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(1) a) $\begin{bmatrix} c(x^2+y^2) & cxy \\ cxy & y^2 \end{bmatrix}$

Given: Strain fields
$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$

Find: Is it possible in a continuous material or not?

$\varepsilon_x = c(x^2+y^2)$ $\quad \varepsilon_{xy} = cxy$

$\varepsilon_y = y^2$

Strain compatibility equations

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} \overset{?}{=} 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}$$

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$$\frac{\partial^2 (c(x^2+y^2))}{\partial y^2} + \frac{\partial^2 (y^2)}{\partial x^2} \overset{?}{=} 2\frac{\partial^2 (cxy)}{\partial x \partial y}$$

$$2c + 0 \overset{\checkmark}{=} 2c$$

POSSIBLE

b) $\begin{bmatrix} cz(x^2+y^2) & cxyz \\ cxyz & y^2z \end{bmatrix}$

Given: Strain fields
$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$

Find: Is it possible in a continuous material or not?

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} \overset{?}{=} 2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y}$$

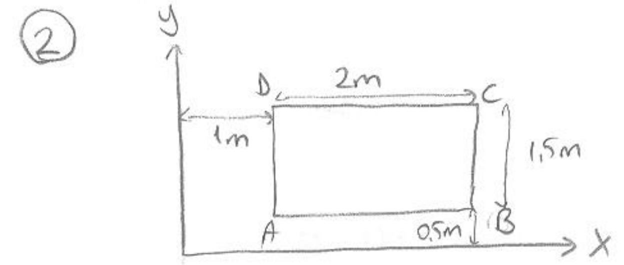
where $\varepsilon_x = cz(x^2+y^2)$
$\varepsilon_y = y^2z$
$\varepsilon_{xy} = cxyz$

$$\frac{\partial^2 (cz(x^2+y^2))}{\partial y^2} + \frac{\partial^2 (y^2z)}{\partial x^2} \overset{?}{=} 2\frac{\partial^2 (cxyz)}{\partial x \partial y}$$

$$2cz + 0 \overset{\checkmark}{=} 2cz$$

POSSIBLE

(2)



Given: $u = c(2x + y^2)$

$v = c(x^2 - 3y^2)$

$c = 10^{-4}$

Find: $l_{AB} = ?$ / subsequent to the loading /

$l_{AD} = ?$

Analysis

$u_A = c\left(2\times 1 + (0{,}5)^2\right) = 2{,}25\times 10^{-4}$

$u_B = u_A + \int_1^3 \frac{\partial u}{\partial x}\,dx = 2{,}25\times 10^{-4} + \int_1^3 c.2\,dx = 2{,}25\times 10^{-4} + 2cx\Big|_1^3 = 6{,}25\times 10^{-4}$

$v_B - v_A = \int_1^3 \frac{\partial v}{\partial x}\,dx = \int_1^3 c.2x = cx^2\Big|_1^3 = 8\times 10^{-4}$ $\quad \alpha_x = \tan\alpha_x = \frac{8\times 10^{-4}}{2{,}0004} = 3{,}999\times 10^{-4}$ (small)

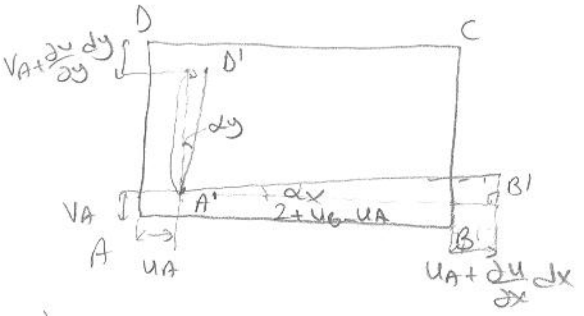
$v_A = c\left(1^2 - 3(0{,}5)^2\right) = 2{,}5\times 10^{-5}$

$v_D = v_A + \int_{0{,}5}^{2} \frac{\partial v}{\partial y}\,dy = 2{,}5\times 10^{-5} + \int_{0{,}5}^{2} c(-6y)\,dy = 2{,}5\times 10^{-5} + (-3y^2c)\Big|_{0{,}5}^{2}$

$= -1{,}1\times 10^{-3}$

$u_D - u_A = \int_{0{,}5}^{2} \frac{\partial u}{\partial y}\,dy = \int_{0{,}5}^{2} c\,2y\,dy = cy^2\Big|_{0{,}5}^{2} = 3{,}75\times 10^{-4}$, $\quad \alpha_y = \tan\alpha_y = \frac{3{,}75\times 10^{-4}}{1{,}4989}$ (small)

$= 2{,}502\times 10^{-3}$



a) $|A'B'| = \left(|AB| + \frac{\partial u}{\partial x}dx\right)\cdot\frac{1}{\cos\alpha_x}$

$|A'B'| = \left(2 + 4\times 10^{-4}\right)\cdot\frac{1}{\cos\alpha_x}$

$= 2{,}00041$

$|A'D'| = \left(|AD| - \frac{\partial v}{\partial y}dy\right)\frac{1}{\cos\alpha_y}$

$= \left(1{,}5 - 1{,}125\times 10^{-3}\right)\cdot\frac{1}{\cos(0{,}0215)}$

$= 1{,}498904$

b) $\Delta\alpha = \alpha_x + \alpha_y$

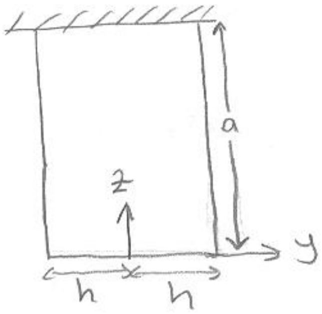
$= 2{,}9019\times 10^{-3}$ rad

$= 0.166°$

c) coordinates of point $A = A'$

$\left(1 + 2{,}25\times 10^{-4},\ 0{,}5 + 2{,}5\times 10^{-4}\right) = \left(1{,}000225,\ 0{,}500025\right)$

(3)



Given: cross section $2h \times b$

specific weight γ

$u = -\frac{\nu\gamma}{E} xz \qquad v = -\frac{\nu\gamma}{E} yz$

$w = \frac{+\gamma}{2E}\left[(z^2 - a^2) + \nu(x^2 + y^2)\right]$

Find: whether this solution satisfies 15 equations of elasticity and the boundary conditions or not.

Assumption: isothropic material (ν = constant)

Analysis: Strain displacement relations.

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -\frac{\nu\gamma}{E} z \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} = -\frac{\nu\gamma}{E} z \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z} = \frac{+\gamma}{2E}(2z)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}(0 + 0) = 0$$

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$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \left[-\frac{\nu\gamma}{E} x + \frac{\gamma\nu 2x}{2E}\right]\frac{1}{2} = 0$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\left[-\frac{\nu\gamma}{E} y + \frac{\nu 2y\gamma}{2E}\right] = 0$$

Stress - Strain Relations

$$\tau_{xy} = 2G \cdot \varepsilon_{xy} = 0 \qquad G = \frac{E}{2(1+\nu)}$$

$$\tau_{xz} = 2G\,\varepsilon_{xz} = 0$$

$$\tau_{yz} = 2G\,\varepsilon_{yz} = 0$$

$$\sigma_{xx} = 2G\,\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) \qquad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$= \frac{E}{1+\nu}\cdot\left(-\frac{\nu\gamma z}{E}\right) + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(-\frac{2\nu\gamma}{E} z + \frac{\gamma}{2E} 2z\right)$$

$$= -\frac{\nu\gamma z}{1+\nu} + \frac{\nu E}{(1+\nu)\cancel{(1-2\nu)}}\left(+\frac{\gamma z}{E}\right)\cancel{(-2\nu+1)}$$

$$= 0$$

$$\sigma_{yy} = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$$

$$= \frac{\cancel{2}E}{\cancel{2}(1+\nu)} \cdot \left(\frac{-\nu\gamma}{\cancel{E}} z\right) + \frac{\nu E}{(\nu+1)\cancel{(1-2\nu)}} \cdot \left(\frac{\gamma z}{\cancel{E}}\right) \cdot \cancel{(-2\nu+1)} = 0$$

$$\sigma_{zz} = 2G\varepsilon_{zz} + \lambda(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})$$

$$= \frac{\cancel{2}E}{\cancel{2}(1+\nu)} \cdot \frac{\gamma z}{\cancel{E}} + \frac{\nu E}{(\nu+1)\cancel{(1-2\nu)}} \cdot \frac{\gamma z}{\cancel{E}} \cancel{(-2\nu+1)} \quad \checkmark$$

$$= \frac{\gamma z + \nu\gamma z}{\nu+1} = \gamma z$$

<u>Stress Equilibrium Equations</u> $\quad P_z = -\gamma \hat{k}$

$$\frac{\partial \sigma_{xx}}{\partial x}^{\to 0} + \frac{\partial \tau_{xy}}{\partial y}^{\to 0} + \frac{\partial \tau_{xz}}{\partial z}^{\to 0} + P_x^{\to 0} \overset{?}{=} 0 \quad \checkmark$$

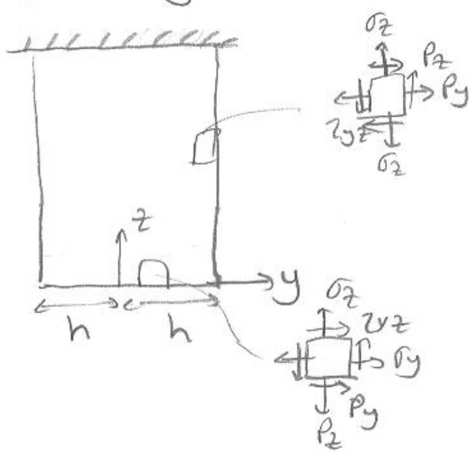
$$\frac{\partial \tau_{xy}}{\partial x}^{\to 0} + \frac{\partial \sigma_{yy}}{\partial y}^{\to 0} + \frac{\partial \tau_{yz}}{\partial z}^{\to 0} + P_y^{\to 0} \overset{?}{=} 0 \quad \checkmark$$

$$\frac{\partial \tau_{xz}}{\partial x}^{\to 0} + \frac{\partial \tau_{xy}}{\partial y}^{\to 0} + \frac{\partial \sigma_{zz}}{\partial z} + P_z \overset{?}{=} 0$$

$$+\gamma - \gamma \overset{\checkmark}{=} 0$$

✓ satisfies stress equilibrium equations

<u>Boundary Conditions</u>



since $P_z = 0$, $P_y = 0$

$\sigma_y = 0$, $\tau_{xy} = 0$ at $y = \pm h$

$\sigma_y = 0$, $\tau_{xy} = 0$ ✓

since $P_z = 0$, $P_y = 0$ at $z = 0$

$\sigma_z = 0$, $\tau_{xz} = 0$ $\qquad \sigma_z = \gamma z \big|_{z=0} = 0$ ✓

$\tau_{xz} = 0$ ✓

This solution satisfies 15 equations of elasticity and boundary conditions.

④ Given stress field for an elastic body

$$\begin{bmatrix} cy^2 & 0 \\ 0 & -cx^2 \end{bmatrix} \qquad c = \text{constant}$$

Find $u(x,y)$ and $v(x,y)$

Assumption : isothropic (ν = constant)

Analysis $\sigma_{xx} = cy^2$, $\tau_{xy} = 0$, $\sigma_{yy} = -cx^2$

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} \Rightarrow \frac{cy^2}{E} + \nu\frac{cx^2}{E}$$

$$\frac{\partial u}{\partial x} = \varepsilon_{xx} \Rightarrow u = \frac{cxy^2}{E} + \frac{\nu c x^3}{3E} + f(y)$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E} = -\frac{cx^2}{E} - \nu\frac{cy^2}{E}$$

$$\frac{\partial v}{\partial y} = \varepsilon_{yy} \Rightarrow v = -\frac{cx^2 y}{E} - \frac{\nu c y^3}{3E} + g(x)$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{2G} = 0$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\frac{2cxy}{E} + f'(y) - \frac{2cxy}{E} + g'(x)\right)$$

$$f'(y) + g'(x) = 0 \qquad f(y) = ky + c_1 \quad g(x) = -kx + c_2$$

k = constant

c_1 = constant

c_2 = constant

$$u = \frac{cxy^2}{E} + \frac{\nu c x^3}{3E} + ky + c_1$$

$$v = -\frac{cx^2}{E} - \frac{\nu c y^2}{E} - kx + c_2$$

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