SOLUTIONS

Problem 1. (Shigley)

6-14 Given: AISI 1006 CD steel, $F = 0.55$ N, $P = 8.0$ kN, and $T = 30$ N · m, applying the DE theory to stress elements A and B with $S_y = 280$ MPa

A:

$$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)(10^3)(0.1)}{\pi(0.020^3)} + \frac{4(8)(10^3)}{\pi(0.020^2)}$$
$$= 95.49(10^6)\text{ Pa} = 95.49\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020^3)} = 19.10(10^6)\text{ Pa} = 19.10\text{ MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = [95.49^2 + 3(19.1)^2]^{1/2} = 101.1\text{ MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{101.1} = 2.77 \quad \textit{Ans.}$$

B:

$$\sigma_x = \frac{4P}{\pi d^3} = \frac{4(8)(10^3)}{\pi(0.020^2)} = 25.47(10^6)\text{ Pa} = 25.47\text{ MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(30)}{\pi(0.020^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)(0.020^2)}\right]$$
$$= 21.43(10^6)\text{ Pa} = 21.43\text{ MPa}$$

$$\sigma' = [25.47^2 + 3(21.43^2)]^{1/2} = 45.02\text{ MPa}$$

$$n = \frac{280}{45.02} = 6.22 \quad \textit{Ans.}$$

Problem 2. (Shigley)

6-39 **(a)** Ignoring stress concentration

$$F = S_y A = 160(4)(0.5) = 320\text{ kips} \quad \textit{Ans.}$$

(b) From Fig. 6-36: $h/b = 1$, $a/b = 0.625/4 = 0.1563$, $\beta = 1.3$

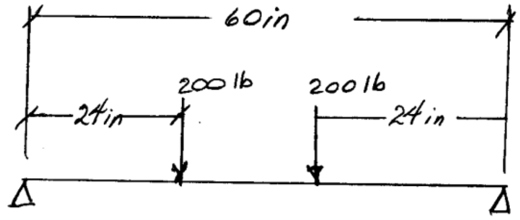
Eq. (6-51)

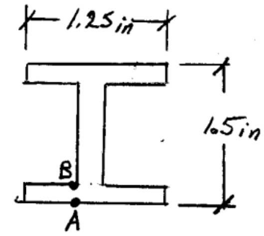
$$70 = 1.3\frac{F}{4(0.5)}\sqrt{\pi(0.625)}$$

$$F = 76.9\text{ kips} \quad \textit{Ans.}$$

Problem 3.

Given:





$S_y = 50$ ksi

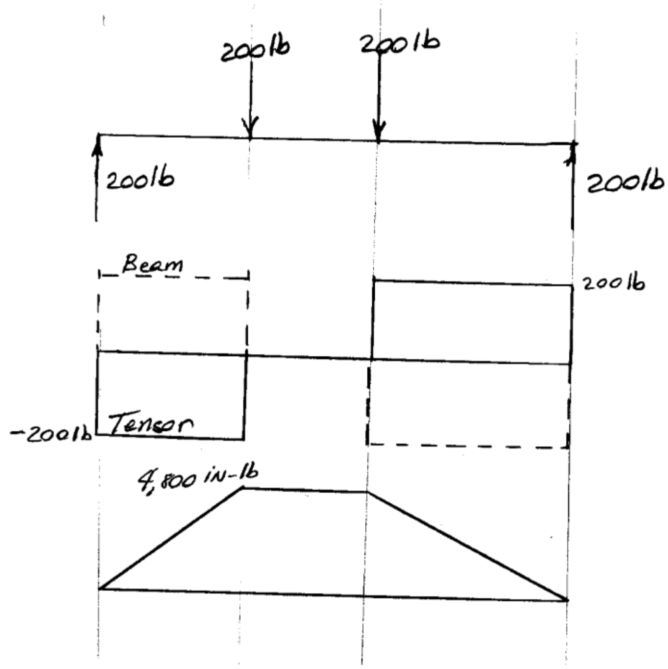
Web and flange thickness = 0.125 in

Find: (a) Bending and transverse shear stress at A & B.

(b) Maximum normal and maximum shear stress.

(c) Factor of safety based on maximum normal and maximum shear stress.

Solution: Find maximum shear force and bending moment.



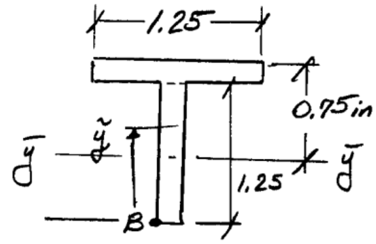
Note that tensor sign convention is used.

$V_{max} = \pm 200$ lb

$M_{max} = 4{,}800$ in-lb

Compute cross section properties

Find Q_B



$$A = (1.25\,in)(0.125\,in) + (1.25\,in)(0.125\,in) = 0.3125\,in^2$$

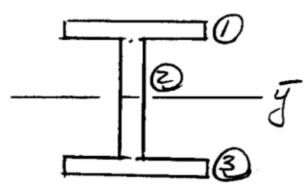
$$\tilde{y} = \frac{(1.25\,in)(0.125\,in)\left(1.25 + \frac{0.125}{2}\right) + (1.25)(0.125)\left(\frac{1.25}{2}\right)}{0.3125}$$

$$= \frac{0.302}{0.3125} = 0.969\,in$$

$$\tilde{y} - \bar{y} = 0.969\,in - 0.625 = 0.344\,in$$

$$Q_B = A(\tilde{y} - \bar{y}) = \underline{\underline{0.1075\,in^3}}$$

Find I



$$I = \sum \bar{I} + (\tilde{y} - \bar{y})^2 A$$

$$= 2\left(\frac{1}{12}(1.25\,in)(0.125\,in)^3 + (1.25\,in)(0.125)(0.75 - 0.125/2)^2\right)$$

$$+ \frac{1}{12}(0.125\,in)(1.25\,in)^3$$

$$= \underline{\underline{0.168\,in^4}}$$

Part A

Bending stress at A $\quad \sigma_A = \dfrac{Mc_A}{I} = \dfrac{(4{,}800 \text{ in-lb})(0.75 \text{ in})}{0.168 \text{ in}^4}$

$$= \underline{\underline{21.4 \text{ ksi}}}$$

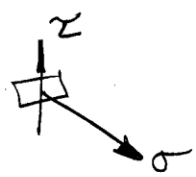
Bending stress at B $\quad \sigma_B = \dfrac{Mc_B}{I} = \dfrac{(4{,}800 \text{ in-lb})(0.625 \text{ in})}{0.168 \text{ in}^4}$

$$= \underline{\underline{17.9 \text{ ksi}}}$$

Transverse shear stress at A $\quad \tau = \dfrac{VQ_A}{It} = 0 \quad$ since $Q_A = 0$

Transverse shear stress at B $\quad \tau = \dfrac{VQ_B}{It} = \dfrac{(200 \text{ lb})(0.1075 \text{ in}^3)}{(0.168 \text{ in}^4)(0.125 \text{ in})}$

$$= \underline{\underline{1.02 \text{ ksi}}}$$



PART B

At point A the principal stresses are

$$\sigma_1 = \sigma = 21.4 \text{ ksi}, \quad \sigma_2 = \sigma_3 = 0 \qquad \underline{\underline{\sigma_{max} = 21.4 \text{ ksi}}}$$

At point B,

$$\sigma_1 = \frac{\sigma_x + \cancel{\sigma_y}}{2} + \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}}{2}\right)^2 + \tau_{xy}^2} = 17.95 \simeq 18.0 \text{ ksi}$$

$$\sigma_2 = \frac{\sigma_x + \cancel{\sigma_y}}{2} - \sqrt{\left(\frac{\sigma_x - \cancel{\sigma_y}}{2}\right)^2 + \tau_{xy}^2} = -0.06 \text{ ksi} \qquad \sigma_{max} = \underline{\underline{18.0 \text{ ksi}}}$$

$$\sigma_3 = 0$$

Maximum shear stress at A $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{21.4}{2} = \underline{\underline{10.7\ ksi}}$

at B $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{18-(-0.06)}{2}$

$= 9.03\ ksi$

PART C

Maximum normal stress theory (Point A)

$$\frac{\sigma_{max}}{S_y} = \frac{1}{N} \Rightarrow N = \frac{S_y}{\sigma_{max}} = \frac{50\ ksi}{21.4\ ksi} = \underline{\underline{2.34}}$$

Maximum shear stress theory (Point A)

$$\frac{\sigma_1 - \sigma_3}{S_y} = \frac{1}{N} \Rightarrow N = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{50\ ksi}{21.4\ ksi} = \underline{\underline{2.34}}$$

Maximum normal stress theory (Point B)

$$\frac{\sigma_{max}}{S_y} = \frac{1}{N} \Rightarrow N = \frac{S_y}{\sigma_{max}} = \frac{50\ ksi}{18.0\ ksi} = \underline{\underline{2.78}}$$

Maximum shear stress theory (Point B)

$$N = \frac{S_y}{\sigma_1 - \sigma_2} = \frac{50\ ksi}{18.06} = \underline{\underline{2.77}}$$