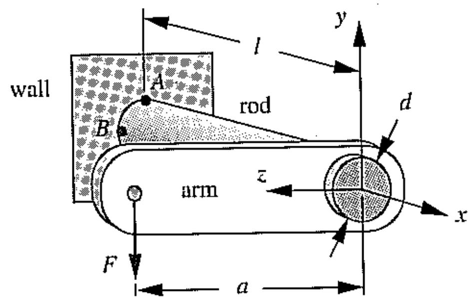


FIGURE 4-30

Bracket for Example 4-9

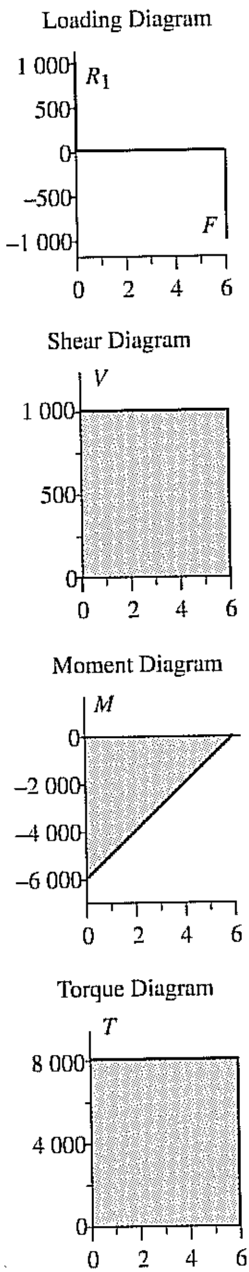


FIGURE 4-31

Loading Diagrams for Bending Shear, Moment, and Torque in Example 4-9

EXAMPLE 4-9

Combined Bending and Torsional Stresses

Problem **Find the most highly stressed locations on the bracket shown in Figure 4-30 and determine the applied and principal stresses at those locations.**

Given **The rod length l = 6 in and arm a = 8 in. The rod outside diameter d = 1.5 in. Load F = 1 000 lb.**

Assumptions **The load is static and the assembly is at room temperature. Consider shear due to transverse loading as well as other stresses.**

Solution **See Figures 4-30 to 4-33.**

1 We will limit our investigation to the rod which is loaded both in bending (as a cantilever beam) and in torsion. (The arm would also need to be analyzed for a complete design.) First, the load distributions over the rod's length need to be determined by drawing shear, moment, and torque diagrams for the rod.

2 The shear and moment diagrams will look similar to those for the cantilever beam in Example 4-5, the difference being that this force is at the end of the beam rather than at some intermediate point. Figure 4-31 shows that the shear force is uniform across the beam length and its magnitude is equal to the applied load $V_{max} = F = 1\,000$ lb. The maximum moment occurs at the wall and its magnitude is $M_{max} = Fl = (1\,000)(6) = 6\,000$ lb-in. (See Example 4-5 for derivations.)

The torque applied to the rod is due to the force F acting at the end of the 8-in arm and is $T_{max} = Fa = (1\,000)(8) = 8\,000$ lb-in. Note that this torque is uniform over the length of the rod as it can only be reacted against by the wall. Figure 4-31 shows all three of these loading functions. It is clear from these plots that the most heavily loaded cross section is at the wall, where all three loads are maximum.

3 We will now take a section through the rod at the wall and examine the stress distributions within it due to these external loads. Figure 4-32a shows the distribution across the section of the normal bending stresses, which are a maximum (+/–) at the outer fibers and zero at the neutral axis. The shear stress due to transverse loading is a maximum at all points in the neutral (xz) plane and zero at the outer fibers (Figure 4-32b).

The shear stress due to torsion is proportional to the radius so is zero at the center and a maximum at all points on the outer surface as shown in Figure 4-32*c*. Note the differences between the distributions of the normal bending stress and the torsional shear stress. The bending stress magnitude is proportional to the distance y from the neutral plane and so is maximum at only the top and bottom of the section, whereas the torsional shear stress is maximal all around the perimeter.

4 We choose two points, A and B of Figure 4-30, to analyze (also shown in Figure 4-33*a*) because they have the worst combinations of stresses. The largest tensile bending stress will be in the top outer fiber at point A, and it combines with the largest torsional shear stress that is all around the outer circumference of the rod. A differential element taken at point A is shown in Figure 4-33*b*. Note that the normal stress (σ_x) acts on the x face in the x direction and the torsional shear stress (τ_{xz}) acts on the x face in the $+z$ direction.

At point B the torsional shear stress has the same magnitude as at point A, but the direction of the torsional shear stress (τ_{xy}) at point B is 90° different than at point A. The shear stress due to transverse loading (τ_{xy}) is a maximum at point B. Note that these shear stresses both act in the $-y$ direction on the x face at point B as shown in Figure 4-33*c*. The transverse and torsional shear stresses then add at point B.

5 Find the normal bending stress and torsional shear stress on point A using equations 4.11*b* (p. 154) and 4.19*b* (p. 161), respectively.

$$\sigma_x = \frac{Mc}{I} = \frac{(Fl)c}{I} = \frac{1\,000(6)(0.75)}{0.249} = 18\,108 \text{ psi} \tag{a}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{(Fa)r}{J} = \frac{1\,000(8)(0.75)}{0.497} = 12\,072 \text{ psi} \tag{b}$$

6 Find the maximum shear stress and principal stresses that result from this combination of applied stresses using equations 4.6 (p. 143).

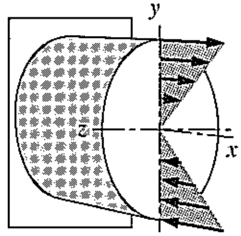
$$\begin{aligned}
\tau_{max} &= \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{\left(\frac{18\,108 - 0}{2}\right)^2 + 12\,072^2} = 15\,090 \text{ psi} \\
\sigma_1 &= \frac{\sigma_x + \sigma_z}{2} + \tau_{max} = \frac{18\,108}{2} + 15\,090 = 24\,144 \text{ psi} \\
\sigma_2 &= 0 \\
\sigma_3 &= \frac{\sigma_x + \sigma_z}{2} - \tau_{max} = \frac{18\,108}{2} - 15\,090 = -6\,036 \text{ psi}
\end{aligned} \tag{c}$$

7 Find the shear due to transverse loading at point B on the neutral axis. The maximum transverse shear stress at the neutral axis of a round rod was given as equation 4.15*c* (p. 159).

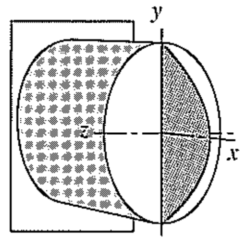
$$\tau_{bending} = \frac{4V}{3A} = \frac{4(1\,000)}{3(1.767)} = 755 \text{ psi} \tag{d}$$

Point B is in pure shear. The total shear stress at point B is the algebraic sum of the transverse shear stress and the torsional shear stress, which both act on the same planes of the differential element.

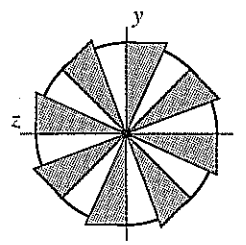
$$\tau_{max} = \tau_{torsion} + \tau_{bending} = 12\,072 + 755 = 12\,827 \text{ psi} \tag{e}$$



(*a*) Bending normal-stress distribution across section



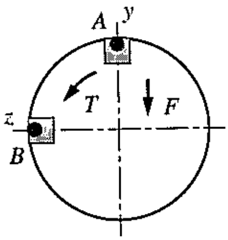
(*b*) Transverse shear-stress distribution across section



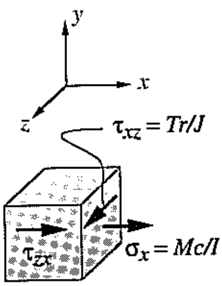
(*c*) Torsional shear-stress distribution across section

FIGURE 4-32

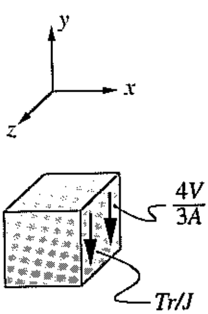
Cross Sections of Rod for Example 4-9



(a) Two points of interest for stress calculations



(b) Stress element at point *A*



(c) Stress element at point *B*

FIGURE 4-33

Stress Elements at Points *A* and *B* within Cross Section of Rod for Example 4-9

which from equation 4.6 or the Mohr's circle can be shown to be equal to the largest principal stress for this point.

8 Point *A* has the larger principal stress in this case, but note that the relative values of the applied torque and moment determine which of these two points will have the higher principal stress. Both points must then be checked. See files EX04-09 on CD.

4.14 SPRING RATES

Every part made of material having an elastic range can behave as a spring. Some parts are designed to function as springs, giving a controlled and predictable deflection in response to an applied load or vice versa. The "springiness" of a part is defined by its spring rate k, which is the load per unit deflection. For rectilinear motion springs,

$$k = \frac{F}{y} \tag{4.27a}$$

where F is the applied load and y is the resulting deflection. Typical units are lb/in or N/m. For angular motion springs the general expression is

$$k = \frac{T}{\theta} \tag{4.27b}$$

where T is the applied torque and θ is the resulting angular deflection. Typical units are in-lb/rad or N-m/rad, or sometimes expressed as in-lb/rev or N-m/rev.

The spring rate equation for any part is easily obtained from the relevant deflection equation, which provides a relationship between force (or torque) and deflection. For example, for a uniform bar in axial tension, the deflection is given by equation 4.8, repeated here and rearranged to define its axial spring rate.

$$y = \frac{Fl}{AE}$$
$$k = \frac{F}{y} = \frac{AE}{l} \tag{4.28}$$

This is a constant spring rate, dependent only on the bar's geometry and its material properties.

For a uniform-section round bar in pure torsion, the deflection is given by equation 4.24 (p. 174), repeated here and rearranged to define its torsional spring rate:

$$\theta = \frac{Tl}{GJ}$$
$$k = \frac{T}{\theta} = \frac{GJ}{l} \tag{4.29}$$

This is also a constant spring rate, dependent only on the bar's geometry and material properties.