AE361 - HW42

1) Known: Cross-sectional area, thickness, and internal pressure of the vessel, loading, welding

Find: a) Normal stress perpendicular to the weld.

b) Shearing stress parallel to the weld.

Analysis: Pick a small element on the side of the pressure vessel.

$\sigma_c = \frac{P}{t} \frac{30}{9 \times 10^{-3}} = 30 \text{ MPa}$

$\sigma_p = \frac{P}{2t} \frac{15}{2 \times 10^{-3}} = 15 \text{ MPa}$

$\sigma_p = -\frac{P}{A_0} \frac{50 \times 10^{-3}}{\sqrt{((50 \times 10^{-3})^2 - (56 \times 10^{-3})^2)}} = 3.4 \text{ MPa}$

So the differential element becomes:

19 MPa left (34 - 15)

30 MPa up

50 MPa
Then for Mohr's Circle,

\[ X(-19,0) \]
\[ Y(30,0) \]

\[ R = \frac{30 - (-19)}{2} = 24.5 \]

To find the normal and shear stresses at the weld location, we have to rotate our element by 32°, which is 64° in Mohr's Circle.

\[ \sigma_{\text{weld}} = 24.5 \sin(64°) = 22.02 \text{ MPa (on x-axis)} \]
\[ \sigma_x = -24.5 \cos(64°) - (-5.5) = -5.24 \text{ MPa} \]
\[ \sigma_y = 24.5 \cos(64°) + 5.5 = 16.29 \text{ MPa} \]
2) **Known**: The orientations and loadings of stress conditions at a point under different conditions.

**Find**: The principal stresses and their orientations at the point under combined loading.

**Schematic**:

![Diagram showing stress conditions](image)

**Analysis**:

1. **will remain the same**.
2. **will be rotated 30° clockwise, so it will be at x-y coordinates**.
3. **will be rotated 60° clockwise**.

For **(2)**,

\[
\sigma_x = \frac{\sigma_x' + \sigma_y'}{2} + \frac{\sigma_x' - \sigma_y'}{2} \cos 2\theta + \tau_{xy}' \sin 2\theta
\]

\[
\sigma_y = \frac{\sigma_x' + \sigma_y'}{2} + \frac{\sigma_x' - \sigma_y'}{2} \cos 2\theta - \tau_{xy}' \sin 2\theta
\]

\[
\tau_{xy} = \frac{\sigma_y - \sigma_x}{2} \sin 2\theta + \tau_{xy}' \cos 2\theta
\]

\[
\sigma_x = 0 + 0 - 40 \times \sin(-60°)
\]

\[
= 0 - 0 + 40 \times \sin(-60°)
\]

\[
= -34.64 \text{ MPa}
\]

\[
\sigma_y = 0 + 0 + 40 \times \sin(-60°)
\]

\[
= 0 + 0 + 40 \times \sin(-60°)
\]

\[
= -34.64 \text{ MPa}
\]
So \( \sigma_2 \) is,

\[
39.64 \text{ MPa} \quad \begin{array}{c}
\downarrow \\
39.64 \text{ MPa}
\end{array}
\]

\[
20 \text{ MPa}
\]

For \( \sigma_3 \),

\[
\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta
\]

\[
= \frac{0 + 0 + 10 \times \sin(-120^\circ)}{2}
\]

\[
= -8.66 \text{ MPa}
\]

\[
\sigma_y = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta
\]

\[
= \frac{0 + 0 + 10 \times \sin(-120^\circ)}{2}
\]

\[
= 8.66 \text{ MPa}
\]

If we combine the three,

\[
1 + 34.64 + 8.66 = 25.98 \text{ MPa}
\]

\[
34.64 - 8.66 = 25.98 \text{ MPa}
\]

\[
5 + 20 + 20 = 45 \text{ MPa}
\]

For Mohr's Circle,

\[
X(25.98, 45)
\]

\[
Y(25.98, -45)
\]

\[
\sigma_{ave} = \frac{25.98 - 25.98}{2} = 0
\]

\[-25.98, -45\]

\[
tan 2\theta = \frac{45}{25.98}
\]

\[
\theta = 35^\circ
\]

\[
R = \sqrt{25.78^2 + 45^2} = 51.96
\]

\[
\sigma_p = \sigma_x = \sigma_y = 25.98 \text{ MPa}
\]

\[
\tau_p = \sigma_x - \sigma_y = 51.96 \text{ MPa}
\]