AE 361 Quiz 6 21/12/2009

Section 01

Sample Problem 5.11 **Determine the Required Diameter of a Steel Connecting Rod**

An industrial machine requires a solid, round connecting rod 1 m long (between pinned ends) that is subjected to a maximum compressive force of 80,000 N. Using a safety factor of 2.5, what diameter is required if steel is used, having properties of $S_y = 689$ MPa, $E = 203$ GPa?

Solution

Known: A 1-m-long steel rod (Figure 5.29) of known elastic modulus, yield strength, and safety factor is compressed by a specified force.

Find: Determine the rod diameter.

Schematic and Given Data:

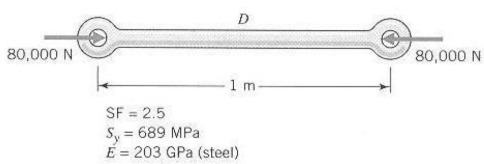


Figure 5.29
Solid round steel connecting rod in compression (used in Sample Problem 5.11).

Assumptions:

1. The rod is straight.
2. The pinned ends act to create an effective rod length of 1 m.
3. The rod does not fail from compressive stress.
4. The buckling capacity of the material corresponds to line *AE* of Figure 5.28.
5. The Euler relationship applies.

Analysis: As assumed, the material corresponds to line *AE* of Figure 5.28, rod construction corresponds to $L_e = L = 1$ m. In addition, tentatively assuming that the Euler relationship applies, we have

$$\frac{P}{A} = \frac{\pi^2 E}{(L_e/\rho)^2}$$

where the *design overload, P*, is 80,000 N × 2.5 or 200,000 N and where A is the cross-sectional area and ρ the radius of gyration. For the solid round section specified here,

$$A = \pi D^2/4, \qquad \rho = D/4$$

Hence,

$$\frac{4P}{\pi D^2} = \frac{\pi^2 E D^2}{16 L_e^2}, \qquad 64PL_e^2 = \pi^3 E D^4$$

$$D = \left(\frac{64PL_e^2}{\pi^3 E}\right)^{1/4} = \left[\frac{64(200{,}000)(1)^2}{\pi^3(203 \times 10^9)}\right]^{1/4} = 0.0378 \text{ m}$$

Comments:

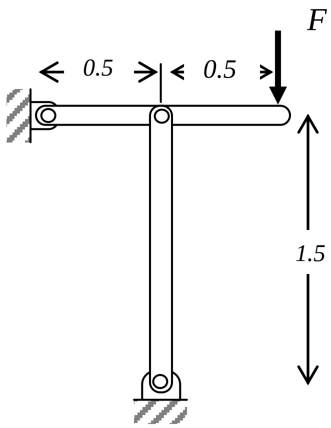
1. The calculated diameter gives a slenderness ratio of

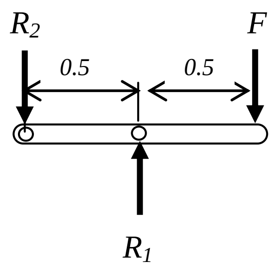
$$\frac{L_e}{\rho} = \frac{1}{0.0378/4} = 106$$

2. Figure 5.28 shows that with the calculated slenderness ratio we are well beyond the tangent point on curve *AE* and into the range where the Euler relationship can indeed be applied. Hence, the final answer (slightly rounded off) is 38 mm.

Section 02

Write an expression for the largest load F that may be applied without causing buckling:





Find the reactions using statics:

$$\sum M_{R_2} = 0$$
$$\Rightarrow R_1 \times 0.5 - F \times 1 = 0$$
$$\Rightarrow R_1 = 2F$$
$$and$$
$$\sum F_{vertical} = 0$$
$$\Rightarrow R_2 = F - R_1$$
$$\Rightarrow R_2 = F$$

Euler's buckling formula gives the critical load for buckling of a pin-loaded column as:

$$P_{cr} = \frac{\Pi^2 EI}{l^2}$$

In our case the compressive load on the column is $R_1=2F$. Letting $P_{cr} \to 2F$ we obtain the largest load F that the structure can withstand without buckling:

$$\Rightarrow \boxed{F = \frac{\Pi^2 EI}{2l^2}} \text{..................}(Ans)$$