Programming Languages: Type Systems

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20 March 2008

Type Systems

Design choices for types:

- monomorphic vs polymorphic type system.
- overloading allowed?
- coercion(auto type conversion) applied, how?
- type relations and subtypes exist?

Polymorphism

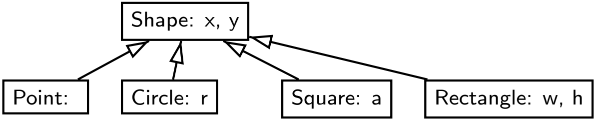
- Monomorphic types: Each value has a single specific type. Functions operate on a single type. C and most languages are monomorphic.
- Polymorphism: A type system allowing different data types handled in a uniform interface:
  1. Ad-hoc polymorphism: Also called overloading. Functions that can be applied to different types and behave differently.
  2. Inclusion polymorphism: Polymorphism based on subtyping relation. Function applies to a type and all subtypes of the type (class and all subclasses).
  3. Parametric polymorphism: Functions that are general and can operate identically on different types

Subtyping

- C types:
  `char` $\subseteq$ `short` $\subseteq$ `int` $\subseteq$ `long`
- Need to define arithmetic operators on them separately?
- Consider all strings, alphanumeric strings, all strings from small letters, all strings from decimal digits.
  Ned to define special concatenation on those types?
- $f : T \rightarrow V\ ,\quad U \subseteq T \Rightarrow f : U \rightarrow V$
- Most languages have arithmetic operators operating on different precisions of numerical values.

Inheritance

- ```
  struct Point { int x, y; };
  struct Circle { int x, y, r; };
  struct Square { int x, y, a; };
  struct Rectangle { int x, y, w, h; };
  ```
- ```
  void move (Point p, int nx, int ny) {
             p.x=nx; p.y=ny;}
  ```
- Moving a circle or any other shape is too different?



Haskell extensible records:

```
import Hugs.Trex; -- Only in -98 mode!!!

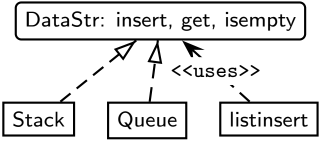
type Shape = Rec (x::Int, y::Int)
type Circle = Rec (x::Int, y::Int, r::Int)
type Square = Rec (x::Int, y::Int, w::Int)
type Rectangle = Rec (x::Int, y::Int, w::Int, h::Int)

move (x=_,y=_|rest) b c = (x=b,y=c|rest)

(a::Shape)=(x=12,y=24)
(b::Circle)=(x=12,y=24,r=10)
(c::Square)=(x=12,y=24,w=4)
(d::Rectangle)=(x=12,y=24,w=10,h=5)

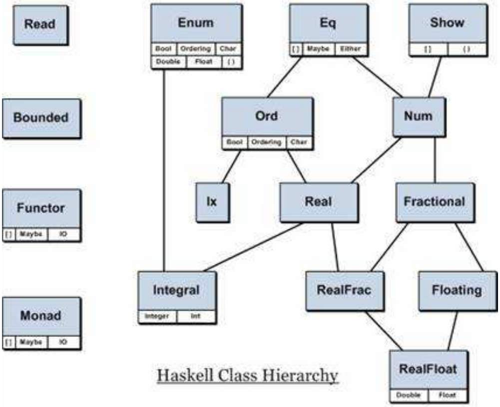
Main> move b 4 5
(r = 10, x = 4, y = 5)
Main> move c 4 5
(w = 4, x = 4, y = 5)
Main> move d 4 5
(h = 5, w = 10, x = 4, y = 5)
```

Haskell Classes

- Subtyping hierarchy based on classes
- An instance implements interface functions of the class
- Functions operating on classes (using interface functions) can be defined
- 

  listinsert :: DataStr a $\Rightarrow$ (a v) $\rightarrow$ [v] $\rightarrow$ (a v)
- Called interface in OO programming

Haskell Class Hieararchy



```
class DataStr a where
    insert :: (a v) -> v -> (a v)
    get :: (a v)-> Maybe (v,(a v))
    isempty :: (a v) -> Bool

instance  DataStr Stack where
    insert x v = push v x
    get x = pop x
    isempty Empty = True
    isempty _ = False

instance DataStr Queue where
    insert x v = enqueue v x
    get x = dequeue x
    isempty EmptyQ = True
    isempty _ = False

insertlist :: DataStr a =>  (a v) -> [v] -> (a v)
insertlist x [] = x
insertlist x (el:rest) = insertlist  (insert x el) rest

data Stack a = Empty | St [a] deriving Show
data Queue a = EmptyQ | Qu [a] deriving Show
```

Parametric Polymorphism

- Polymorphic types: A value can have multiple types. Functions operate on multiple types uniformly
- `identity x = x` function. type: $\alpha \rightarrow \alpha$
  `identity 4` : 4, `identity "ali"` : "ali" , `identity (5,"abc")` : (5,"abc")
  $int \rightarrow int$, $String \rightarrow String$, $int \times String \rightarrow int \times String$
- `compose f g x = f (g x)` function
  type: $(\beta \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta) \rightarrow \alpha \rightarrow \gamma$
  `compose square double 3` : 36,
  $(int \rightarrow int) \rightarrow (int \rightarrow int) \rightarrow int \rightarrow int$.
  `compose listsum reverse [1,2,3,4]` : 10
  $([int] \rightarrow int) \rightarrow ([int] \rightarrow [int]) \rightarrow [int] \rightarrow int$

- ```
  filter f [] = []
  filter f (x:r) = if (f x) then x:(filter f r) else (filter r)
  ```
  $(\alpha \rightarrow Bool) \rightarrow [\alpha] \rightarrow [\alpha]$
  `filter ((<) 3) [1,2,3,4,5,6]` : [4,5,6]
  $(int \rightarrow Bool) \rightarrow [int] \rightarrow [int]$
  `filter identity [True, False, True, False]` : [True,True]
  $(Bool \rightarrow Bool) \rightarrow [Bool] \rightarrow [Bool]$
- Operations are same, types are different.
- Types with type variables: polytypes
- Most functional languages are polymorphic
- Object oriented languages provide polymorphism through inheritance
  ```

Overloading

- Overloading: Using same identifier for multiple places in same scope
- Example: Two different functions, two distinct types, same name.
- Polymorphic function: one function that can process multiple types.
- C++ allows overloading of functions and operators.

```
typedef struct Comp { double x, y; } Complex;
double mult(double a, double b) { return a*b; }
Complex mult(Complex s, Complex u) {
    Complex t;
    t.x = s.x*u.x - s.y*u.y;
    t.y = s.x*u.y + s.y*u.x;
    return t;
}
Complex a,b; double x,y; ... ; a=mult(a,b) ; x=mult(y,2.1);
```

- Binding is more complicated. not only according to name but according to name and type
- Function type:

  $\texttt{name} : \ parameters \rightarrow result$

- Context dependent overloading :
  Overloading based on function name, parameter type and return type.
- Context independent overloading : Overloading based on function name and parameter type. No return type!

Context dependent overloading

- Which
  type does the expression calling the function expects (context) ?

```
int f(double a) { ....① }
int f(int a) { ....② }
double f(int a) { ....③ }
double x,y;
int a,b;
```

- a=f(x); ① (x double)
  a=f(a); ② (a int, assign int)
  x=f(a); ③ (a int, assign double)
  x=2.4+f(a); ③ (a int, mult double)
  a=f(f(x)); ②(①) (x double, f(x):int, assign int)
  a=f(f(a)); ②(②) or ①(③) ???
- Problem gets more complicated. (even forget about coercion)

Context independent overloading

- Context dependent overloading is more expensive.
- Complex and confusing. Useful as much?
- Most overloading languages are context independent.
- Context independent overloading forbids ② and ③ functions defined together.
- "`name: parameters`" part should be unique in "`name: parameters` → `result`", in the same scope
- Overloading is not much useful. So languages avoid it.

Use carefully:

Overloading is useful only for functions doing same operations. Two functions with different purposes should not be given same names. Confuses programmer and causes errors

- Is variable overloading possible? What about same name for two types?

Coercion

- Making implicit type conversion for ease of programming.

```
double x;      int k;
x = k+4.2;      /* x = (double) k + 4.2 */
k = x+3.45;     /* k=(int) (x+3.45); */
k = x+2;        /* k=x+(double)2; */
k = x+k-2;      /* k=(int)(x+ (double)k - (double)2) ; */
```

- C makes *int* ↔ *double* coercions and pointer coercions (with warning)
- Are other type of coercions are possible? (like A * → A, A → A *). Useful?
- May cause programming errors: `x=k=3.25` : `x` becomes 3.0
- Coercion + Overloading: too complex.
- Most newer languages quit coercion completely (Strict type checking)

Type Inference

- Type system may force user to declare all types (C and most compiled imperative languages), or
- Language processor infers types. How?
- Each expression position provide information (put a constraint) on type inference:
  - Equality $e = x, x :: \alpha, y :: \beta \;\Rightarrow\; \alpha \equiv \beta$
  - Expressions $e = a + f\ x$, $+ :: Num \rightarrow Num \rightarrow Num \;\Rightarrow$ $a :: Num, f :: \alpha \rightarrow Num, e :: Num$
  - Function application $e = f\ x \;\Rightarrow\; e :: \beta, x :: \alpha, f :: (\alpha \rightarrow \beta)$
  - Type constructors $f\ (x : r) = t \;\Rightarrow\; x :: \alpha, t :: \beta, f :: ([\alpha] \rightarrow \beta)$
- Inference of all values start from the most general type (i.e: any type α)
- Type inference finds the most general type satisfying the constraints.