Three Phase Systems

by

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Electrical and Electronics Engineering Department
Three Phase Systems

Why Three Phase?

The reasons for using Three Phase

The main reasons for using three phase systems are;

1. The kVA rating of the three phase equipment (i.e. machinery or transformer) is 150% greater than that of a single phase equipment with the same frame structure (weight, size).

Single Phase  Three Phase

This part of the metal work is not utilized (wasted)
Three Phase Systems

Why Three Phase?

The Reasons for using Three Phase

Please note that most of the iron part in the stator and rotor are not utilized (wasted)

This part of the metal work is not utilized (wasted)
Three Phase Systems

Why Three Phase?

The Reasons for using Three Phase

Please note that most of the iron part in the stator and rotor are now fully utilized (not wasted)

This part is now fully utilized (not wasted)

Three Phase
Three Phase Systems

Why Three Phase?

The Reasons for using Three Phase

The main reasons for using three phase systems are:

2. Conductor volume in a three phase system is about 25-40% less than that of a single phase two-wire system with the same kVA rating.

**Current:** $I = \frac{1.000.000 \text{ VA}}{34.500 \text{ V} \times 0.85}$

$= 34.10 \text{ Amp}$

Cross section = $6 \text{ mm}^2$

Cond. volume = $1000 \times 2 \times 6 \times 10^{-6} = 0.012 \text{ m}^3$

**Current:** $I = \frac{1.000.000 \text{ VA}}{\sqrt{3} \times 34.500 \text{ V} \times 0.85}$

$= 19.69 \text{ Amp}$

Cross section = $2.5 \text{ mm}^2$

Cond. volume = $1000 \times 3 \times 2.5 \times 10^{-6} = 0.0075 \text{ m}^3$
Three Phase Systems

Three Phase Voltages

Circuit - a

\[ V_a(t) = V_{\text{max}} \cos \omega t \]
\[ I_a(t) = I_{\text{max}} \cos(\omega t - \theta) \]

Circuit - b

\[ V_b(t) = V_{\text{max}} \cos(\omega t - 120^\circ) \]
\[ I_b(t) = I_{\text{max}} \cos(\omega t - 120^\circ - \theta) \]

Circuit - c

\[ V_c(t) = V_{\text{max}} \cos(\omega t - 240^\circ) \]
\[ I_c(t) = I_{\text{max}} \cos(\omega t - 240^\circ - \theta) \]

Circuit - a

\[ Z_a = 3 + j4 \ \Omega \]

\[ \angle Z_a = \theta = \tan^{-1} \left( \frac{4}{3} \right) \]
\[ = 53.13^\circ \]

Circuit - b

\[ Z_b = 3 + j4 \ \Omega \]

\[ \angle Z_b = \theta = \tan^{-1} \left( \frac{4}{3} \right) \]
\[ = 53.13^\circ \]

Circuit - c

\[ Z_c = 3 + j4 \ \Omega \]

\[ \angle Z_c = \theta = \tan^{-1} \left( \frac{4}{3} \right) \]
\[ = 53.13^\circ \]
Three Phase Systems

Three Phase Voltages

Circuit - a

\[ V_a(t) = V_{\text{max}} \cos(wt) \]
\[ I_a(t) = I_{\text{max}} \cos(wt - 53.13^\circ) \]

Circuit - b

\[ V_b(t) = V_{\text{max}} \cos(wt - 120^\circ) \]
\[ I_b(t) = I_{\text{max}} \cos(wt - 120^\circ - 53.13^\circ) \]

Circuit - c

\[ V_c(t) = V_{\text{max}} \cos(wt - 240^\circ) \]
\[ I_c(t) = I_{\text{max}} \cos(wt - 240^\circ - 53.13^\circ) \]
Three Phase Systems

Three Phase Voltages

Circuit - a

\[ V_a(t) = V_{max} \cos \omega t \]
\[ I_a(t) = I_{max} \cos(\omega t - 53.13^\circ) \]

Circuit - b

\[ V_b(t) = V_{max} \cos(\omega t - 120^\circ) \]
\[ I_b(t) = I_{max} \cos(\omega t - 120^\circ - 53.13^\circ) \]

Circuit - c

\[ V_c(t) = V_{max} \cos(\omega t - 240^\circ) \]
\[ I_c(t) = I_{max} \cos(\omega t - 240^\circ - 53.13^\circ) \]
Three Phase Systems

Connection of Three Phase Voltages

Now, re-draw the above three phase circuits in an alternative form as shown on the RHS.

Please note that the three circuits are still electrically unconnected.
Three Phase Systems

Connection of Three Phase Voltages

Phase Currents (Amp)

\[
\begin{align*}
I_a(t) & = V_{\text{max}} \cos wt \\
I_b(t) & = V_{\text{max}} \cos (wt - 120^\circ) \\
I_c(t) & = V_{\text{max}} \cos (wt - 240^\circ)
\end{align*}
\]

\[I_a(t) + I_b(t) + I_c(t)\]
Three Phase Systems

Three Phase Voltage Waveforms

Circuit - a

\[ V_a(t) = V_{\text{max}} \cos wt \]

Circuit - b

\[ V_b(t) = V_{\text{max}} \cos( wt -120^\circ ) \]

Circuit - c

\[ V_c(t) = V_{\text{max}} \cos( wt -240^\circ ) \]

Phase Voltages (Volts)

21.2 kV

Time (msec)

20 / 3 = 6.67 msec

20 msec
Three Phase Systems

Three Phase Voltage and Current Phasors

Three phase voltage and current phasors may be drawn as shown on the RHS.

\[
\tan^{-1}\left(\frac{X}{R}\right) = \frac{V - I}{I} = -53.13^\circ \leftrightarrow -53.13^\circ / 360^\circ \times 20 \text{ sec.}
\]
Three Phase Systems

Balanced Three Phase Circuits

**Definition**

Please note that voltage and current phasors for each phase are $120^\circ$ displaced from each other

(a) $|V_a| = |V_b| = |V_c|$
(b) $V_a - V_b = V_b - V_c = V_c - V_a = 120^\circ$
(c) $|I_a| = |I_b| = |I_c|$
(d) $I_a - I_b = I_b - I_c = I_c - I_a = 120^\circ$

or

(e) $V_a + V_b + V_c = 0$
(f) $I_a + I_b + I_c = 0$
Three Phase Systems

Balanced Three Phase Circuits

**Definition**

A three phase system satisfying the above condition is said to be "balanced"

In a balanced three phase system,
- sum of phase currents is zero,
- sum of phase voltages is zero

\[ V_a + V_b + V_c = 0 \]
\[ I_a + I_b + I_c = 0 \]
Three Phase Systems

Balanced Three Phase Circuits

**Balance Condition**

\[ V_a + V_b + V_c = 0 \]

\[ I_a + I_b + I_c = 0 \]

*Please note that the central point does not move*
Three Phase Systems

Balanced Three Phase Circuits

In a balanced three phase system, sum of currents at any instant is always zero, i.e.

\[ I_a + I_b + I_c = 0 \]

\[ \tan^{-1} \left( \frac{X}{R} \right) = -53.13^\circ \]

\[ I_a + I_b + I_c = 0 \]

Phase Currents (Amp)

\[ V_a(t) = V_{max} \cos wt \]

\[ V_b(t) = V_{max} \cos(wt - 120^\circ) \]

\[ V_c(t) = V_{max} \cos(wt - 240^\circ) \]

\[ I_a(t) \]

\[ I_b(t) \]

\[ I_c(t) \]
In a balanced three phase system, no current will flow if we connect the ground wires of the above three circuits

\[ I_a + I_b + I_c = 0 \]
Three Phase Systems

Three Phase Measurement-Energy Analyzer

Three Phase Energy Analyzer

Energy analyzer shown on the RHS is capable of reading and recording three phase voltages and currents in rms, peak and time - waveform and transmitting the resulting data to computer.
Three Phase Systems

Three Phase Measurement-Energy Analyser

EPR-04S

- Cosφ
- Aktif güç
- Reaktif güç
- Görünür güç
- Aktif enerji
- Reaktif enerji
- Dijital giriş
- Enerji pulse çıkışı
- Demand
- 2 Ayrı enerji kaydı
- RS-485 haberleşme
- Toplam aktif, reaktif ve görünür güç

CT-25 Akım Trafosu (1-120A)
Three Phase Systems

Three Phase Circuit Breaker

Three Phase Circuit Breaker

Three phase low voltage circuit breaker is a device that breaks the three phases of power service automatically or manually.

This dashed line implies that poles operate in “gang” manner.

Three Phase Load

“Gang” Mechanism

- a
- b
- c

Otomatik Sigortalar

Kaçak Akım Röleleri

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Three Phase Systems

Three Phase Measurement

Three Phase Power Analyzer

Device shown on the RHS is capable of reading and recording three phase voltages and currents in rms, peak and time-waveform and transmitting the resulting data to computer.

Clamp type Current Transformers
## Three Phase Systems

### Three Phase Circuit

<table>
<thead>
<tr>
<th>Phase - a</th>
<th>Phase - b</th>
<th>Phase - c</th>
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</table>

![Three Phase Circuit Diagram](image)

\( V_a \) \( I_a \) \( V_b \) \( I_b \) \( V_c \) \( I_c \)
## Three Phase Systems

### Three Phase Circuit Connection

<table>
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</table>

\[
I_a(t) + I_b(t) + I_c(t) = 0
\]
Three Phase Systems

Three Phase Circuit Connection

Basic Diagram

\[ I_a(t) + I_b(t) + I_c(t) = 0 \]
Three Phase Systems

Three Phase Cable

Basic Diagram

\[ I_a(t) + I_b(t) + I_c(t) = 0 \]
Three Phase Systems

Turkish 380 kV System
Three Phase Systems

Ankara 154 kV Ring

Please note that the lines form a closed ring
Three Phase Systems

Istanbul Anatolian Side MV System
Three Phase Systems

Three Phase Synchronous Generator

Phase Voltage (volts)

Time (msec)
Three Phase Systems

Three Phase Generation System

Three Phase Generator
Three Phase Systems

Karakaya Hydroelectric Plant – 1800 MW
Three Phase Systems

Hydroelectric Plant - Sectional View

Configuration
Three Phase Systems

Hydroelectric Plant - Sectional View

Typical Hydroelectric Plant
Three Phase Systems

Atatürk Hydroelectric Plant; 8 x 300 = 2400 MW
Three Phase Systems

Hydroelectric Plant - Water Turbine
Three Phase Systems

Generation of AC Voltage - Synchronous Generator

Bagnell Dam on Ozarks Lake

Turbine - Generator Set

Water Flow
Three Phase Systems

Atatürk Dam Generator Room
Three Phase Systems

Itaipu Power Plant - 12500 MW
Stator Mounting Ceremony
Three Phase Systems

Combined Cycle Power Plant
Three Phase Systems

Thermal (Coal) Power Plant
Three Phase Systems

Parallel Operation of Plants (Double Bus Configuration)

Open circuit breaker

Units

Main Bus

Spare Bus

Load and Feeder Group
Three Phase Systems

Phase and Line Voltages

Definition

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage

\[ I_a(t) + I_b(t) + I_c(t) = 0 \]
Three Phase Systems

Phase and Line Voltages

**Definition**

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage

Birds prefer neutral wire since the voltage is zero

\[ I_a(t) + I_b(t) + I_c(t) = 0 \]
Three Phase Systems

Phase and Line Voltages

**Definition**

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage

Neutral wire acts as lightning arrestor

\[ I_a(t) + I_b(t) + I_c(t) = 0 \]
Three Phase Systems

Phase and Line Voltages

**Definition**

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage

<table>
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<th>$V_{an}$ = Phase Voltage</th>
<th>$V_{ab}$ = Line Voltage</th>
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<td>$V_{an}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{ab}$</td>
<td></td>
<td></td>
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</table>

Diagram:

- Three phase conductors (a, b, c) connected.
- Ground (n) connected to the circuit.
- Phases a, b, c connected in a loop.
- Voltage symbol $\pm$ indicating direction.

**Equations:**

- $V_{an}$ = Phase Voltage
- $V_{ab}$ = Line Voltage
Three Phase Systems

Relation between Phase and Line Voltages

**Definition**

\[ V_{ab} = V_a - V_b = V_a + (-V_b) \]

- \( V_{an} \) = Phase Voltage
- \( V_{ab} \) = Line Voltage

\[ V_a - V_b = V_a + (-V_b) \]

\[ V_c \]

\[ V_b \]

\[ V_a \]

\[ 30^\circ \]
### Three Phase Systems

#### Relation between Phase and Line Voltages

**Definition**

\[
V_{ab} = V_a - V_b = V_a + (-V_b)
\]

\[
\left| V_{ab} \right| / 2 = \left| V_a \right| \cos 30 \approx \left| V_a \right| \frac{\sqrt{3}}{2}
\]

\[
\left| V_{ab} \right| = \left| V_a \right| \cdot \sqrt{3}
\]

**Line Voltage** = \( \sqrt{3} \) x **Phase Voltage**
Three Phase Systems

Example - Turkish HV System

Example (Turkish HV System)

| Line Voltage | = | \sqrt{3} x | Phase Voltage |

Phase voltage (rms) = 220 kV
Line voltage (rms) = 220 x \sqrt{3} = 380 kV

\[ V_{ab} / 2 = \frac{|V_a + (-V_b)|}{2} \]
Three Phase Systems

Star (Y) Connection

**Definition**

Star-connected configuration is the one with all the neutral points are connected to a common ground point.

**Alternative Representation**

| Line Voltage | = \( \sqrt{3} \times \) | Phase Voltage |

\[V_{ac} = \text{Line Voltage} \]

\[V_{an} = \text{Phase Voltage} \]
### Three Phase Systems

**Star (Y) Connected Generator**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Alternative Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star-connected generator is the one with all the generator neutral points are connected to a common ground point</td>
<td>[</td>
</tr>
</tbody>
</table>

- \( V_{ac} = \text{Line Voltage} \)
- \( V_{an} = \text{Phase Voltage} \)

![Diagram of Star Connected Generator](image-url)
Three Phase Systems

**Delta (Δ) Connected Generator**

**Definition**
Delta-connected generator is the one with the generator terminals are so connected that they form a triangle (delta)-configuration.

**Configuration**
| Line Voltage | = | Phase Voltage |

Vac = Line Voltage
# Three Phase Systems

## Currents in a Delta (Δ) Connected Generator

<table>
<thead>
<tr>
<th>Definition</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_a = I_{ca} - I_{ab}$</td>
<td></td>
</tr>
<tr>
<td>$I_b = I_{ab} - I_{bc}$</td>
<td></td>
</tr>
<tr>
<td>$I_c = I_{bc} - I_{ca}$</td>
<td></td>
</tr>
</tbody>
</table>

**Configuration**

- $I_{ca} = $ Phase Current
- $I_a = $ Line Current
- $I_c = $ Line Current
- $I_{ab} = $ Line Current
- $I_{bc} = $ Line Current

**Diagram:**

[Diagram showing currents in a delta connected generator with arrows indicating currents $I_a$, $I_b$, $I_c$, $I_{ab}$, and $I_{bc}$.]
Three Phase Systems

Currents in a Delta (Δ) Connected Generator

Relation between Line and Phase Currents

\[ I_a = I_{ca} + (-I_{ab}) \]

- \( I_{ba} = \text{Phase Current} \)
- \( I_a = \text{Line Current} \)

\[ I_a = I_{ca} + (-I_{ab}) \]

\[ V_{ab} \]
\[ V_{ca} \]
\[ V_{bc} \]
Three Phase Systems

Currents in a Delta (Δ) Connected Generator

**Definition**

\[ I_a = I_{ca} + (-I_{ab}) \]

\[ \frac{|I_{\text{line}}|}{2} = |I_{\text{phase}}| \times \cos 30^\circ \]

\[ = |I_{\text{phase}}| \times \sqrt{3}/2 \]

**Line Current** = \( \sqrt{3} \times |\text{Phase Current}| \)

**Relation between Line and Phase Currents**

\[ \hat{I}_{\text{line}} = \hat{I}_{ba} - \hat{I}_{ac} \]

\[ \hat{I}_{\text{line}}/2 \]

\[ \hat{V}_{ab} \]

\[ \hat{V}_{bc} \]

\[ \hat{V}_{ca} \]

\[ \hat{I}_{ab} \]

\[ \hat{I}_{bc} \]

\[ \hat{I}_{ca} \]
### Three Phase Systems

#### Summary

**Star (Y) Connected System**

- Line Voltage $| = \sqrt{3} \times | Phase Voltage $ |
- Line Current = Phase Current

**Delta (Δ) Connected System**

- Line Voltage = Phase Voltage
- Line Current $| = \sqrt{3} \times | Phase Current $ |

**Electrical Symbols**

- $V_{an}$ = Phase Voltage
- $I_{ba} = Phase Current$
- $V_{ab}$ = Line Voltage
- $I_{a} = Line Current$
- $I_{bc} = Phase Current$
- $I_{ab} = Line Current$
- $I_{ca} = Line Current$
- $I_{b} = Line Current$
Three Phase Systems

Star-Star (Y-Y) Connected Systems

**Definition**

A star-star connected system is the one with both ends are star connected.

**Configuration**

Please note that neutral wire does not carry any current for a balanced Y-Y System.

Please note that the neutral wire is not really essential, since it carries no current. Hence it can be placed.
Three Phase Systems

Star-Delta (Y-Δ) Connected Systems

**Definition**
A star-delta connected system is the one with the source is star and load is delta connected

**Configuration**
Please note that neutral wire does not exist in a system with one or both ends delta.
### Delta-Star (Δ - Y) Connected Systems

#### Definition

A delta-star connected system is the one with the source is delta and load is star connected.

#### Configuration

Please note that neutral wire does not exist in a system with one or both ends delta.

![Diagram of Delta-Star Connected System]

- **Ia**: Current through the a phase
- **Ib**: Current through the b phase
- **Ic**: Current through the c phase
- **Ibc**: Current through the bc phase
- **Ica**: Current through the ca phase
- **Iab**: Current through the ab phase
- **n**: Neutral point

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Three Phase Systems

Three Phase Power in Star Connected Loads

**Star (Y) Connected System**

<table>
<thead>
<tr>
<th>Line Voltage</th>
<th>= $\sqrt{3}$ x</th>
<th>Phase Voltage</th>
</tr>
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<tr>
<td>Line Current = Phase Current</td>
<td></td>
<td></td>
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</tbody>
</table>

### Power Per Phase

$$S_{ph} = P_{ph} + j Q_{ph} = V_{ph} I_{ph}^*$$

Since we have three phases with identical power consumption, total power consumption becomes

$$S_{3-ph} = 3 \times S_{ph} = 3 V_{ph} I_{ph}^* = 3 \left( V_{line} / \sqrt{3} \right) I_{line}$$

$$S_{3-ph} = \sqrt{3} V_{line} I_{line}$$
Three Phase Systems

Three Phase Active Power in Star Connected Loads

**Star (Y) Connected System**

Remember that:

\[ P_{ph} = S_{ph} \cos \theta \]

Since we have three phases with identical power consumption, total power consumption becomes

\[ P_{3-ph} = 3 \times S_{ph} \cos \theta \]

\[ = 3 V_{ph} I_{ph} \cos \theta \]

\[ = 3 \left( \frac{V_{line}}{\sqrt{3}} \right) I_{line} \cos \theta \]

\[ P_{3-ph} = \sqrt{3} V_{line} I_{line} \cos \theta \]

**Line Voltage** \( = \sqrt{3} \times \) **Phase Voltage**

**Line Current** = **Phase Current**
Three Phase Systems

Three Phase Reactive Power in Star Connected Loads

**Star (Y) Connected System**

Remember that:

\[ Q_{ph} = S_{ph} \sin \theta \]

Since we have three phases with identical power consumption, total power consumption becomes:

\[ Q_{3-ph} = 3 \times S_{ph} \sin \theta = 3 V_{ph} I_{ph} \sin \theta = 3 \left( \frac{V_{line}}{\sqrt{3}} \right) I_{line} \sin \theta \]

\[ Q_{3-ph} = \sqrt{3} V_{line} I_{line} \sin \theta \]

<table>
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\[ V_{ac} = \text{Line Voltage} \]
\[ V_{an} = \text{Phase Voltage} \]
Three Phase Reactor

Remember that:

\[ Q_{3-ph} = 3 \times S_{ph} \sin \theta \]
\[ = 3 \, V_{ph} \, I_{ph} \sin \theta \]
\[ = 3 \left( \frac{V_{line}}{\sqrt{3}} \right) I_{line} \sin \theta \]

\[ Q_{3-ph} = \sqrt{3} \, V_{line} \, I_{line} \sin \theta \]

Line Voltage \( = \sqrt{3} \times \) Phase Voltage

Line Current = Phase Current

Phase - a

Phase - b

Phase - c
Three Phase Systems

Three Phase Reactor

Small-Size Three Phase Reactor

Remember that:

\[ Q_{3-ph} = 3 \times S_{ph} \sin \theta \]
\[ = 3 V_{ph} I_{ph} \sin \theta \]
\[ = 3 \left( \frac{V_{line}}{\sqrt{3}} \right) I_{line} \sin \theta \]

\[ Q_{3-ph} = \sqrt{3} V_{line} I_{line} \sin \theta \]

Line Voltage \( = \sqrt{3} \times \) Phase Voltage

Line Current = Phase Current
Three Phase Systems

Three Phase Transformer

Primary Side (Delta)

- Primary Side Phase - a
- Primary Side Phase - b
- Primary Side Phase - c

Voltages

Primary Side

\[ V_{\text{line}} = 34500 \text{ Volts} \]
\[ V_{\text{phase}} = V_{\text{line}} = 34500 \text{ Volt} \]

Secondary Side

\[ V_{\text{line}} = 380 \text{ Volts} \]
\[ V_{\text{phase}} = 380 / \sqrt{3} = 220 \text{ Volts} \]
Three Phase Systems

Three Phase Transformer

Primary Side (Delta)

Primary Side Phase - a
Primary Side Phase - b
Primary Side Phase - c

Secondary Side (Star)

Secondary Side Phase - a
Secondary Side Phase - b
Secondary Side Phase - c

Please note that almost all distribution transformers are delta-star connected.
### Three Phase Power

#### (Overview)

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
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<tr>
<td>$P_{\text{prim. - a}}$</td>
<td>$V_a I_a \cos \theta$</td>
</tr>
<tr>
<td>$P_{\text{prim. - b}}$</td>
<td>$V_b I_b \cos \theta$</td>
</tr>
<tr>
<td>$P_{\text{prim. - c}}$</td>
<td>$V_c I_c \cos \theta$</td>
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$$
P_{\text{prim. - Total}} = V_a I_a \cos \theta + V_b I_b \cos \theta + V_c I_c \cos \theta
$$

$$
= 3 \ V_{\text{phase}} I_{\text{phase}} \cos \theta
$$

$$
= 3 \ V_{\text{line}} I_{\text{line}} / \sqrt{3} \cos \theta
$$

$$
= \sqrt{3} \ V_{\text{line}} I_{\text{line}} \cos \theta
$$
Three Phase Systems

Three Phase Transformer

Three-Phase Power (Overview)

Power on the Primary Side

\[ P_{\text{Prim. - Total}} = \sqrt{3} V_{\text{Prim.- line}} I_{\text{Prim.-line}} \cos \theta \]
\[ Q_{\text{Prim. - Total}} = \sqrt{3} V_{\text{Prim.- line}} I_{\text{Prim.-line}} \sin \theta \]
\[ S_{\text{Prim. - Total}} = \sqrt{3} V_{\text{Prim.- line}} I_{\text{Prim.-line}} \]

Power on the Secondary Side

\[ P_{\text{Sec. - Total}} = \sqrt{3} V_{\text{Sec.- line}} I_{\text{Sec.-line}} \cos \theta \]
\[ Q_{\text{Sec. - Total}} = \sqrt{3} V_{\text{Sec.- line}} I_{\text{Sec.-line}} \sin \theta \]
\[ S_{\text{Sec. - Total}} = \sqrt{3} V_{\text{Sec.- line}} I_{\text{sec.-line}} \]
Three Phase Systems

Example: Star (Y) Connected Load

| Line Voltage | = $\sqrt{3} \times$ | Phase Voltage |

Line Current = Phase Current

135 MVA shunt reactor delivered to Nevada Power Company by VA TECH ELIN

Line Voltage

$V_{ac} = \text{Line Voltage}$

$V_{an} = \text{Phase Voltage}$

Phase Voltage
You do not seem to understand the line voltage in a Y-Connected Load ...
### Three Phase Systems

#### Three Phase Power in Delta Connected Loads

**Delta (Δ) Connected System**

**Power Per Phase**

\[ S_{ph} = P_{ph} + j Q_{ph} = V_{ph} I_{ph}^* \]

Since we have three phases with identical power consumptions, three phase (total) power consumption becomes

\[ S_{3-ph} = 3 \times S_{ph} = 3 \times V_{ph} I_{ph}^* = 3 \times V_{line} \left( I_{line} / \sqrt{3} \right) \]

\[ S_{3-ph} = \sqrt{3} \times V_{line} I_{line} \]
Three Phase Systems

Three Phase Power in Delta Connected Loads

Delta (Δ) Connected System

Power Per Phase

\[ P_{ph} = V_{ph} I_{ph} \cos \theta \]

Since we have three phases with identical power consumptions, three phase (total) power consumption becomes

\[ P_{3-ph} = 3 \times P_{ph} \]

\[ = 3 \ V_{ph} I_{ph} \cos \theta \]

\[ = 3 \ V_{line} \left( \frac{I_{line}}{\sqrt{3}} \right) \cos \theta \]

\[ P_{3-ph} = \sqrt{3} \ V_{line} I_{line} \cos \theta \]
Three Phase Systems

Three Phase Power in Delta Connected Loads

**Delta (Δ) Connected System**

**Power Per Phase**

\[ Q_{ph} = V_{ph} I_{ph} \sin \theta \]

Since we have three phases with identical power consumptions, three phase (total) power consumption becomes

\[ Q_{3-ph} = 3 \times Q_{ph} = 3 \times V_{ph} I_{ph} \sin \theta = 3 \times V_{line} \left( \frac{I_{line}}{\sqrt{3}} \right) \sin \theta \]

\[ Q_{3-ph} = \sqrt{3} \times V_{line} I_{line} \sin \theta \]
### Three Phase Power - Summary

#### Star (Y) Connected System

- \( S_{3-ph} = \sqrt{3} V_{\text{line}} I_{\text{line}} \)

#### Delta (Δ) Connected System

- \( P_{3-ph} = \sqrt{3} V_{\text{line}} I_{\text{line}} \cos \theta \)
- \( Q_{3-ph} = \sqrt{3} V_{\text{line}} I_{\text{line}} \sin \theta \)

**Three phase power expressions are identical for star and delta connections.**
Three Phase Systems

Star - Delta Conversion

Formulation

A star connected load can be converted to a delta connected load as follows:

\[ z_{ab} = \frac{(z_a z_b + z_b z_c + z_c z_a)}{z_c} \]
\[ z_{ca} = \frac{(z_a z_b + z_b z_c + z_c z_a)}{z_b} \]
\[ z_{bc} = \frac{(z_a z_b + z_b z_c + z_c z_a)}{z_a} \]

Please note that the neutral node is now eliminated.
Three Phase Systems

Star - Delta Conversion

If all impedances of star are identical, then the formula reduces to the following simple form

$$z_\Delta = \frac{z_Y^2 + z_Y^2 + z_Y^2}{z_Y} = 3z_Y$$

Please note that the neutral node is eliminated by transformation, i.e. no. of nodes is reduced by one.

[Diagram showing star and delta configurations with impedances labeled]
Three Phase Systems

Delta - Star Conversion

Formulation

A delta connected load can be converted to a star connected load as follows:

\[ z_a = \frac{z_{ba} z_{ac}}{z_{ba} + z_{ac} + z_{cb}} \]
\[ z_b = \frac{z_{cb} z_{ba}}{z_{ba} + z_{ac} + z_{cb}} \]
\[ z_c = \frac{z_{ac} z_{cb}}{z_{ba} + z_{ac} + z_{cb}} \]
Three Phase Systems

Delta - Star Conversion

If all impedances of delta are identical, then the formula reduces to the following special simple form:

\[ z_Y = \frac{z_\Delta^2}{z_\Delta + z_\Delta + z_\Delta} = \frac{z_\Delta}{3} \]
Solution Procedure for Three Phase Problems

**Procedure**

1. First convert all the Δ-connected loads, if any, to Y-connected loads by employing the Delta - Star Conversion Technique given in the previous section,

2. Find the source voltages / phase by dividing all the line voltages of the sources by $\sqrt{3}$ for the Y-connected sources

3. Decompose the given three phase system into three independent (electrically unconnected) single phase systems

*Line Voltage = 380 kV*
*Phase Voltage = 220 kV*
4. Then solve one of these single phase systems, i.e. in particular, the one which corresponds to phase-a,

5. Calculate active and reactive powers and power losses per phase,

6. Finally, multiply;
   a) all these active and reactive powers by three in order to find the three phase powers,
   b) all voltages by $\sqrt{3}$ in order to find the resulting line voltages
Solve the following three phase system for line currents and line losses.

- Line Voltage = 380 kV
- Phase Voltage = 220 kV

\[ r + jx = 1 + j2 \, \Omega \]
\[ r + jx = 9 + j12 \, \Omega \]

\[ I_L = \text{Line current} \]

\[ V_{s\text{line}} = 6.3 \, \text{kV (rms)} \]
Three Phase Systems

Example

Solution

1. First convert all the Δ-connected loads to Y-connected loads, if any, by employing the Delta - Star Conversion Technique given in the previous section

\[ Z_\Delta / 3 = (9 + j 12) / 3 = 3 + j 4 \ \Omega \]
Three Phase Systems

Example

Problem

Solve the following three phase system for line currents and line losses

Line Voltage = 380 kV

Phase Voltage = 220 kV

\[ I_L = \text{Line current} \]

\[ r + jx = 1 + j2 \Omega \]

\[ Z_{\Delta}/3 = 3 + j4 \Omega \]

\[ V_{s\text{line}} = 6.3 \text{ kV (rms)} \]

Neutral Wire

Phase Voltage = 220 kV

Line Voltage = 380 kV
Three Phase Systems

Example

Solution

2. find the source voltages / phase
3. Divide all the line voltages by $\sqrt{3}$ for Y-connected sources in order to

$$V_{s\text{ phase}} = \frac{V_{s\text{ line}}}{\sqrt{3}}$$

$$= \frac{6300}{1.732}$$

$$= 3637.41 \text{ Volts (rms)}$$

Vs(line) = 6.3 kV
3. Decompose the given three phase system into three independent (electrically unconnected) single phase systems

Solution

\[ V_a(t) = V_{\max} \cos wt \]
\[ I_a(t) \]

\[ V_c(t) = V_{\max} \cos(wt - 240^\circ) \]
\[ I_c(t) \]

\[ V_b(t) = V_{\max} \cos(wt - 120^\circ) \]
\[ I_b(t) \]
Three Phase Systems

Example

Solution

4. Then, solve one of the resulting three single phase systems, i.e. in particular, the one which corresponds to phase-a, with zero phase angle, due to its simplicity.

\[ \begin{align*}
I_a + jx &= 1 + j2 \Omega \\
V_a &= 3637.41 / 0^\circ \text{ Volts} \\
Z_{\Delta / 3} &= 3 + j4 \Omega
\end{align*} \]

\[ L_{\text{line}} = \frac{V_{a - \text{phase}}}{Z_{\text{tot}}} = \frac{3637.41 / 0^\circ}{[(1 + j2) + (3 + j4)]} \]

\[ = \frac{3637.41 / 0^\circ}{(4 + j6)} = 3637.41 / 0^\circ / 8.9442 / 56.31^\circ \]

\[ = 406.67 / -56.31^\circ \text{ Amp} \]
Example

Solution

5. Now, calculate;
   a) Active and reactive power losses

\[ r + jx = 1 + j2 \ \Omega \]
\[ I_a = 406.67 \angle -56.31^\circ \text{ Amp} \]
\[ V_a = 3637.41 \angle 0^\circ \text{ Volts} \]
\[ Z_\Delta / 3 = 3 + j 4 \ \Omega \]

**Line Losses:**

\[ r_{line} I^2 = 1 \times 406.67^2 = 165380 \text{ Watts / phase} \]
\[ = 165.38 \text{ kWs / phase} \]
\[ x_{line} I^2 = 2 \times 406.67^2 = 330760 \text{ Vars / phase} \]
\[ = 330.76 \text{ kVARs / phase} \]
Three Phase Systems

Example

Solution

6. Finally, multiply;
   a) Active and reactive power losses by three in order to find the three phase power losses

Three phase power losses:
Active power loss $= 3 \times \text{active power loss /phase}$
$= 3 \times 165.38 = 496.14 \text{ kWs}$

Reactive Power loss $= 3 \times 330.76 = 992.28 \text{ kVARs}$
Solution

7. Now, calculate the active and reactive powers consumed by the load;

Active and reactive power consumptions:

\[ r_{load} I_a^2 = 3 \times 406.67^2 = 496140 \text{ Watts / phase} \]
\[ = 496.14 \text{ kWs / phase} \]

\[ x_{load} I_a^2 = 4 \times 406.67^2 = 661520 \text{ Vars / phase} \]
\[ = 661.52 \text{ kVARs / phase} \]
Three Phase Systems

Example

Solution

8. Now, calculate the three phase active and reactive powers consumed by the load;

\[ I_a = 406.67 \angle -56.31^\circ \text{ Amp} \]
\[ r + jx = 1 + j2 \Omega \]
\[ V_a = 3637.41 \angle 0^\circ \text{ Volts} \]
\[ Z_\Delta / 3 = 3 + j4 \Omega \]

Three phase active and reactive power consumptions:

\[ 3 \times r_{load} I^2 = 3 \times 496.14 = 1488.42 \text{ kWs} \]
\[ 3 \times x_{load} I^2 = 3 \times 661.52 = 1984.56 \text{ kVARs} \]
Example

Solution

9. Now, calculate the load voltage / phase

Load Voltage/phase (Y-connected load)

\[ V_{\text{Load/phase}} = \frac{Z_{\Delta}}{3} \times I_a \]

\[ = (3 + j4) \times 406.67/-56.31^\circ \]

\[ = 5/53.13^\circ \times 406.67/-56.31^\circ \]

\[ = 2033.35/-3.18^\circ \text{ Volts/phase} \]
Three Phase Systems

Example

Solution

10. Now, calculate the load voltage / line

\[ V_{\text{Load}/\text{phase}} = 3637.41 \, \text{Volts} \]

\[ V_a = 3637.41 \angle 0^\circ \, \text{Volts} \]

\[ V_{\text{load} / \text{phase}} = 3637.41 \angle 0^\circ \, \text{Volts} \]

\[ Z_{\Delta} / 3 = 3 + j 4 \, \Omega \]

\[ Z_{\Delta} / 3 = 3 + j 4 \, \Omega \]

\[ I_a = 406.67 \angle -56.31^\circ \, \text{Amp} \]

\[ I_a = 406.67 \angle -56.31^\circ \, \text{Amp} \]

\[ r + jx = 1 + j2 \, \Omega \]

\[ \frac{V_{ab}}{} = \frac{V_a}{\theta} + 30^\circ \]

\[ V_{ab} = \frac{V_a}{\theta} + 30^\circ \]

\[ V_{ab} = \frac{V_a}{\theta} + 30^\circ \]

Load Voltage / line (Y-connected load)

\[ V_{\text{load} / \text{line}} = \sqrt{3} \times 2033.35 \angle -3.18^\circ + 30^\circ \]

\[ V_{\text{load} / \text{line}} = 3521.76 \angle 26.82^\circ \, \text{Volts / line} \]
Example

Solution

Please note that phase voltage across the delta is the same as line voltage, i.e.

\[ V_{\text{load/line}} = 3521.76 \, \sqrt{26.82^\circ} \, \text{Volts} \]

Load Voltage / line (Y-connected load)

\[ V_{\text{Load/line}} = V_{\text{Load/phase}} = 3521.76 \, /26.82^\circ \, \text{Volts/line} \]
Three Phase Systems

Did everybody understand three phase systems?