

Final - January 14, 2010

Name and Surname:

Student ID:

Department:

Signature:

You should show your work. You will lose points if you do not show your work. If the question asks you to find a vector, you should find both its magnitude and direction. The questions can contain unnecessary information of insufficient information. If the question contains insufficient information, make necessary assumptions. Any unnecessary assumptions will cost you points.

Discussion

Answer the following question with words only. You do not need to give a quantitative answer, a qualitative answer is enough. You will lose points if you use equations.

1. Consider straight, infinite conducting wires. The conductivity of the material used to make the wires is σ . Assume that the wires are placed side by side to form an infinite plane (the wires are insulated from one another). Define the x axis to be along the wires, the y axis to lie in this plane defined by the wires and perpendicular to the x axis, and the z axis to be in the direction of $\hat{x} \times \hat{y}$. If an electromagnetic wave is incident on this plane, what can you say about the polarization of the transmitted wave. (20 points)

Short Questions

2. Sketch the magnetic field lines of point charge moving with a constant velocity \vec{v} (10 points)
3. Consider two current carrying wires that are lying on a plane. What will be the direction of the forces if the currents are running in the (i) same (ii) opposite directions? What happens if the wires are perpendicular to each other? (10 points)

Explicit Calculations:

In answering following problems, show your steps in detail. You should also explain why you do a specific step.

4. Consider an infinite non-conducting plane. Inside this plane is embedded a non-conducting sphere of radius R such that the plane goes through the center of the sphere. The system as a whole has a uniform surface charge density σ . Using Gauss' Law, find the electric field at an arbitrary point in space. Show explicitly that away from the sphere, the electric field reduces to the electric field of an infinite plane. (15 points)
5. Consider a ferromagnetic material square doughnut. A wire is wound N_1 times on one side of the square and another wire is wound N_2 times on the other side of the square. Calculate an approximate expression for the mutual inductance of the system. (In your calculations, express the approximations that you have made) (15 points).
6. Show that, in electrostatics, the electric field at a point tip of a conductor diverges. For this aim, follow the following procedure:
 - (a) Consider a cone defined by the equation $\theta = \beta$. Any point that has azimuthal angle larger than β is inside the conductor. Find the most general solution for the electrostatic potential in spherical coordinate. (10 points)
 - (b) For points close to the tip, r is very small and hence only the leading terms is sufficient to describe the behaviour. For $\beta = 45^\circ$, $\beta = 90^\circ$, $\beta = 135^\circ$ and $\beta = 170^\circ$, estimate, using the figure, the value of the power of the leading term. (10 points)
 - (c) Using the leading term, argue that the electric field diverges close to the top for a pointed tip. (10 points)
7. Suppose that in a region of space, the electric field is given by $\vec{E}(\vec{r}, t) = \vec{E}_0 e^{-i(\omega t - \vec{k} \cdot \vec{r})}$.
 - (a) What should be the magnetic field? (10 points)
 - (b) What is the momentum density of the field averaged over one period? (5 points)
 - (c) What is the energy density of the field averaged over one period? (5 points)

Some useful formulas:

Associated Legendre Polynomials $P_l^m(x)$ are solutions of the differential equation:

$$\frac{d}{dx} \left[(1-x^2) \frac{dy}{dx} \right] + \left[l(l+1) - \frac{m^2}{1-x^2} \right] y = 0 \quad (1)$$

that are finite for $-1 \leq x \leq 1$. Bessel functions $J_n(x)$ are solutions of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0 \quad (2)$$

that are finite at $x = 0$. Maxwell's Equations in vacuum:

$$\begin{aligned} \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\ \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{B} &= 0 \end{aligned}$$