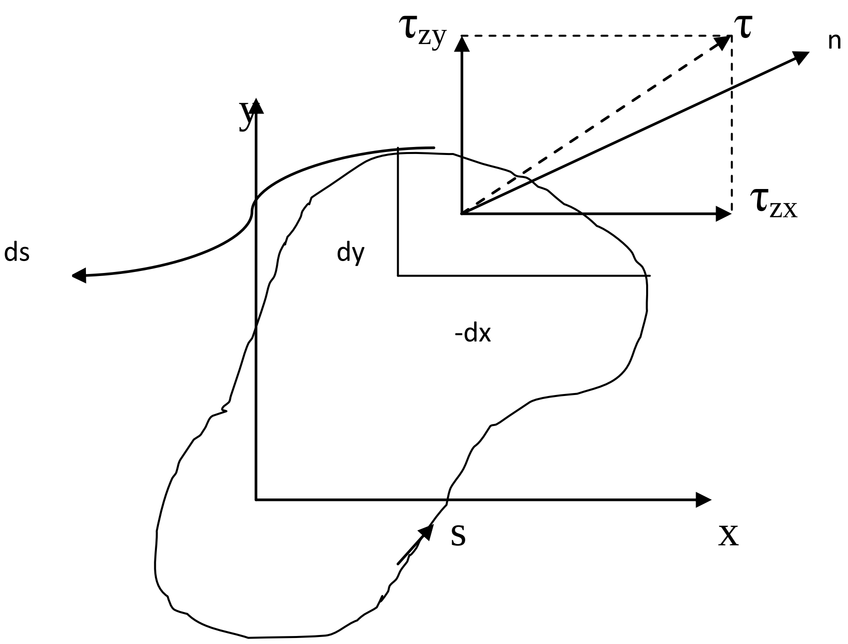
Stress function

As in the case of a beam the torsion problem formulated in the preceding is commonly solved by introducing a single stress function. If a function. If a function $\phi(x,y)$, the Prandtl stress function is assumed to exist, such that

$$\tau_{zx}=\frac{\partial\phi}{\partial y} \quad , \qquad \tau_{zy}=-\frac{\partial\phi}{\partial x}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=H$$

Boundary conditions



cosine of the angle between z and a unit normal n to the surface is zero that is cos(n,z) =0 we have

$$\tau_{zx}l+\tau_{zy}m=0$$

τ must be tangent to the surface

$$l=\cos(n,x)=\frac{dy}{ds} \quad , \qquad m=\cos(n,y)=-\frac{dx}{ds}$$

so on the boundary we have $\frac{\partial\phi}{\partial y}\frac{dy}{ds}+\frac{\partial\phi}{\partial x}\frac{dx}{ds}=\frac{\partial\phi}{\partial s}=0$ (on the boundary)

References

Lecture notes

Ugural - Frenster