METU

Three Phase Systems

by
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Electrical and Electronics Engineering Department

Three Phase Systems

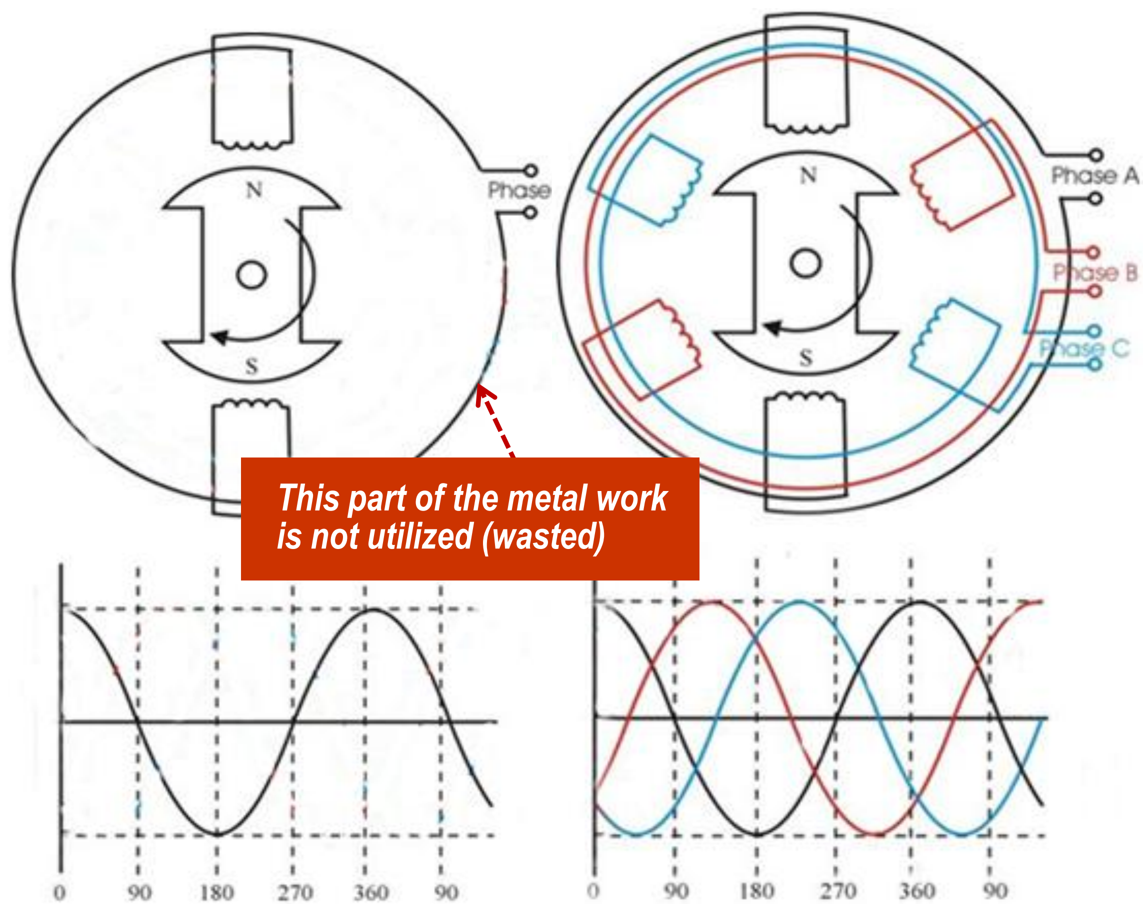
Why Three Phase ?

The reasons for using Three Phase

The main reasons for using three phase systems are;

1. The kVA rating of the three phase equipment (i.e. machinary or transformer) is is 150 % greater than that of a single phase equipment with the same frame structure (weight , size)

Single Phase | **Three Phase**

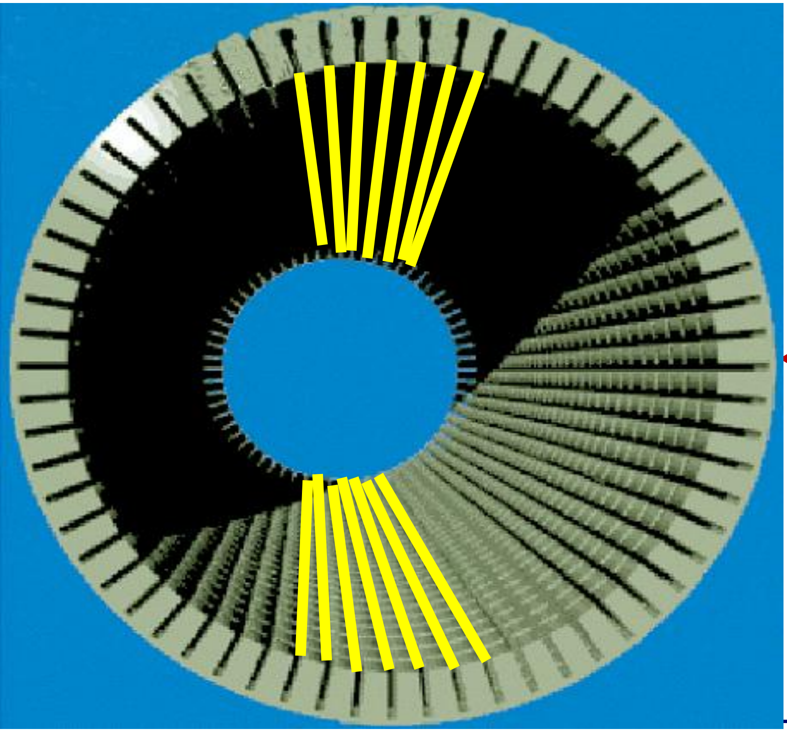


Why Three Phase ?

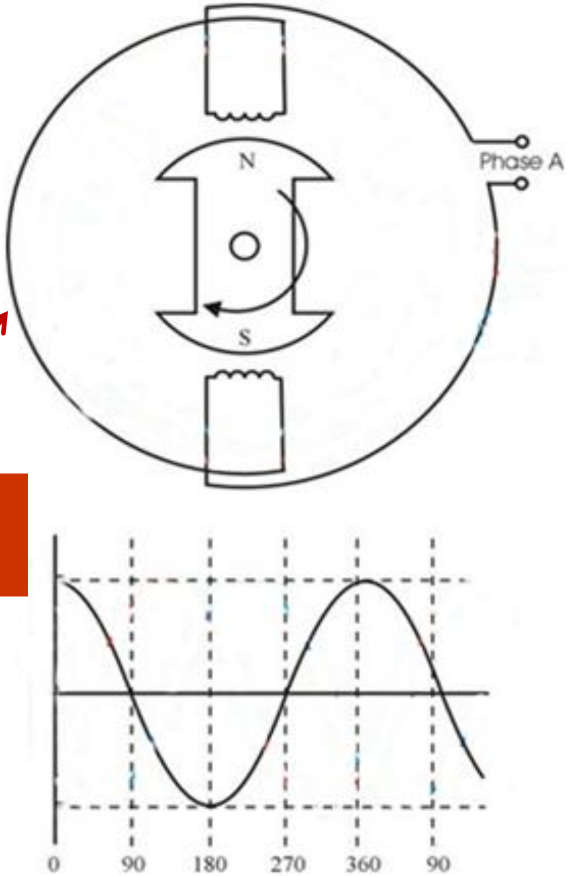
The Reasons for using Three Phase

Single Phase

Please note that most of the iron part in the stator and rotor are not utilized (wasted)



This part of the metal work is not utilized (wasted)

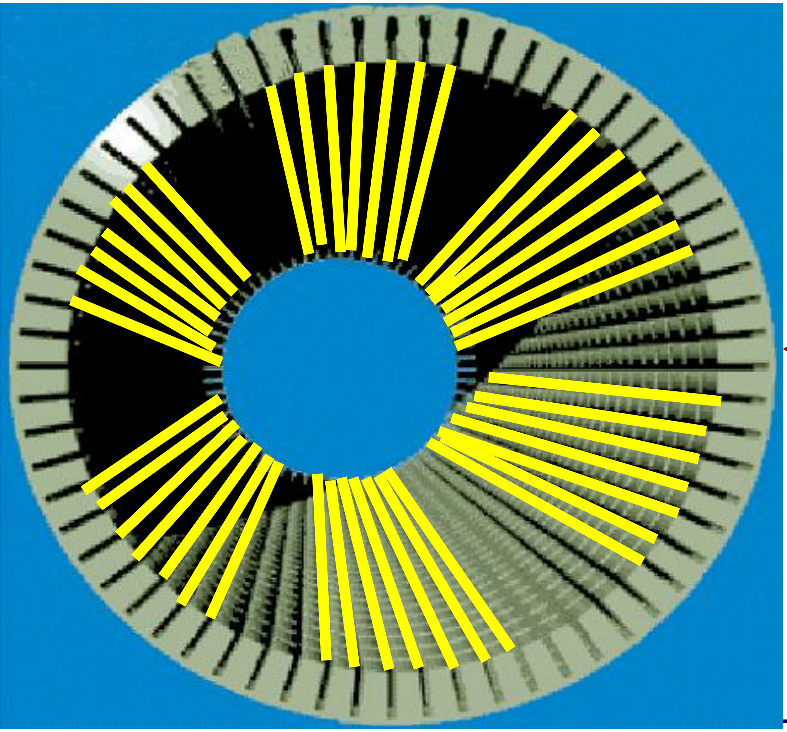


Three Phase Systems

Why Three Phase ?

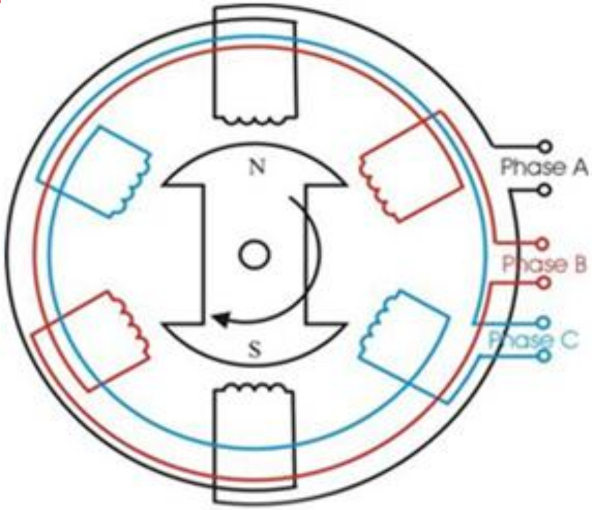
The Reasons for using Three Phase

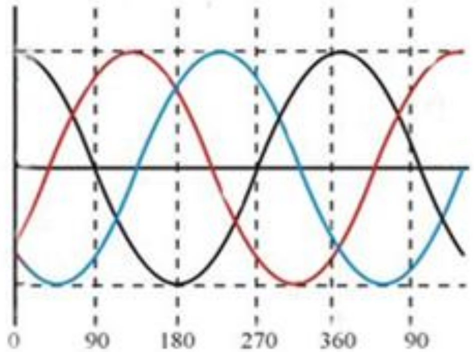
Please note that most of the iron part in the stator and rotor are now fully utilized (not wasted)



This part is now fully utilized (not wasted)

Three Phase





Three Phase Systems

Why Three Phase ?

The Reasons for using Three Phase

The main reasons for using three phase systems are;

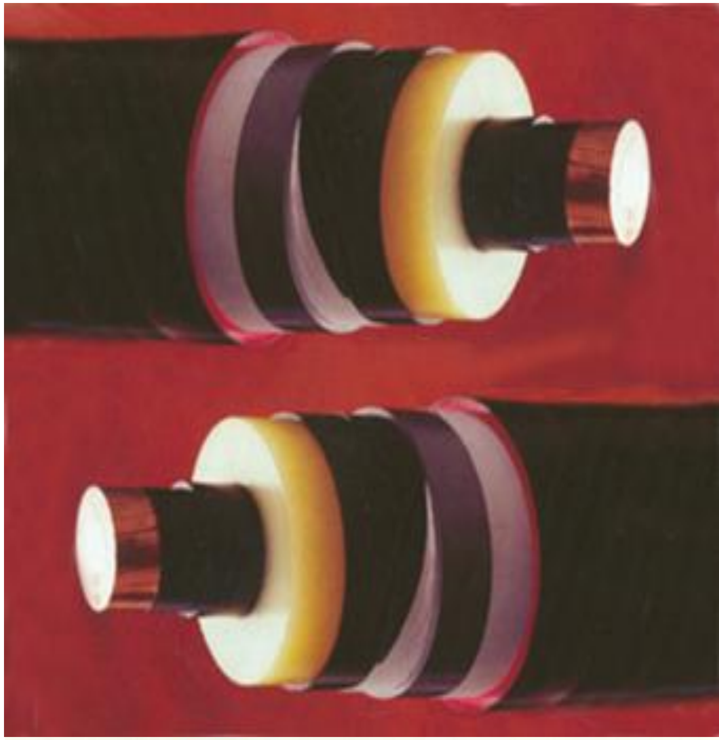
2. Conductor volume in a three phase system is about 25-40 % less than that of a single phase two-wire system with the same kVA rating.

Current: I = 1.000.000 VA / (34.500 V x 0.85)
= 34,10 Amp
Cross section = 6 mm²
Cond. volume = 1000 m x 2 x 6 x 10^{-6} = 0,012 m³

no. of conductors

Current: I = 1.000.000 VA / ($\sqrt{3}$ x 34.500 V x 0.85)
= 19,69 Amp
Cross section = 2,5 mm²
Cond. volume = 1000 x 3 x 2.5 x 10^{-6} = 0.0075 m³

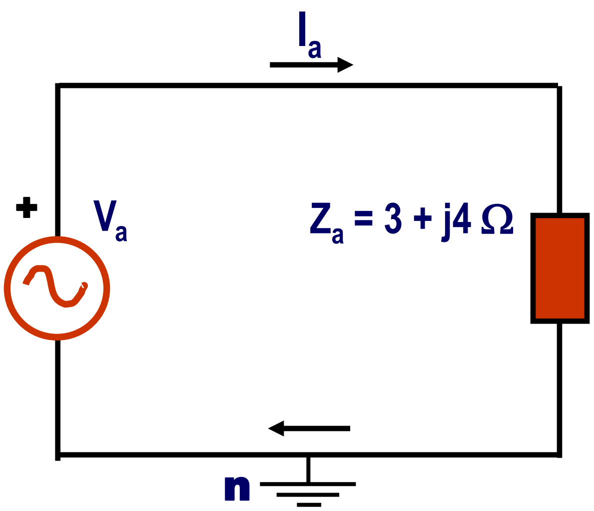
no. of conductors

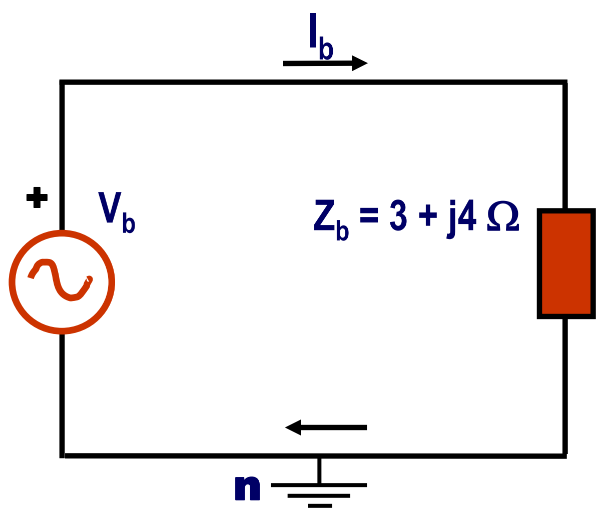


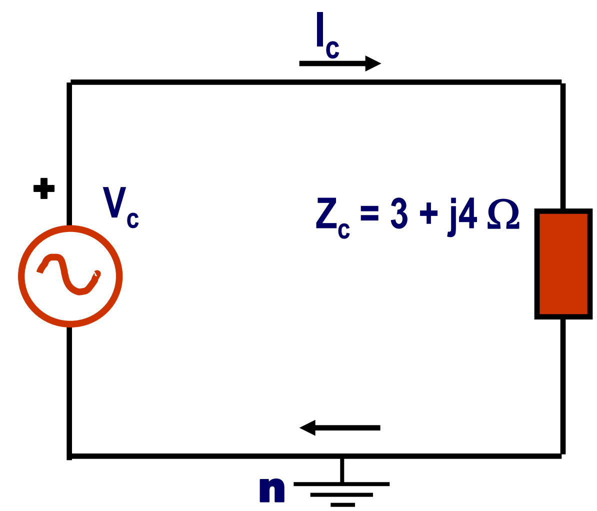
Three Phase Systems

Three Phase Voltages

Circuit - a	Circuit - b	Circuit - c
$V_a(t) = V_{max} \cos \ wt$ $I_a(t) = I_{max} \cos(wt - \theta)$	$V_b(t) = V_{max} \cos(wt - 120^o)$ $I_b(t) = I_{max} \cos(wt - 120^o - \theta)$	$V_c(t) = V_{max} \cos(wt - 240^o)$ $Ic(t) = I_{max} \cos(wt - 240^o - \theta)$







$\angle Z_a = \theta = Tan^{-1} (4 / 3)$ $= 53.13^o$	$\angle Z_b = \theta = Tan^{-1} (4 / 3)$ $= 53.13^o$	$\angle Z_c = \theta = Tan^{-1} (4 / 3)$ $= 53.13^o$

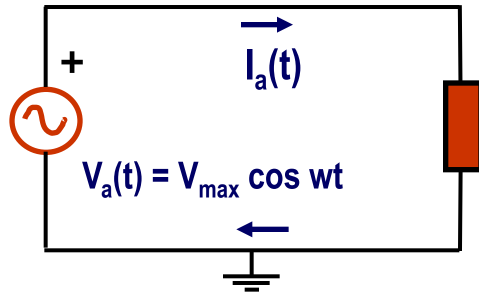
Three Phase Systems

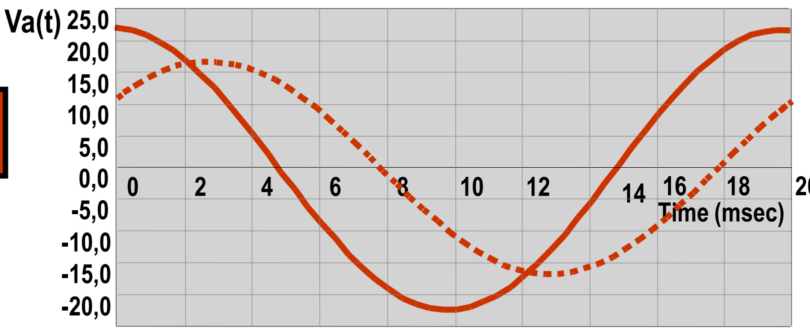
Three Phase Voltages

Circuit - a

$V_a(t) = V_{max} \cos\ wt$

$I_a(t) = I_{max} \cos(\ wt - 53.13^o\ )$

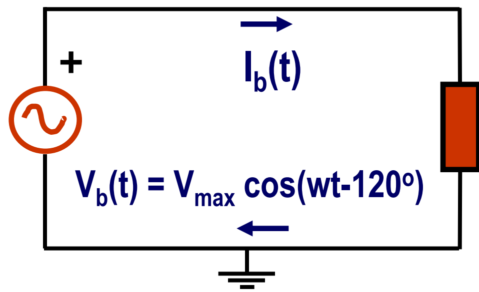


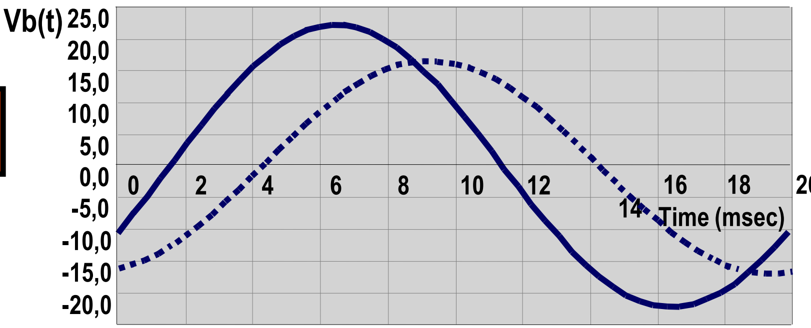


Circuit - b

$V_b(t) = V_{max} \cos(\ wt - 120^o\ )$

$I_b(t) = I_{max} \cos(wt - 120^o - 53.13^o)$

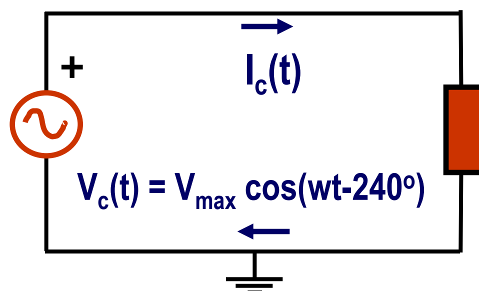


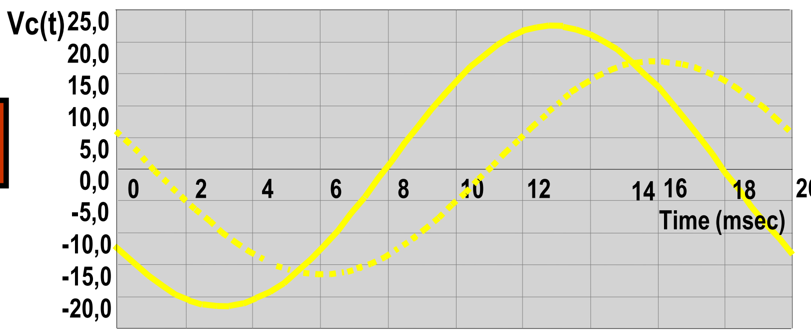


Circuit - c

$V_c(t) = V_{max} \cos(\ wt - 240^o\ )$

$I_c(t) = I_{max} \cos(wt - 240^o - 53.13^o)$





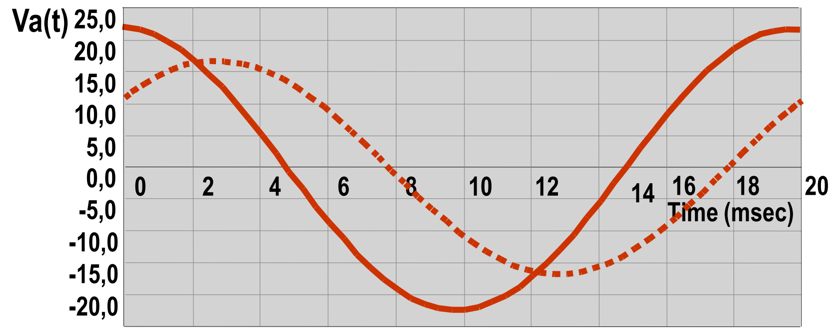
Three Phase Systems

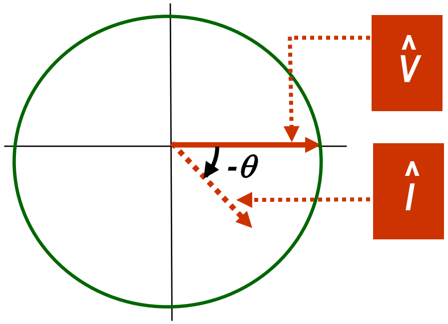
Three Phase Voltages

Circuit - a

$V_a(t) = V_{max} \cos\ wt$

$I_a(t) = I_{max} \cos(wt - 53.13^o)$

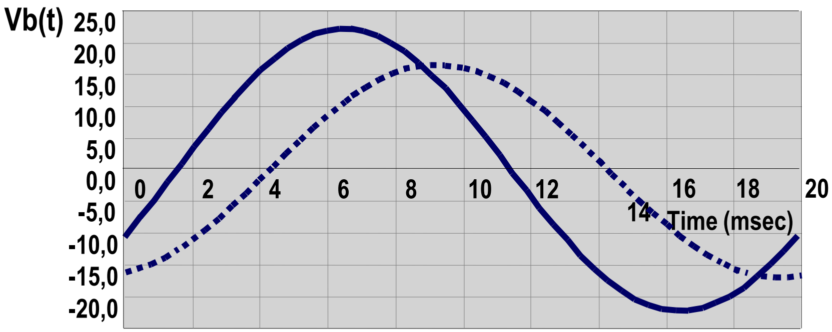


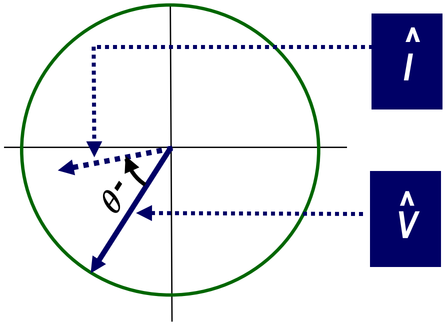


Circuit - b

$V_b(t) = V_{max} \cos(wt - 120^o)$

$I_b(t) = I_{max} \cos(wt - 120^o - 53.13^o)$

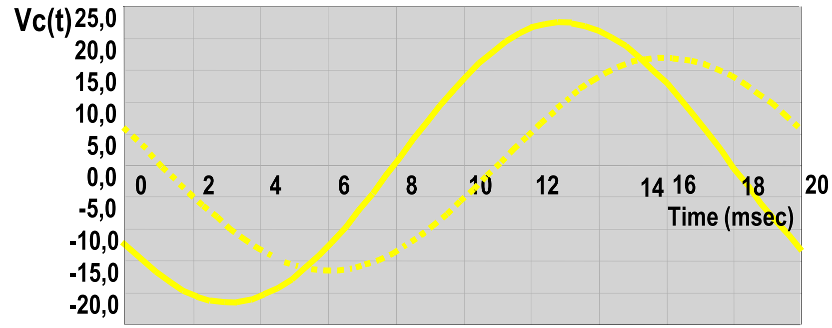


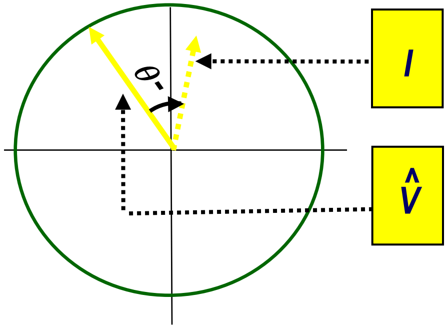


Circuit - c

$V_c(t) = V_{max} \cos(wt - 240^o)$

$I_c(t) = I_{max} \cos(wt - 240^o - 53.13^o)$

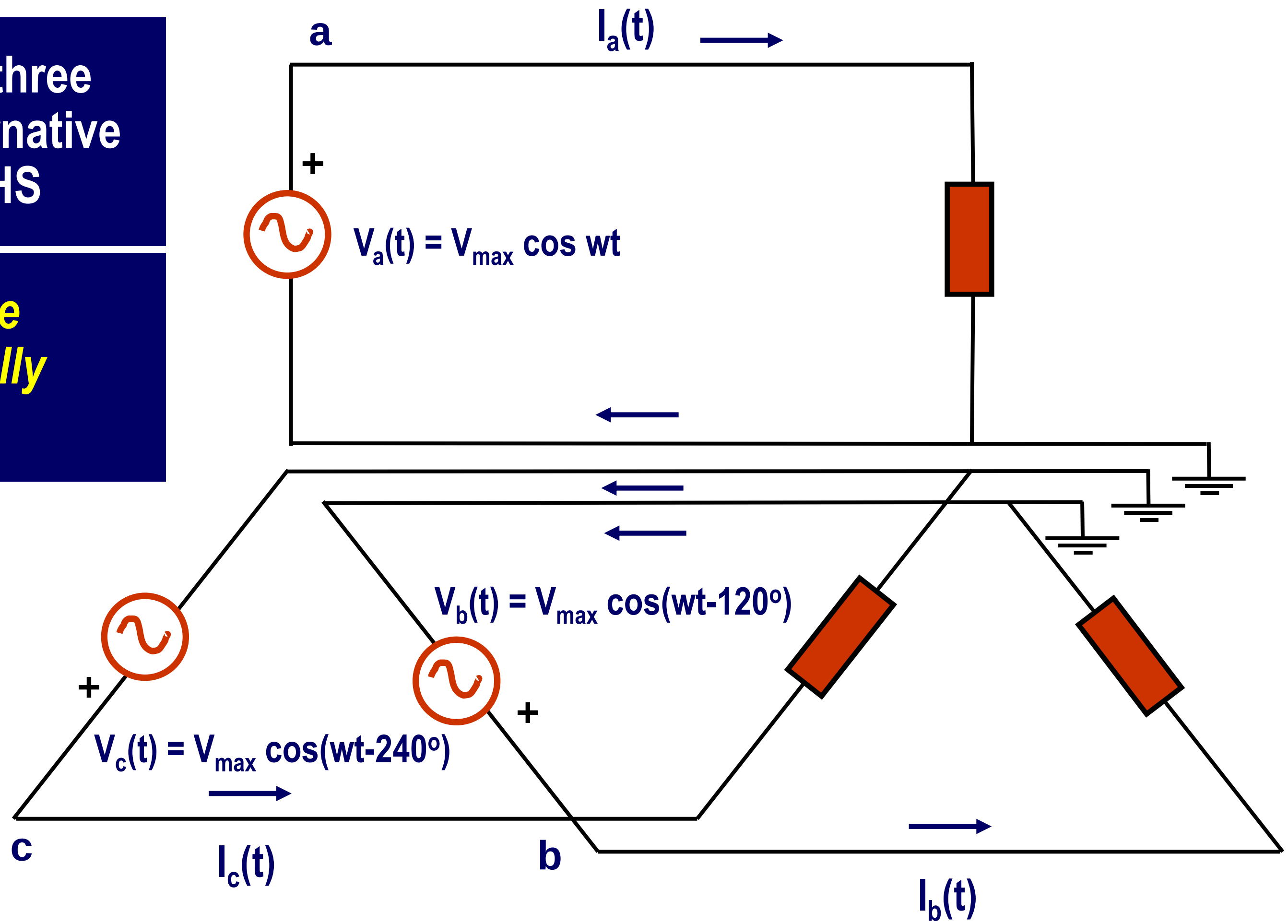




Connection of Three Phase Voltages

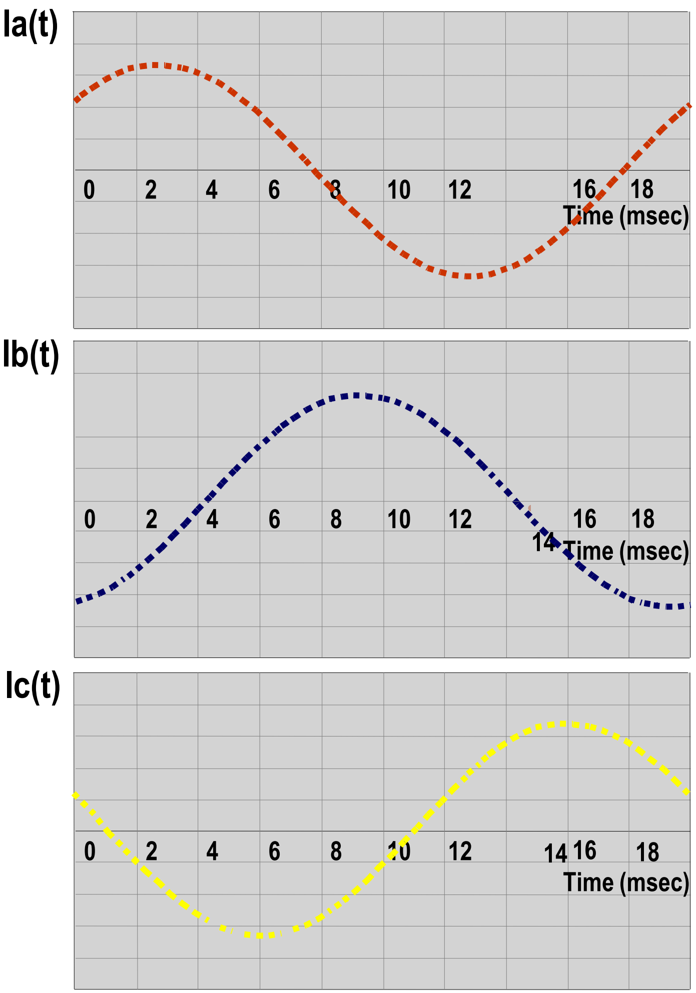
Now, re-draw the above three phase circuits in an alternative form as shown on the RHS

Please note that the three circuits are still electrically unconnected

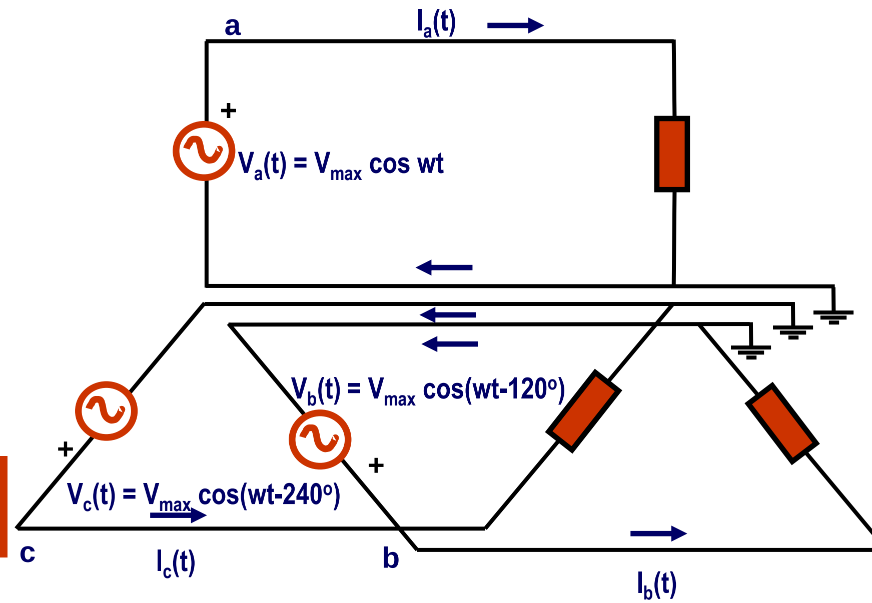


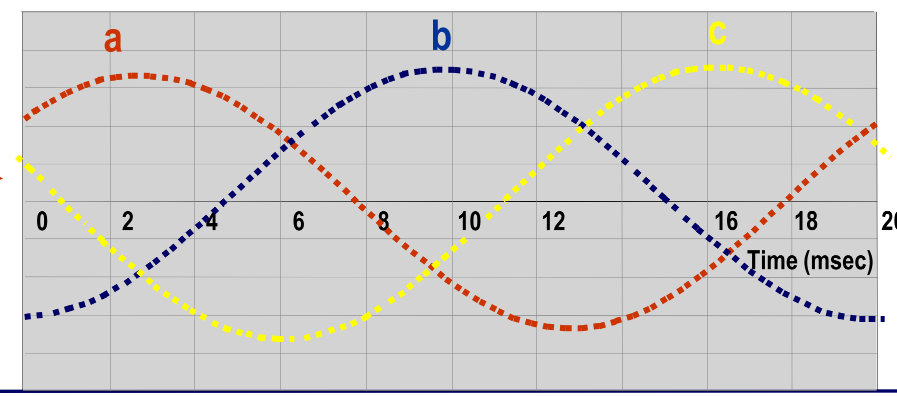
Connection of Three Phase Voltages

Phase Currents (Amp)



$I_a + I_b + I_c$

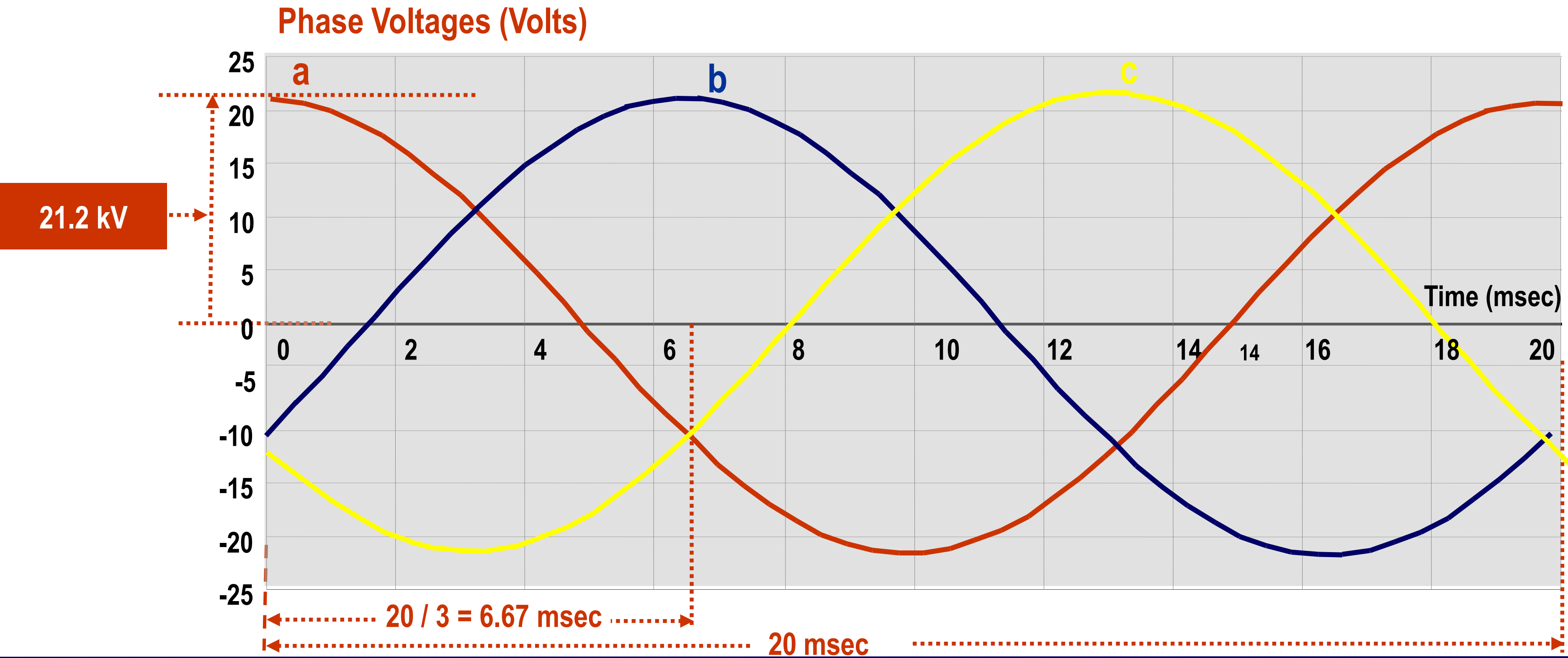




Three Phase Systems

Three Phase Voltage Waveforms

Circuit - a	Circuit - b	Circuit - c
$V_a(t) = V_{max} \cos\ wt$	$V_b(t) = V_{max} \cos(wt - 120^o)$	$V_c(t) = V_{max} \cos(wt - 240^o)$

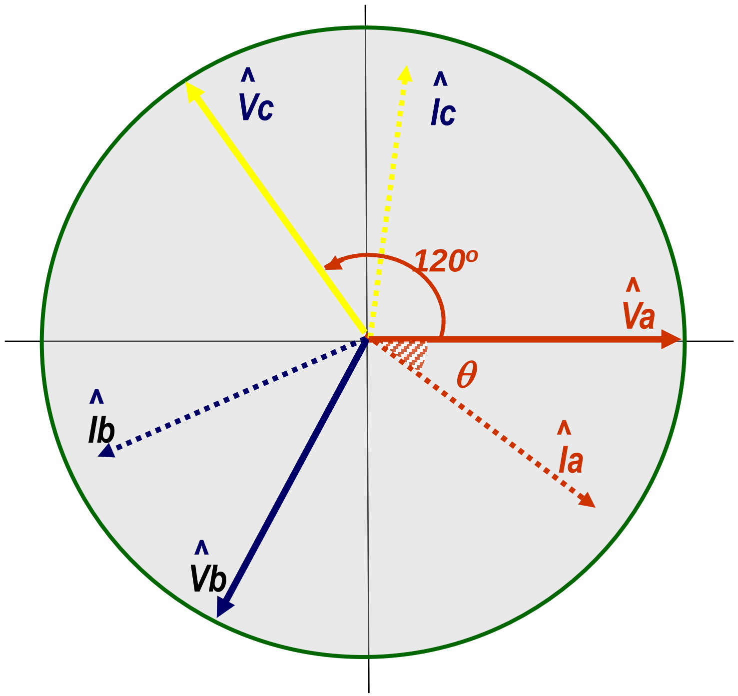


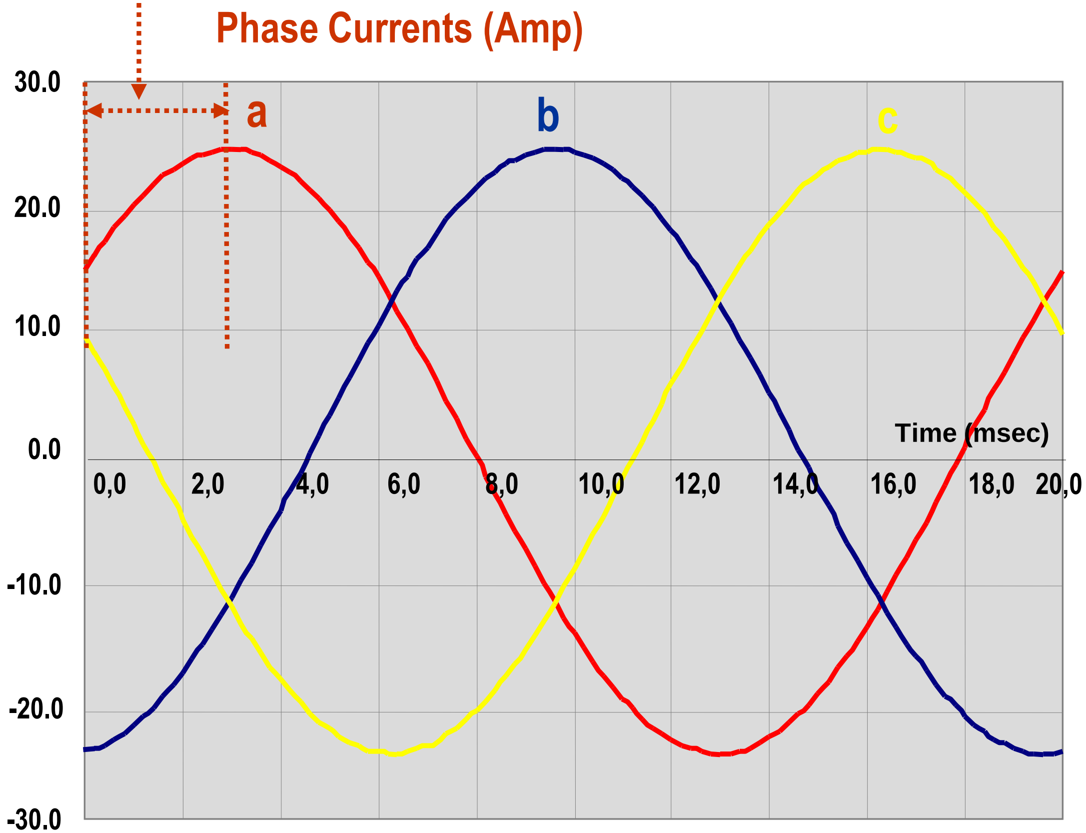
Three Phase Systems

Three Phase Voltage and Current Phasors

Three phase voltage and current phasors may be drawn as shown on the RHS

$Tan^{-1} (X/R) = \angle V - \angle I = -53.13^o \leftrightarrow -53.13^o /360^o \times 20$ sec.





Three Phase Systems

Balanced Three Phase Circuits

Definition

Please note that voltage and current phasors for each phase are 120° displaced from each other

(a) $|V_a| = |V_b| = |V_c|$

(b) $\angle V_a - \angle V_b = \angle V_b - \angle V_c = \angle V_c - \angle V_a = 120^o$

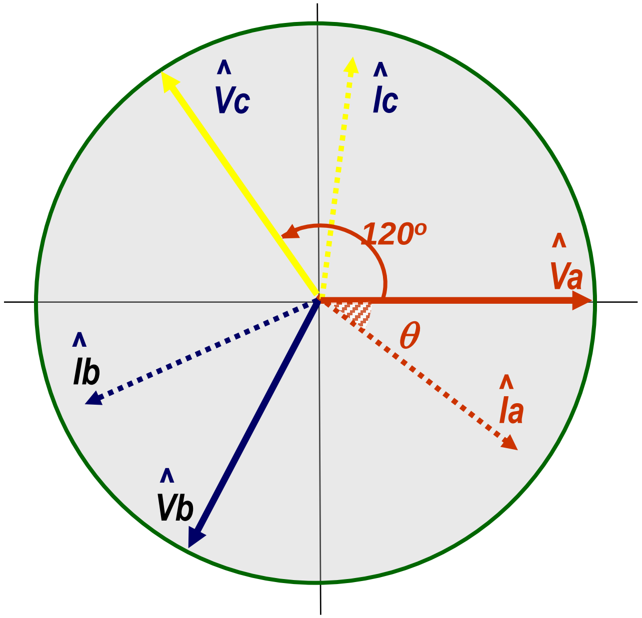
(c) $|I_a| = |I_b| = |I_c|$

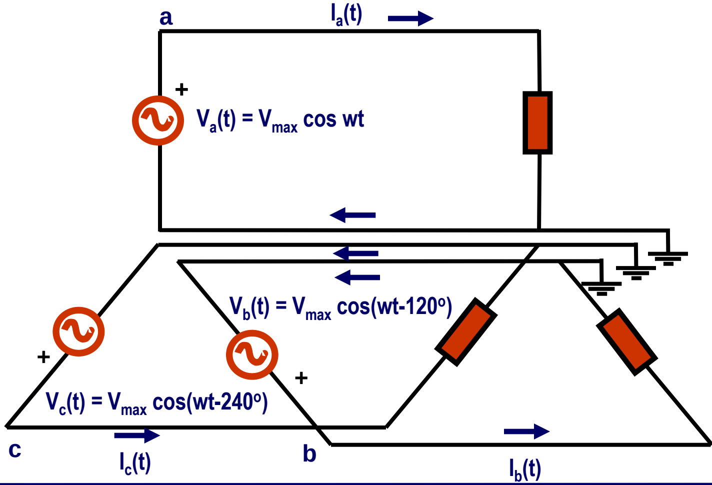
(d) $\angle I_a - \angle I_b = \angle I_b - \angle I_c = \angle I_c - \angle I_a = 120^o$

or

(e) $V_a + V_b + V_c = 0$

(f) $I_a + I_b + I_c = 0$





Balanced Three Phase Circuits

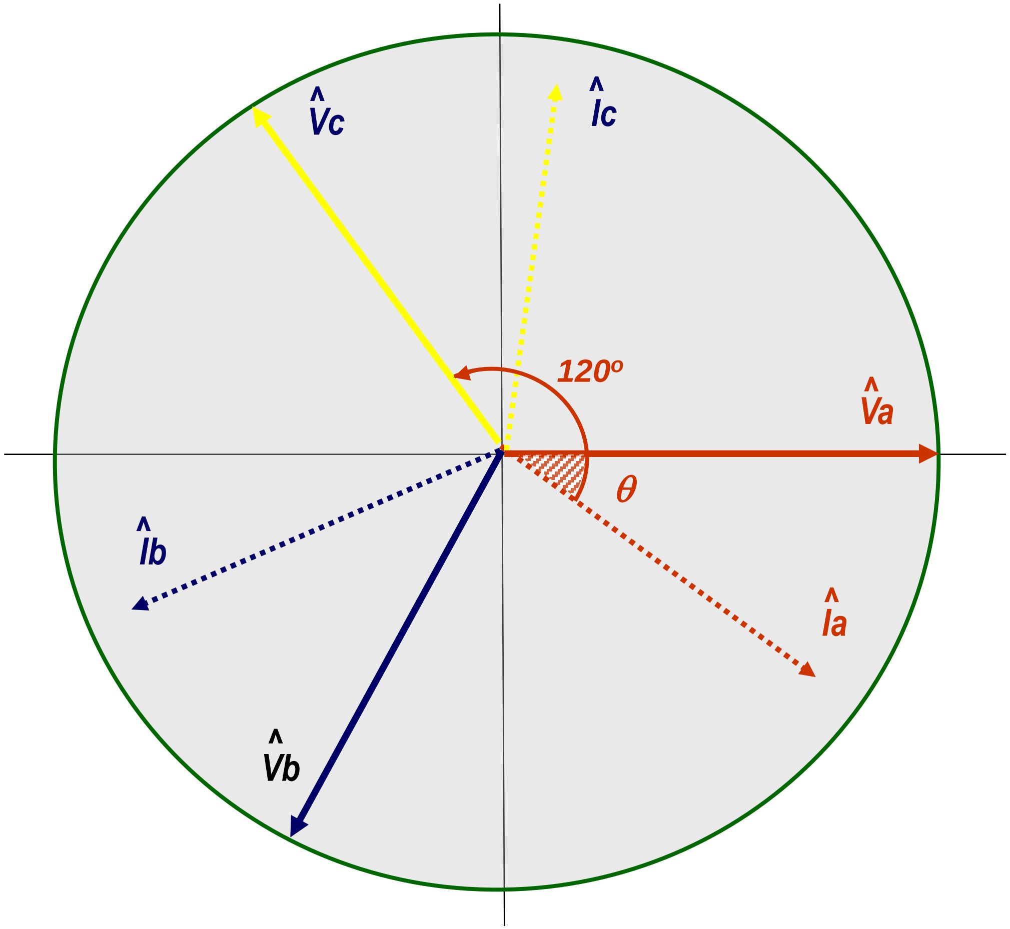
Definition

A three phase system satisfying the above condition is said to be "balanced"

In a balanced three phase system,
- sum of phase currents is zero,
- sum of phase voltages is zero

$$V_a + V_b + V_c = 0$$

$$I_a + I_b + I_c = 0$$



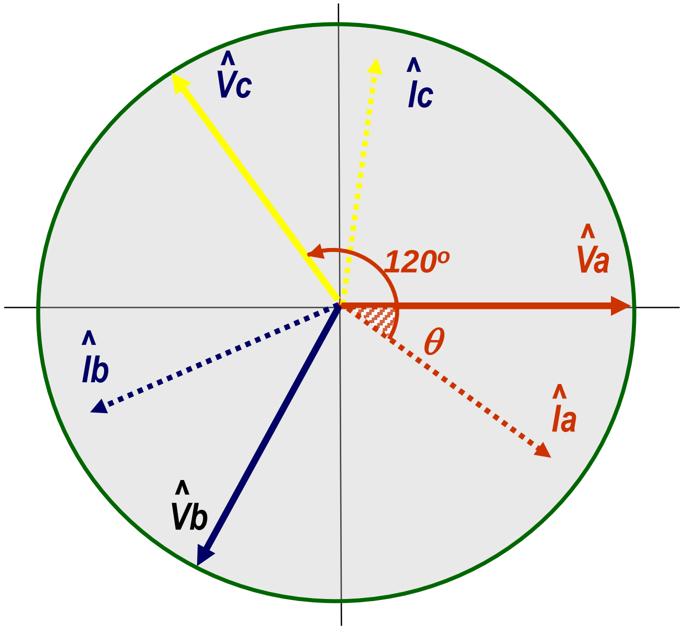
Balanced Three Phase Circuits

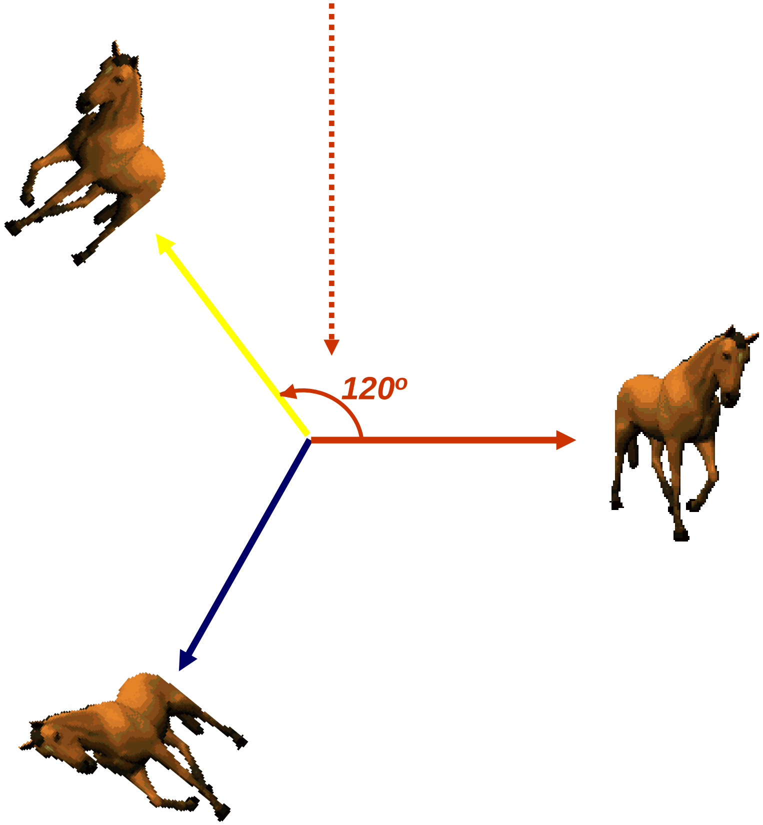
Balance Condition

$$V_a + V_b + V_c = 0$$

$$I_a + I_b + I_c = 0$$

Please note that the central point does not move





Three Phase Systems

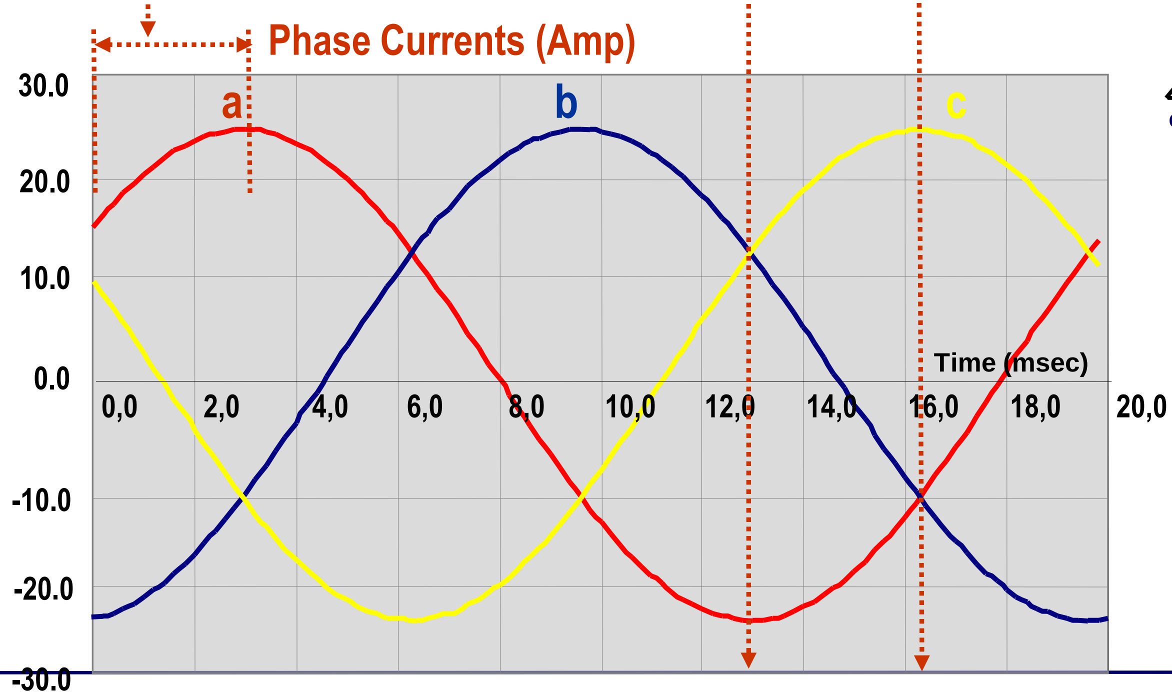
Balanced Three Phase Circuits

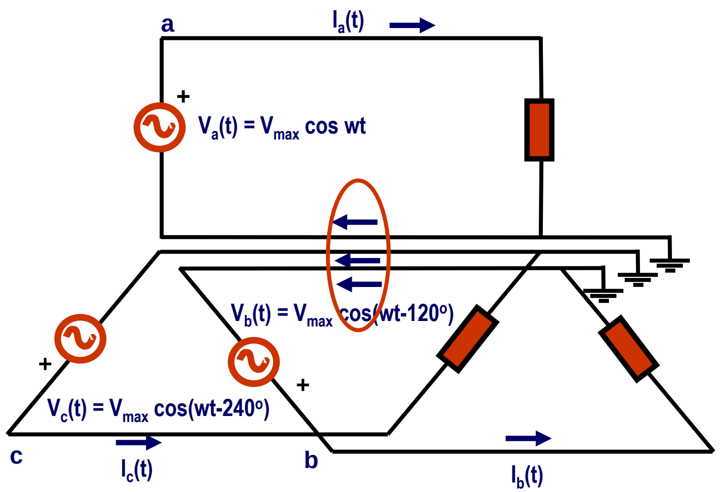
In a balanced three phase system, sum of currents at any instant is always zero, i.e.

$$I_a + I_b + I_c = 0$$

$Tan^{-1} (X / R) = -53.13^o$

$I_a + I_b + I_c = 0$

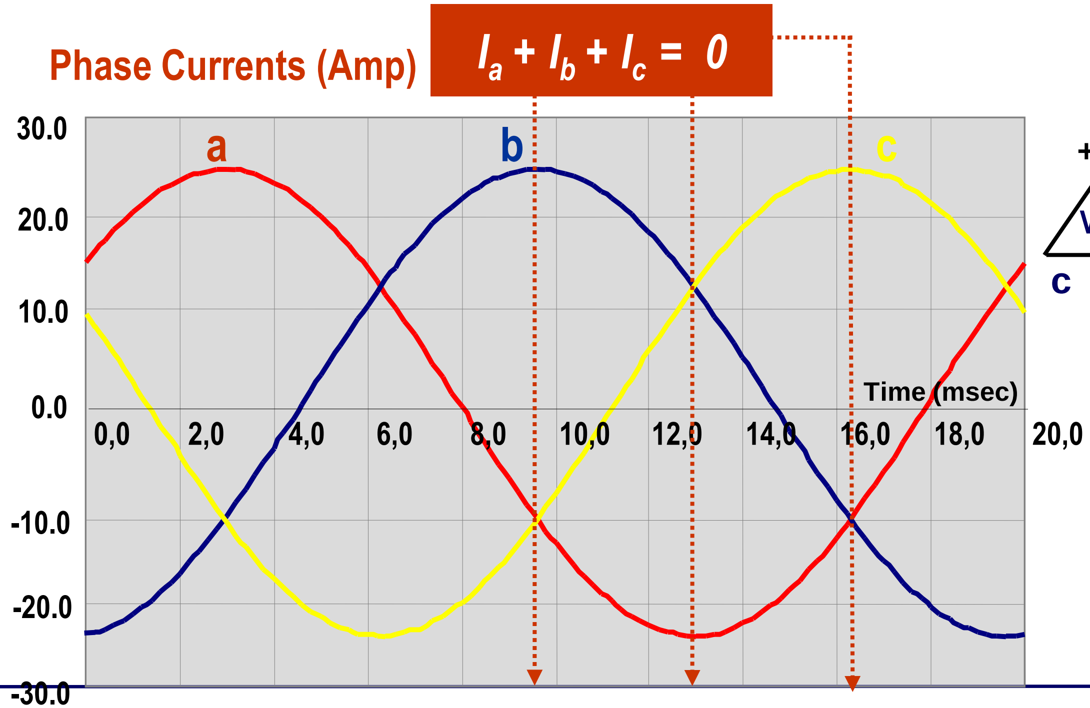


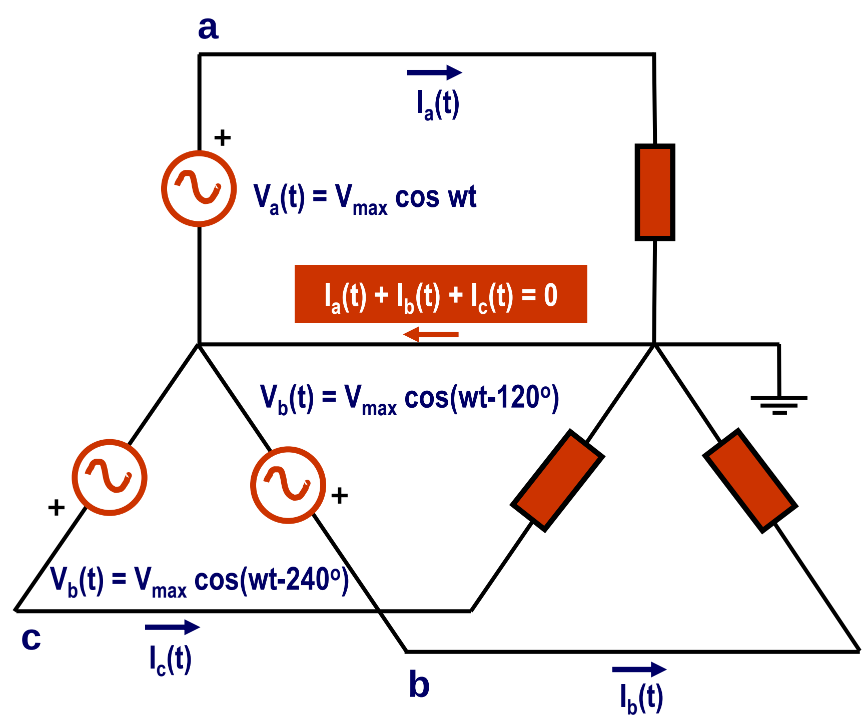


Balanced Three Phase Circuits

In a balanced three phase system, no current will flow if we connect the ground wires of the above three circuits

$$I_a + I_b + I_c = 0$$





Three Phase Measurement-Energy Analzer

Three Phase Energy Analyzer

Energy analyzer shown on the RHS is capable of reading and recording three phase voltages and currents in rms, peak and time - waveform and transmitting the resulting data to computer

Three Phase Measurement-Energy Analzer

EPR-04S

- Cosφ
- Aktif güç
- Reaktif güç
- Görünür güç
- Aktif enerji
- Reaktif enerji
- Dijital giriş
- Enerji pulse çıkışı
- Demand
- 2 Ayrı enerji kaydı
- RS-485 haberleşme
- Toplam aktif, reaktif ve görünür güç

CT-25 Akım Trafosu (1-120A)

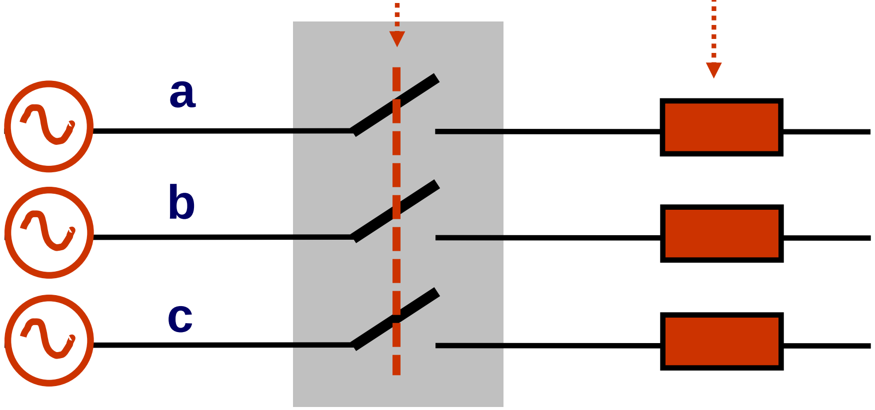
Three Phase Circuit Breaker

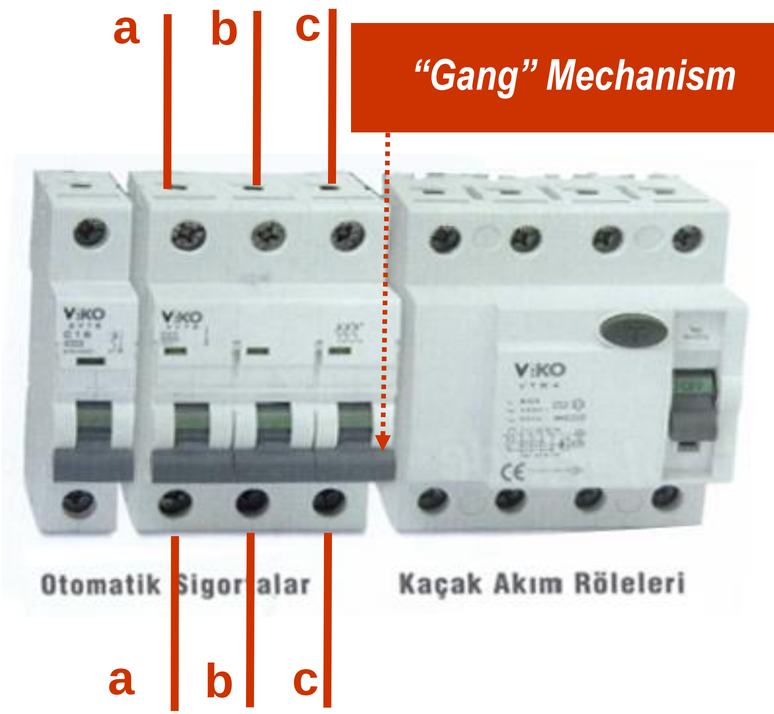
Three Phase Circuit Breaker

Three phase low voltage circuit breaker is a device that breaks the three phases of power service automatically or manually

This dashed line implies that poles operate in "gang" manner

Three Phase Load





Three Phase Systems

Three Phase Measurement

Three Phase Power Analyzer

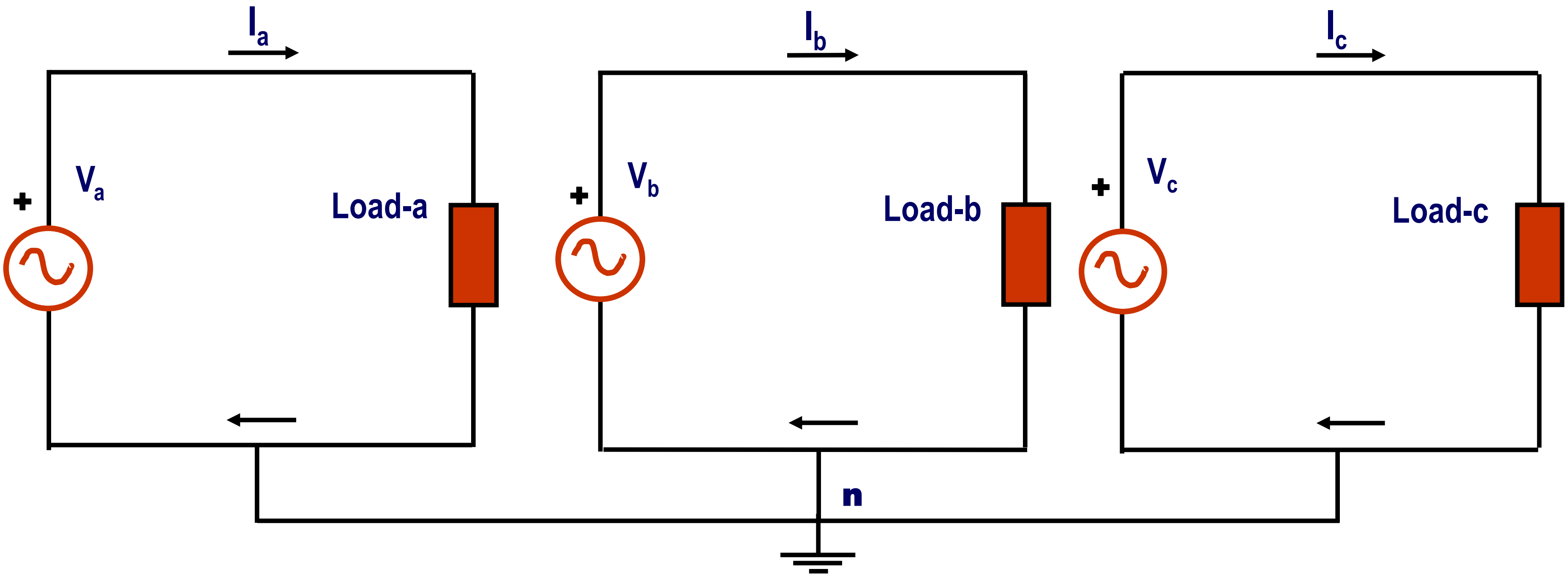
Device shown on the RHS is capable of reading and recording three phase voltages and currents in rms, peak and time-waveform and transmitting the resulting data to computer

Clamp type Current Transformers

Three Phase Systems

Three Phase Circuit

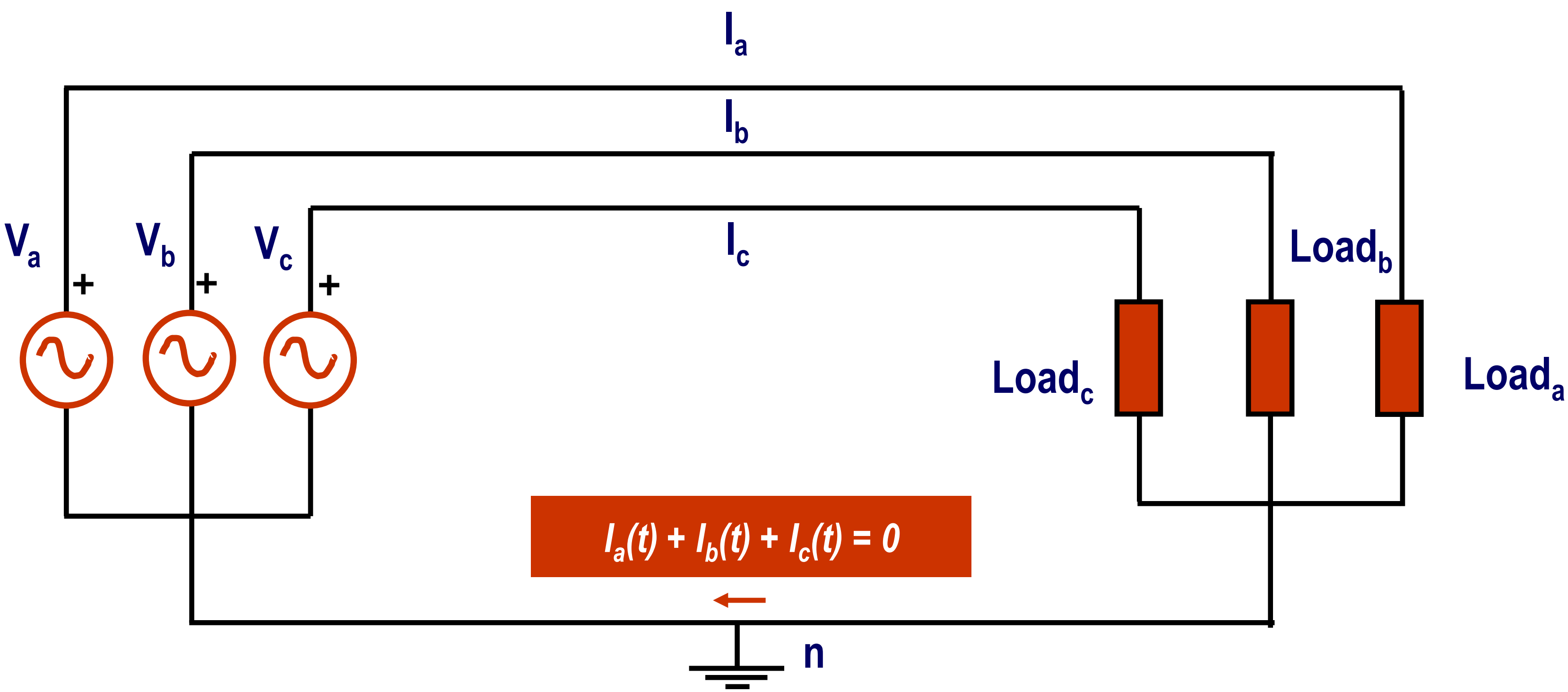
Phase - a	Phase - b	Phase - c
$V_a(t) = V_{max} \cos \ wt$	$V_b(t) = V_{max} \cos(wt - 120^o)$	$V_c(t) = V_{max} \cos(wt - 240^o)$



Three Phase Systems

Three Phase Circuit Connection

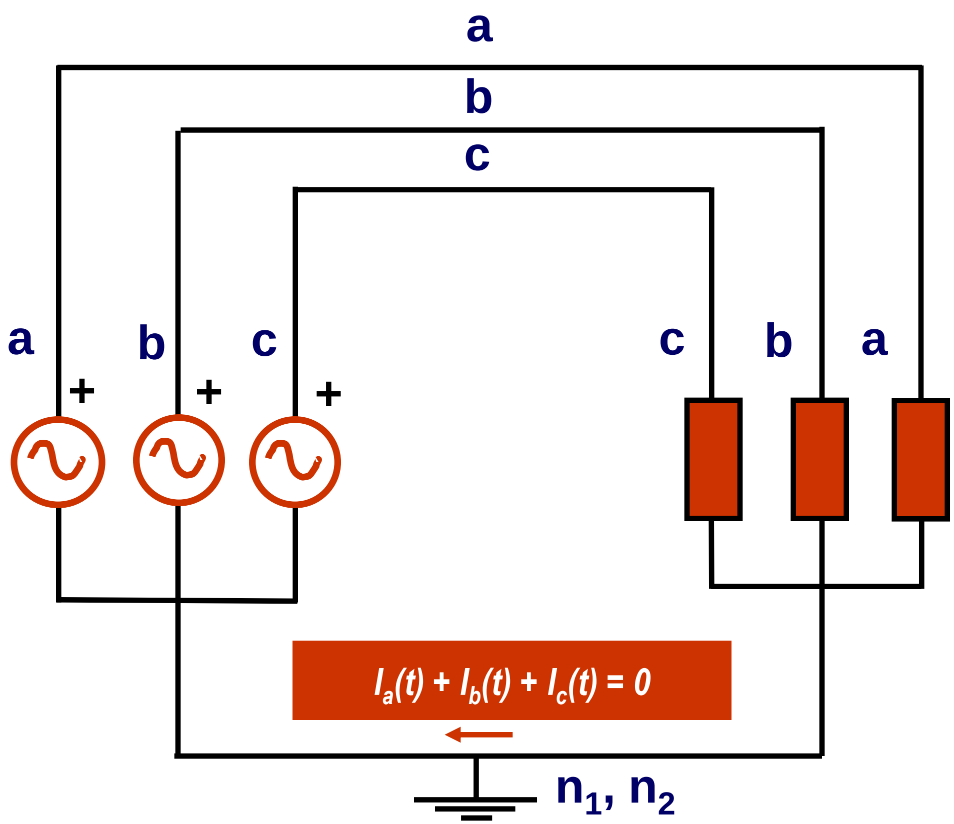
Phase - a	*Phase - b*	*Phase - c*
$V_a(t) = V_{max} \cos \; wt$	$V_b(t) = V_{max} \cos(wt - 120^o)$	$V_c(t) = V_{max} \cos(wt - 240^o)$

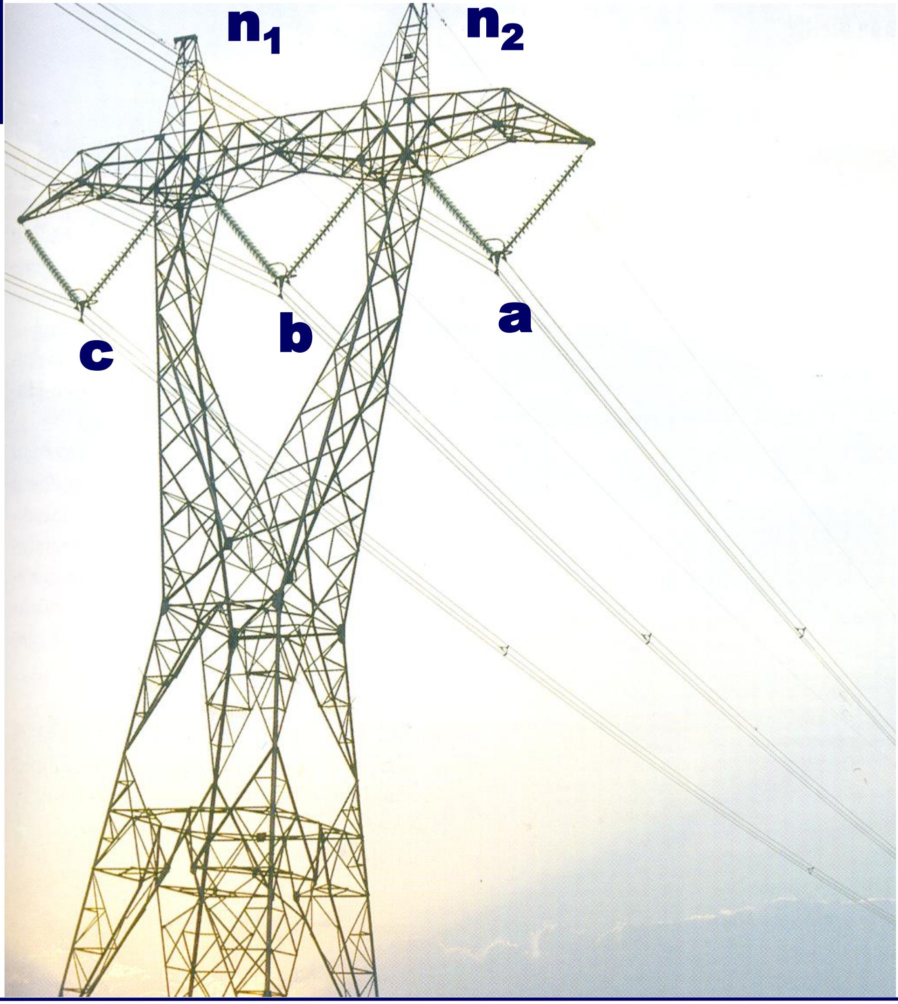


$I_a(t) + I_b(t) + I_c(t) = 0$

Three Phase Circuit Connection

Basic Diagram

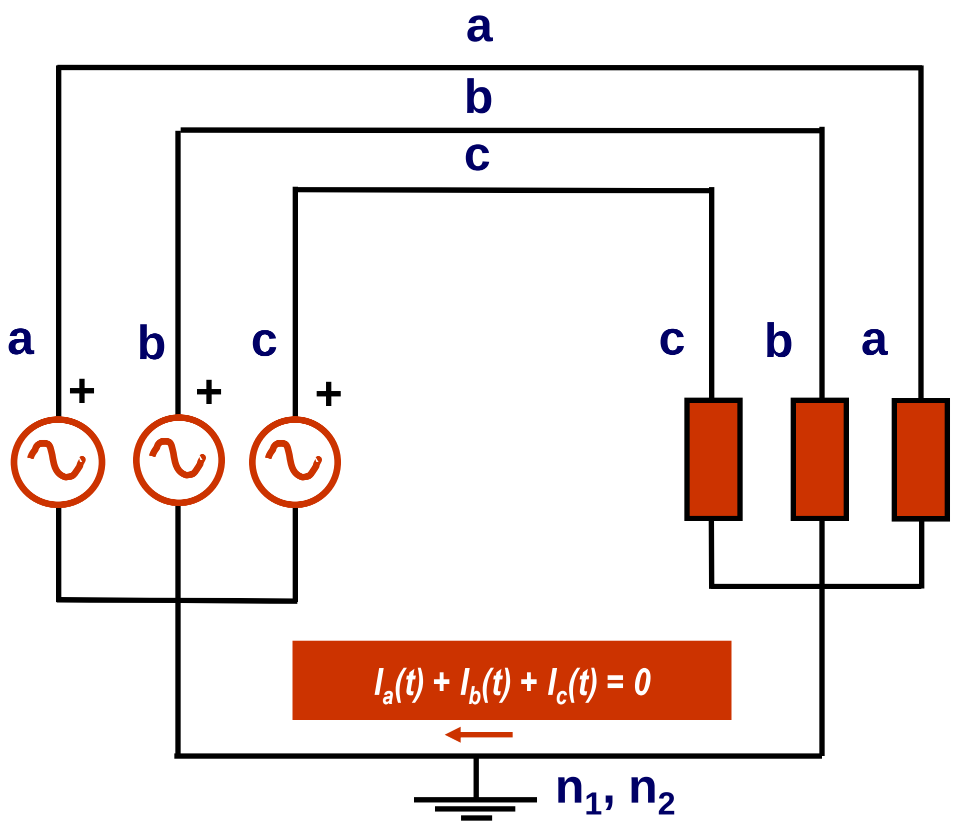




Three Phase Systems

Three Phase Cable

Basic Diagram



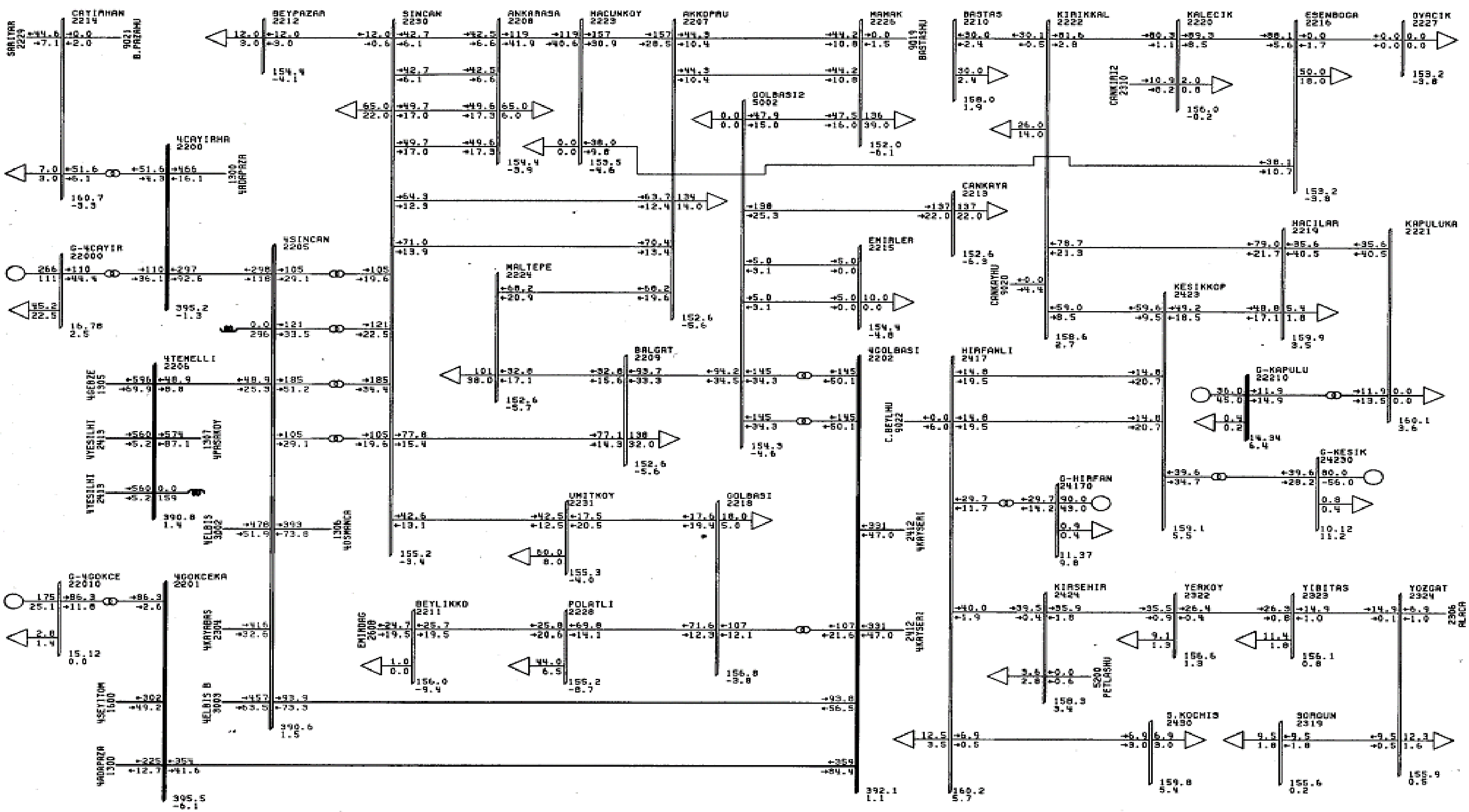
Three Phase Systems

Turkish 380 kV System

METU

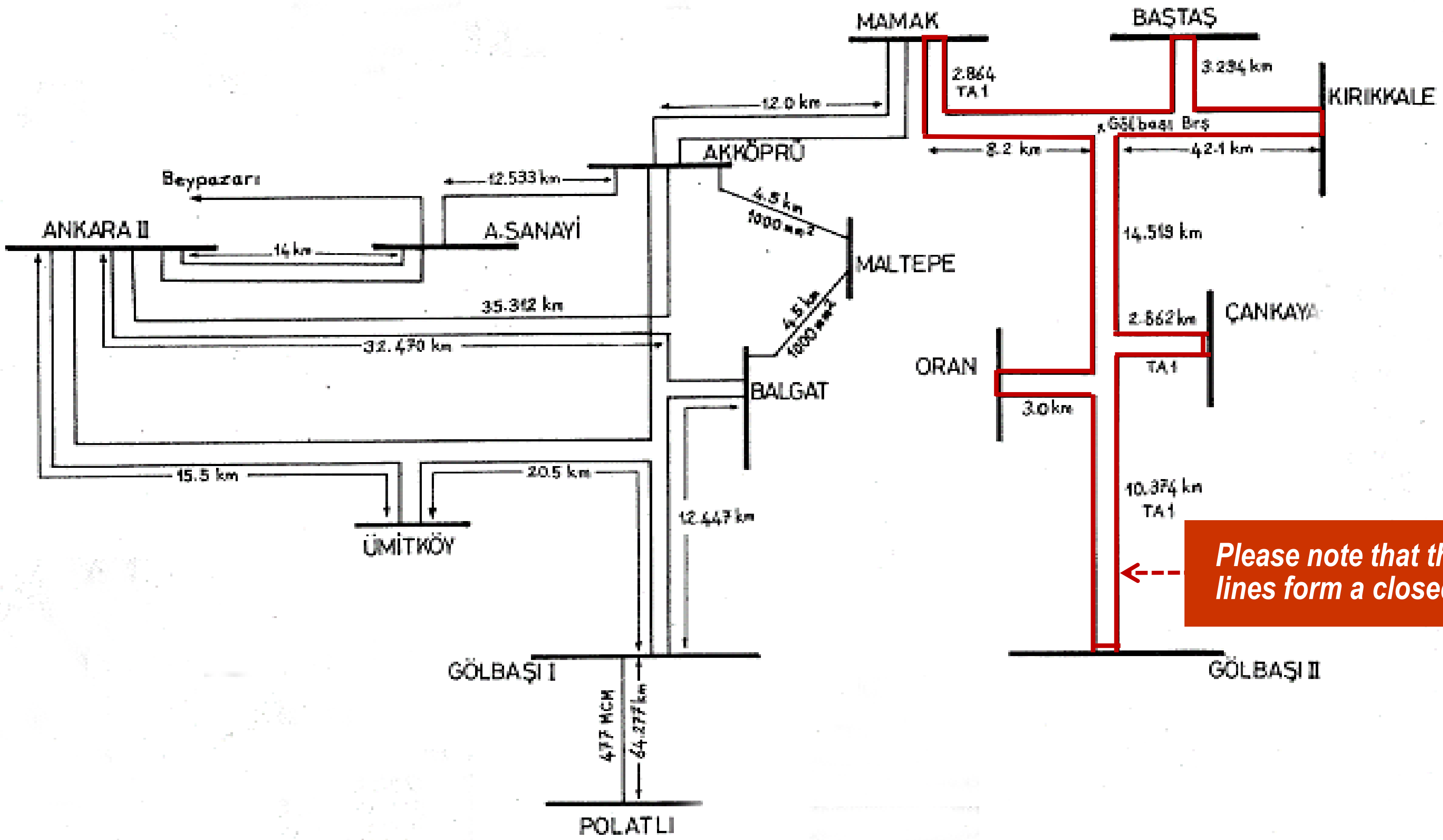
Three Phase Systems

Part of Turkish 154 kV System



Three Phase Systems

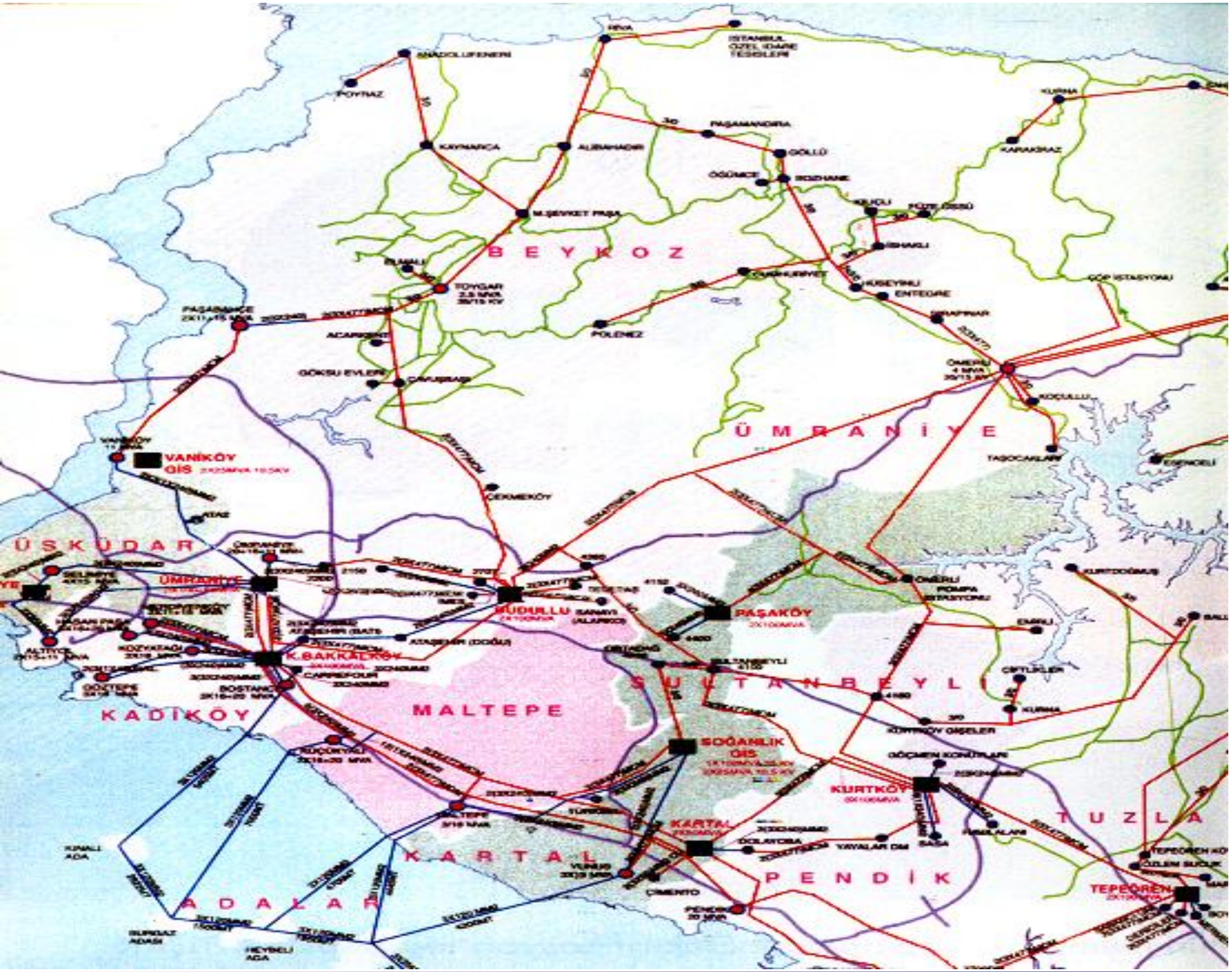
Ankara 154 kV Ring



Please note that the lines form a closed ring

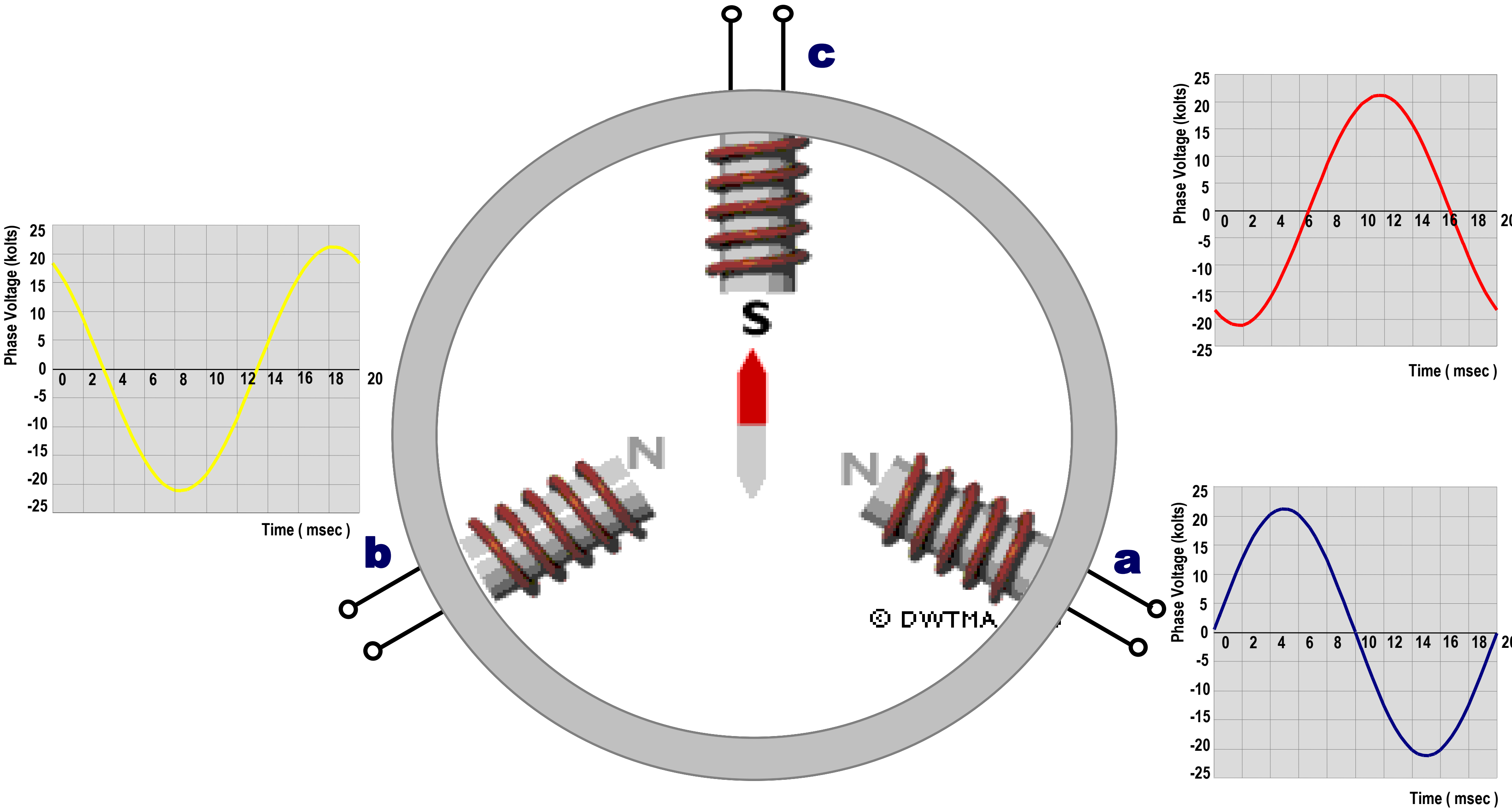
Three Phase Systems

Istanbul Anatolian Side MV System



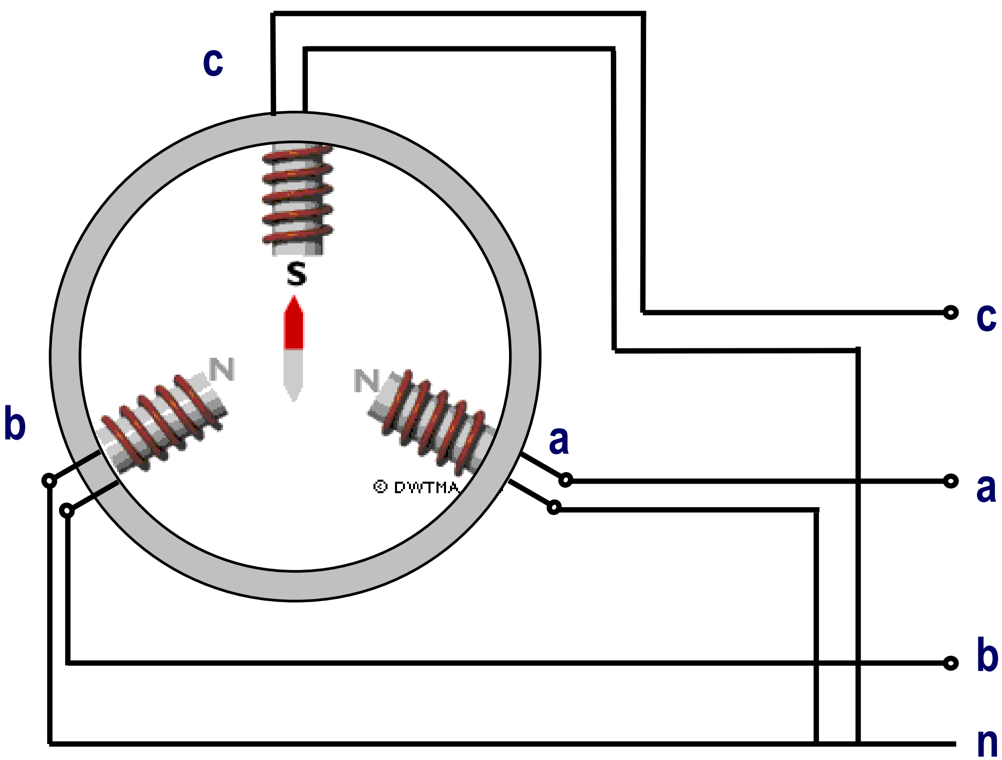
Three Phase Systems

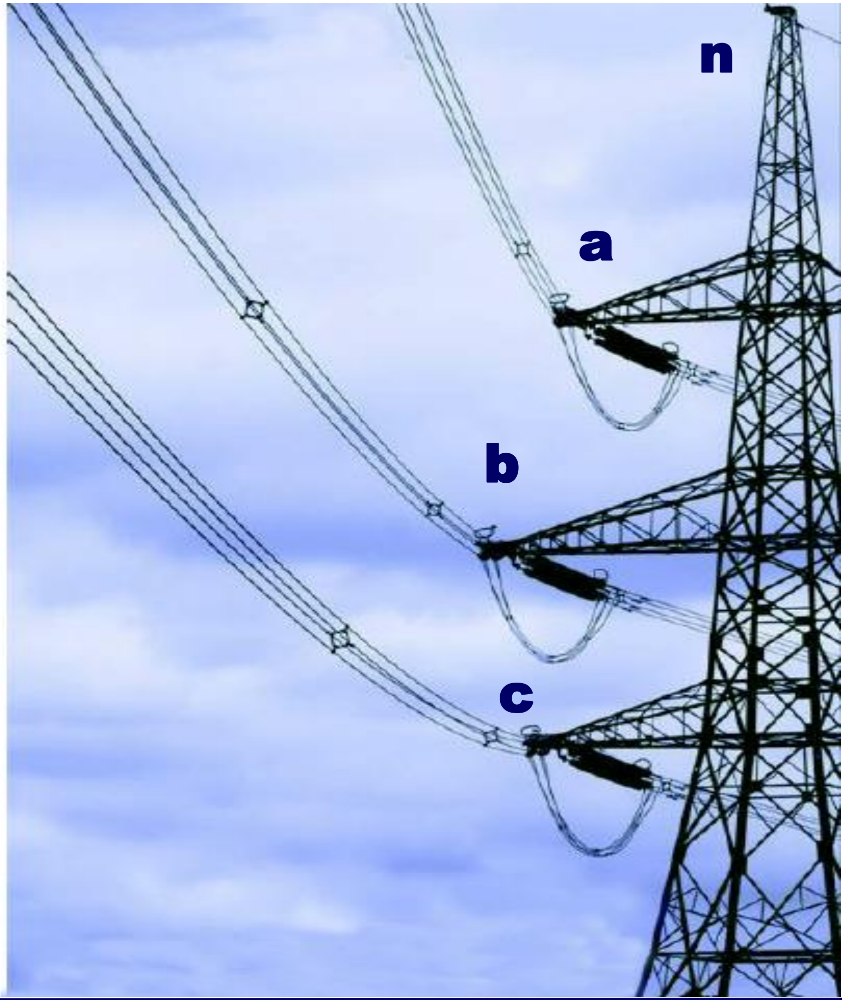
Three Phase Synchronous Generator



Three Phase Generation System

Three Phase Generator



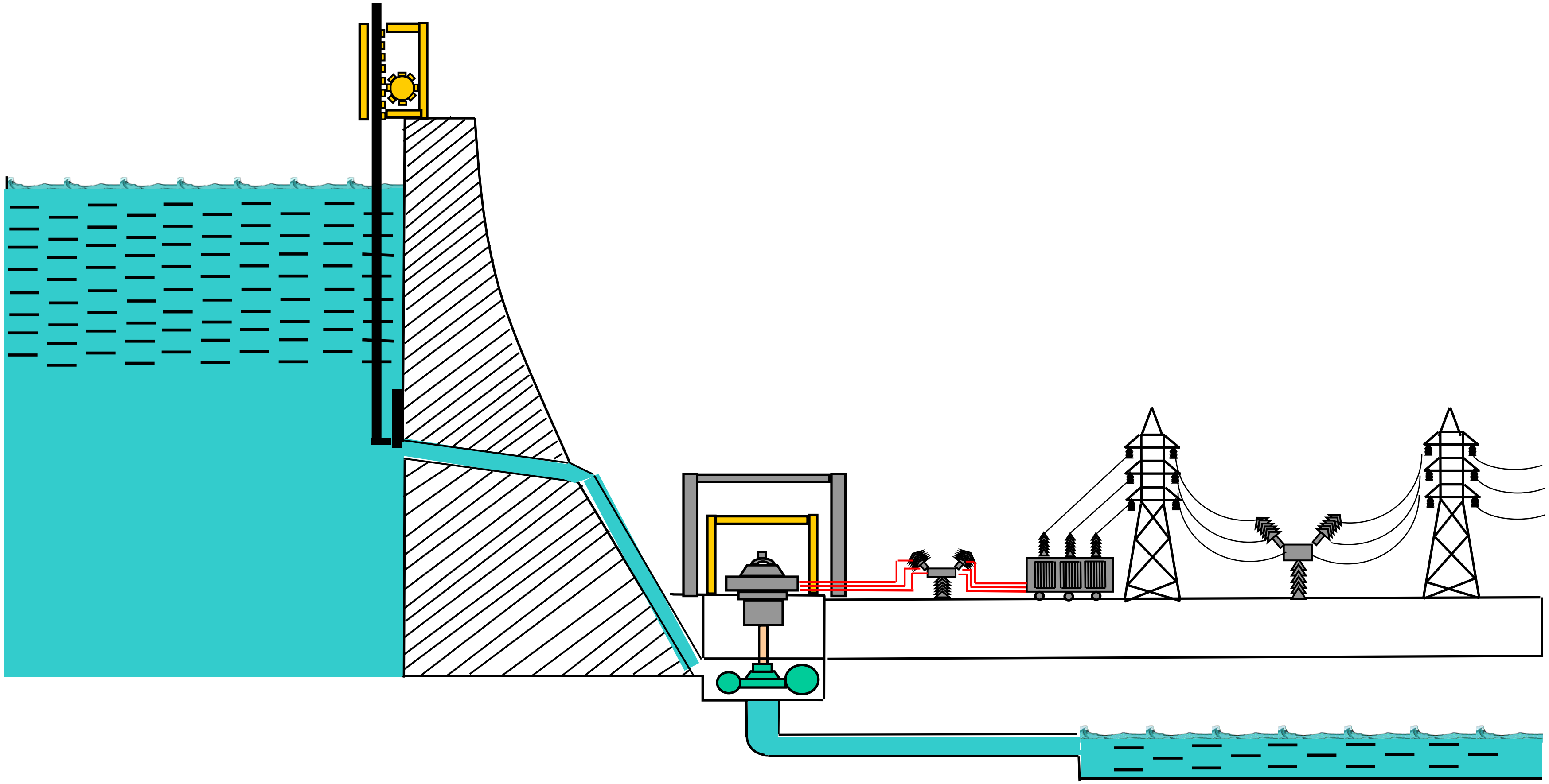


Three Phase Systems

Karakaya Hydroelectric Plant – 1800 MW

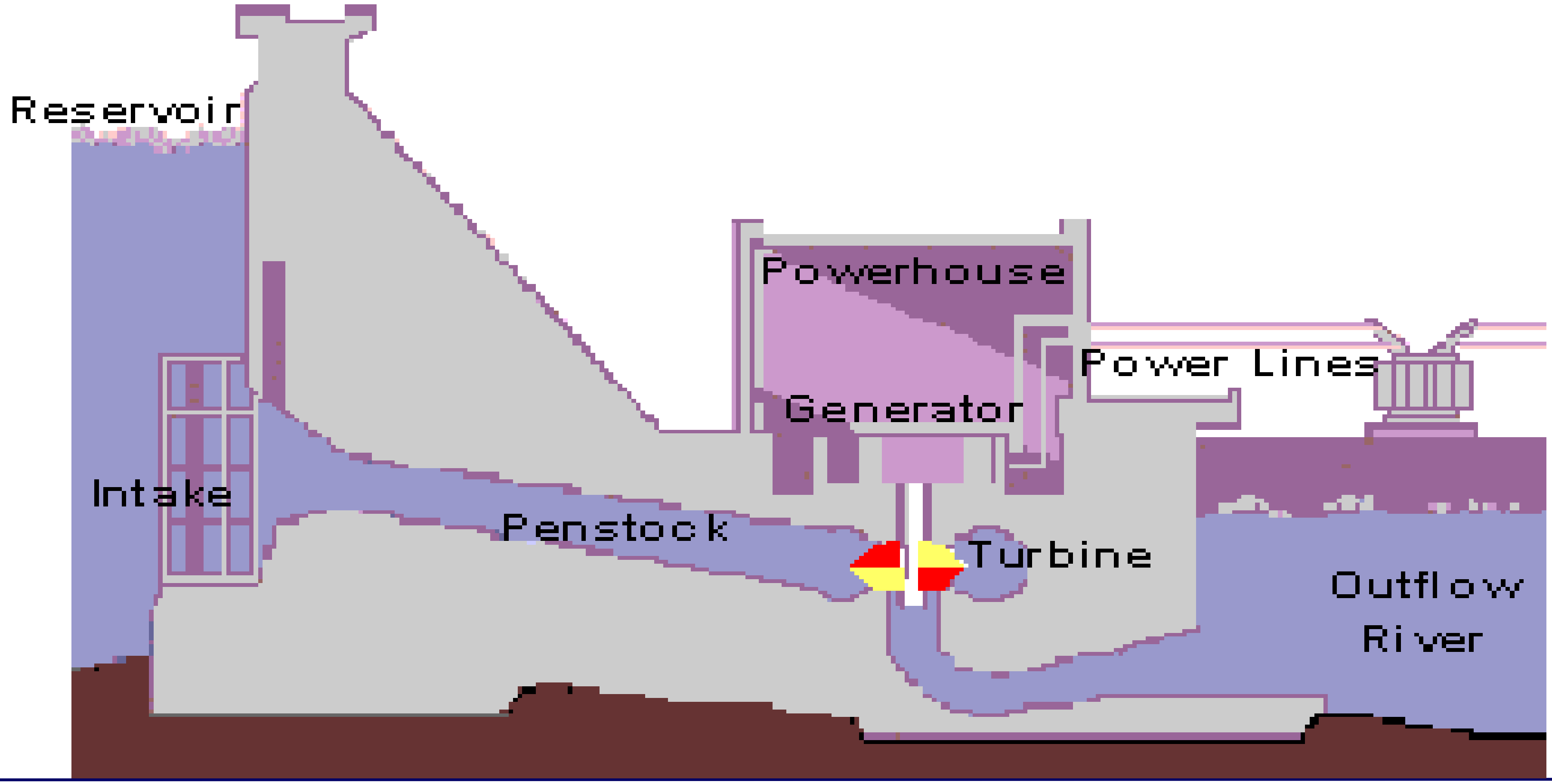
Hydroelectric Plant - Sectional View

Configuration



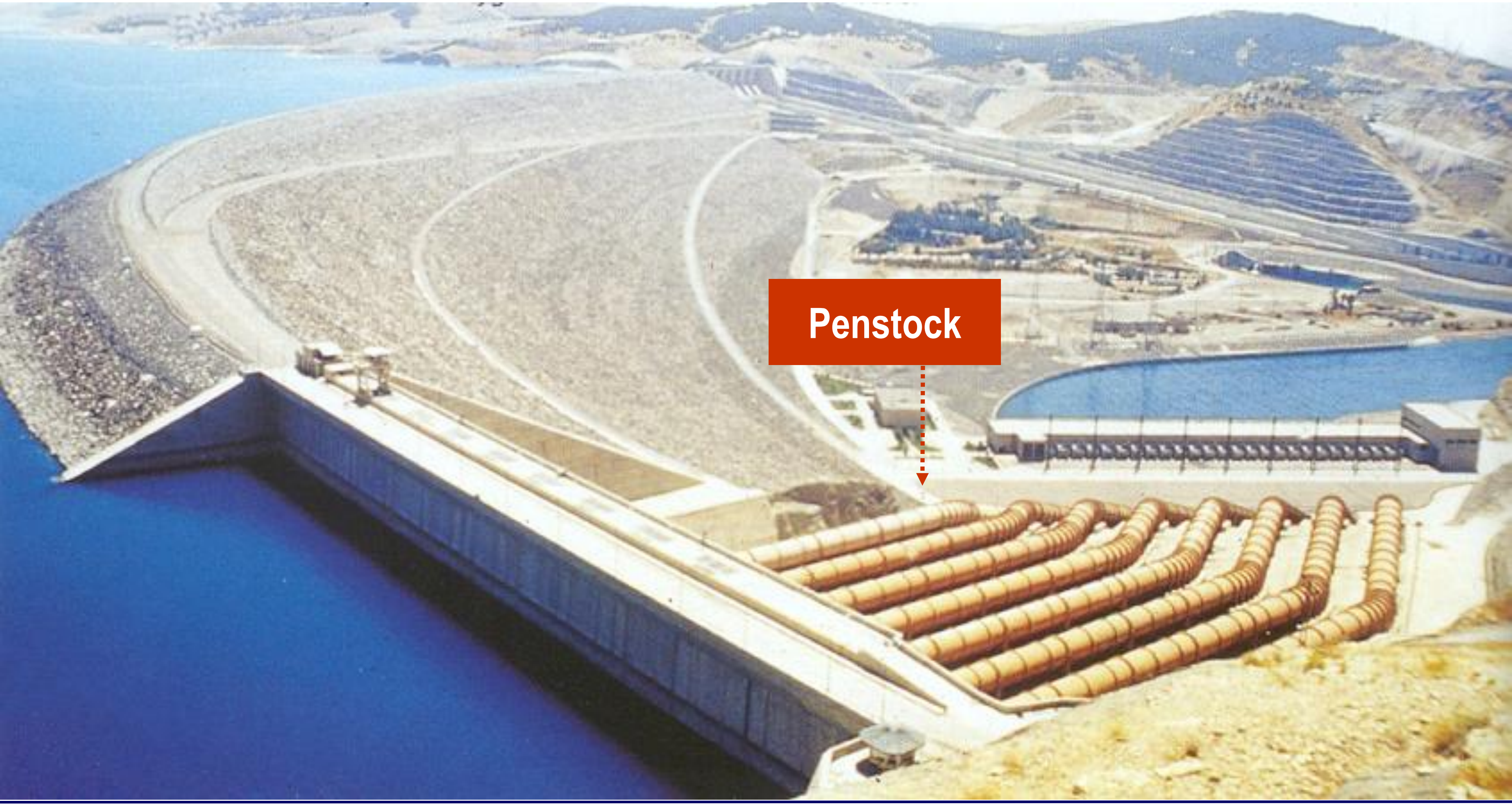
Hydroelectric Plant - Sectional View

Typical Hydoelectric Plant



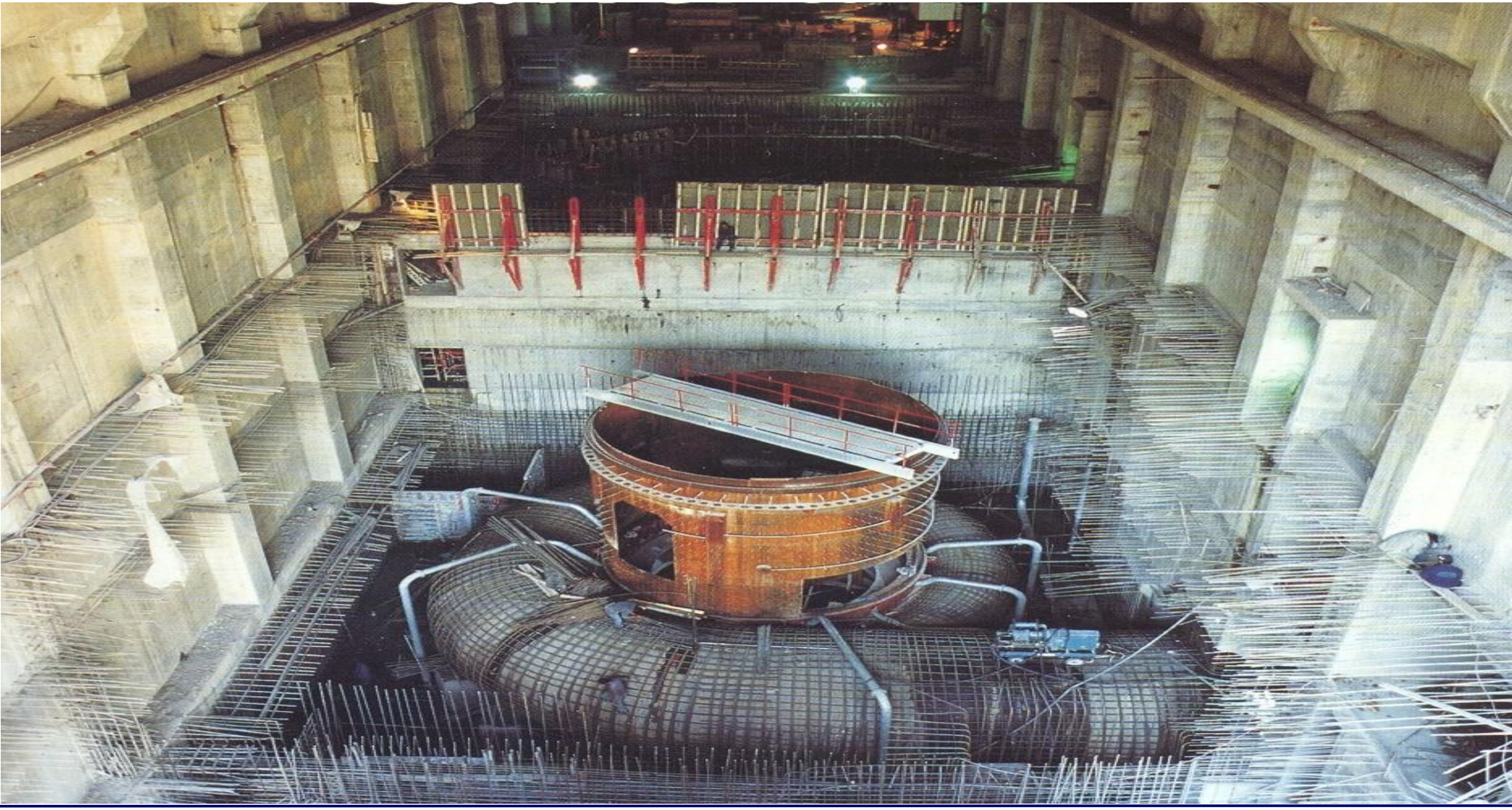
Three Phase Systems

Atatürk Hydroelectric Plant; 8 x 300 = 2400 MW



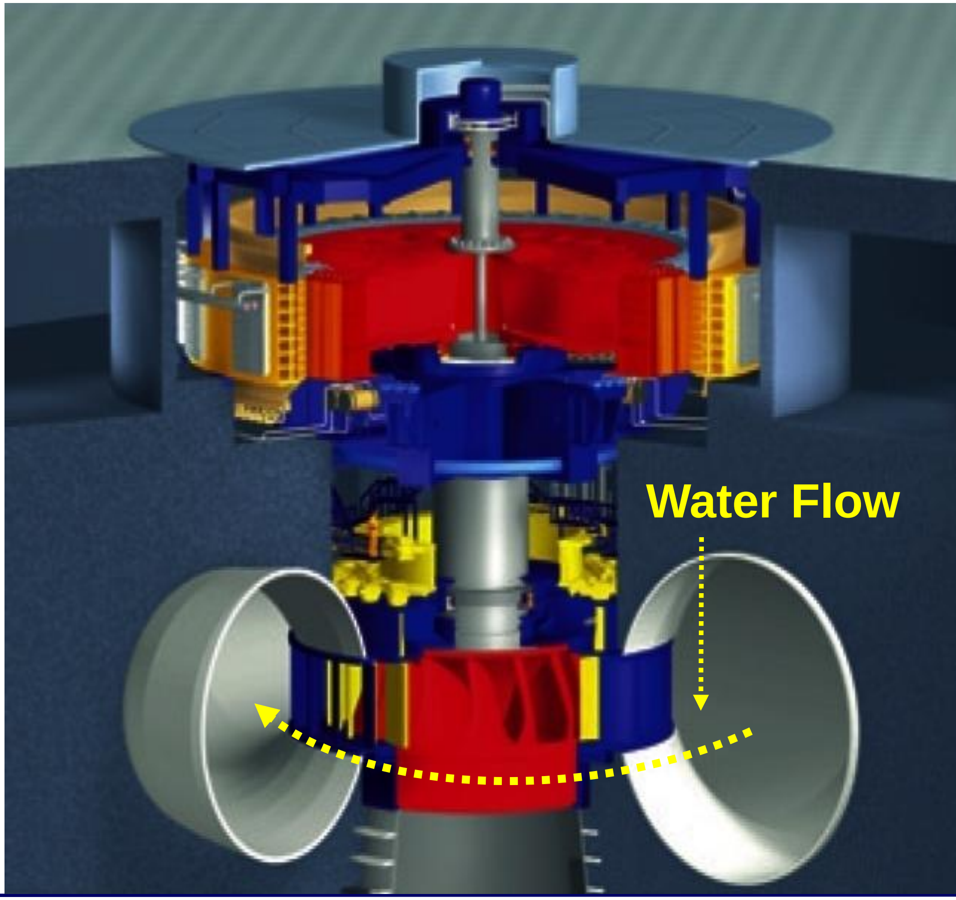
Three Phase Systems

Hydroelectric Plant - Water Turbine



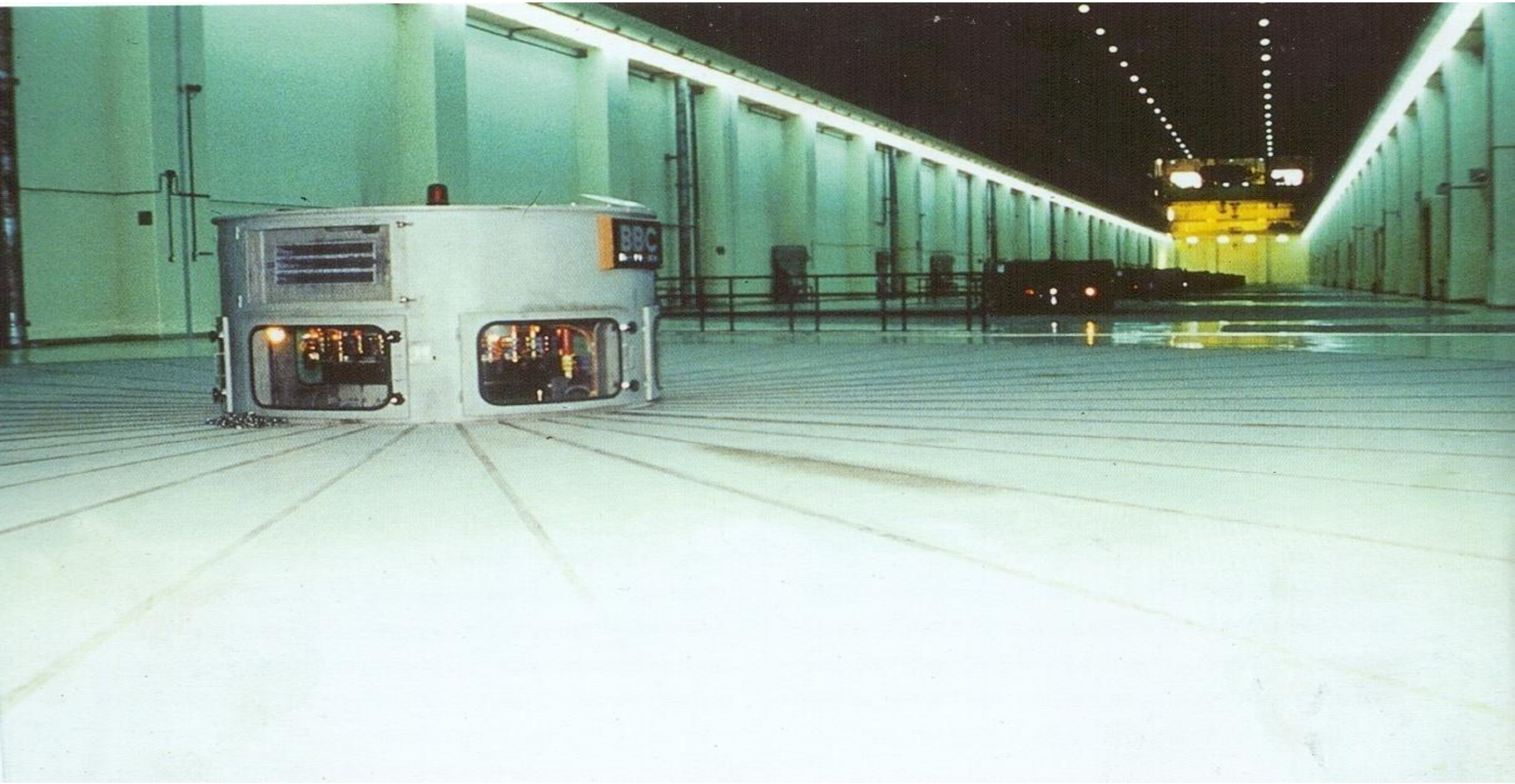
Generation of AC Voltage - Synchronous Generator

Bagnell Dam on Ozarks Lake	Turbine - Generator Set



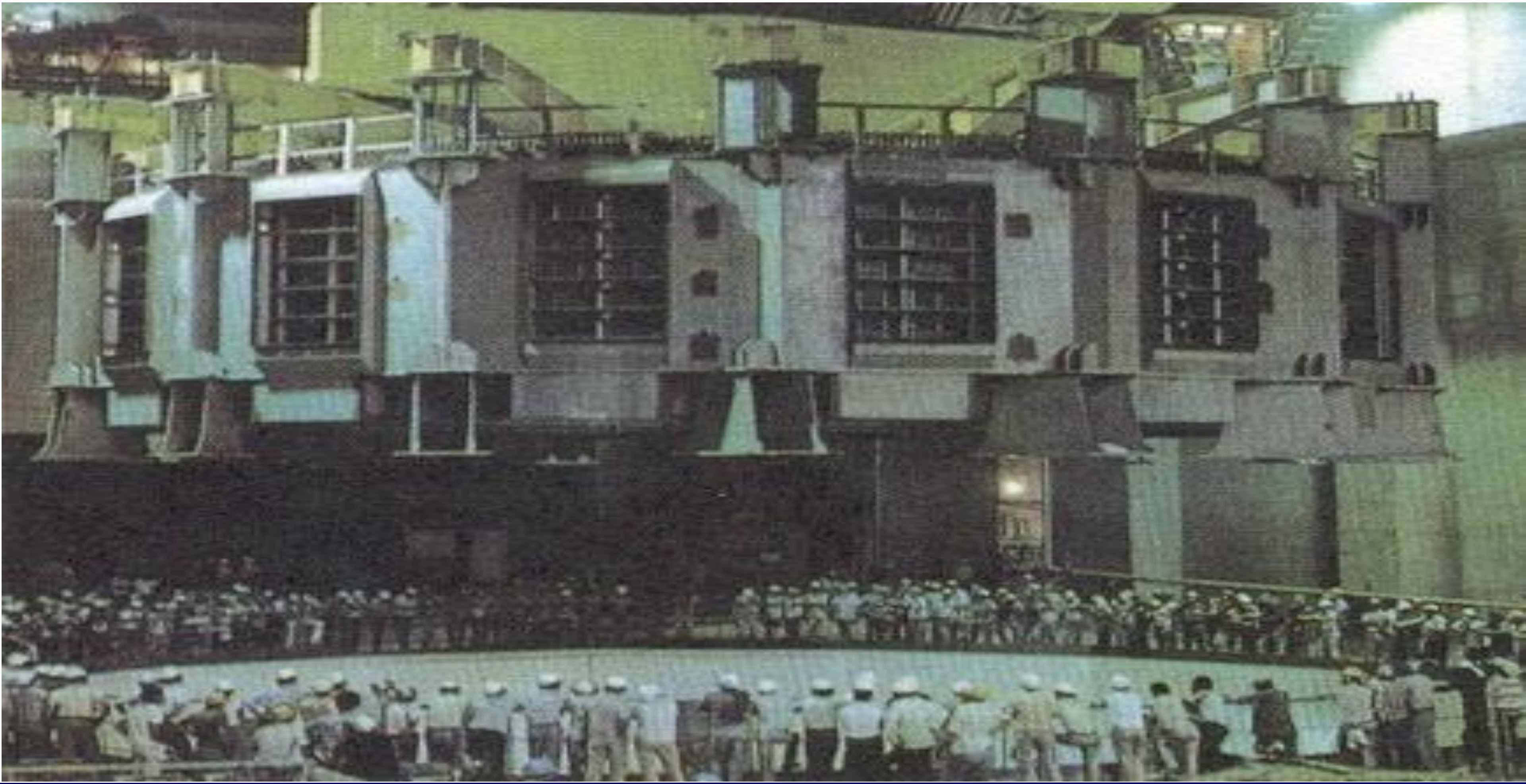
Three Phase Systems

Atatürk Dam Generator Room



Three Phase Systems

Itaipu Power Plant - 12500 MW
Stator Mounting Ceremony



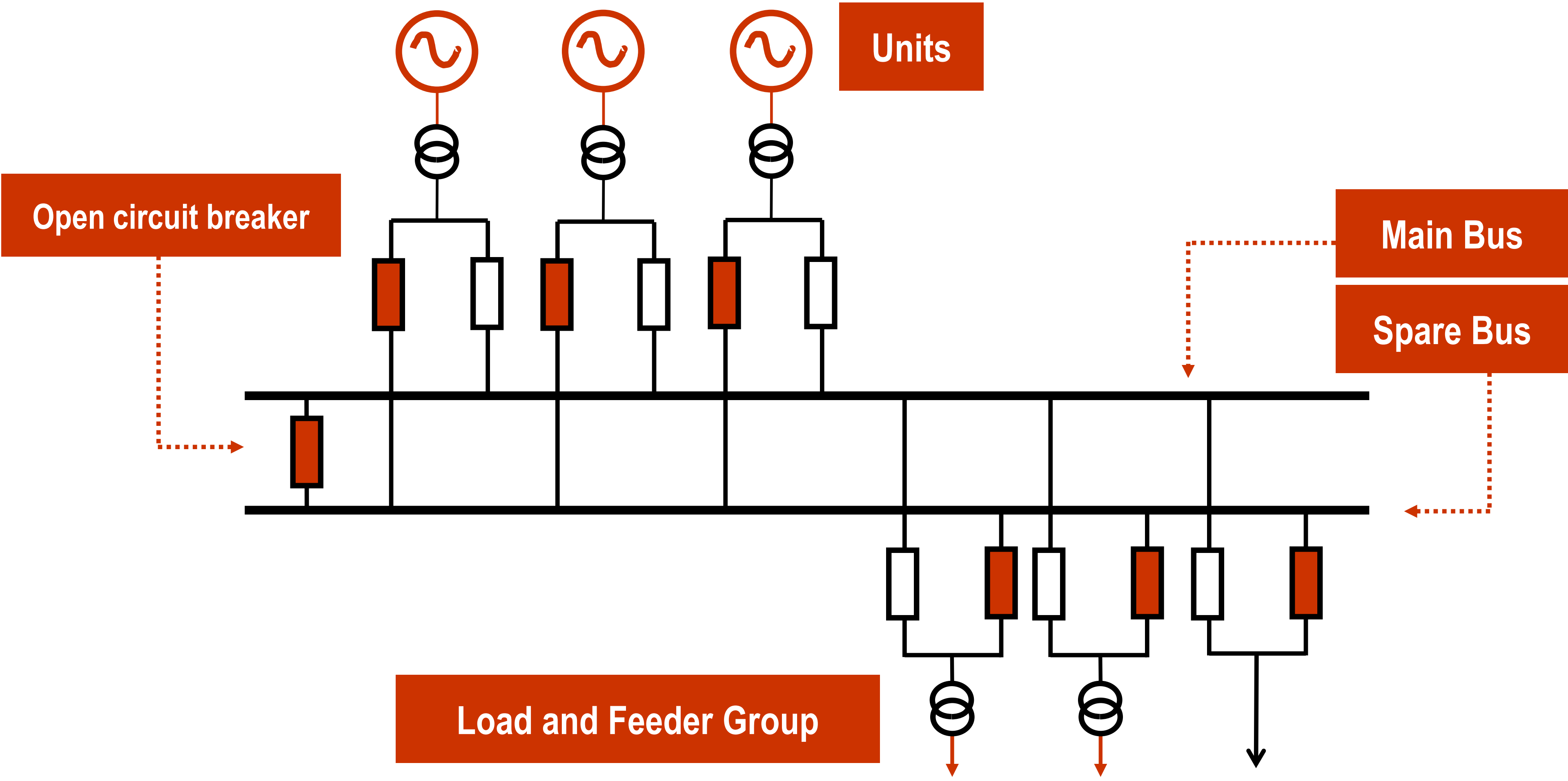
Combined Cycle Power Plant

Three Phase Systems

Thermal (Coal) Power Plant

Three Phase Systems

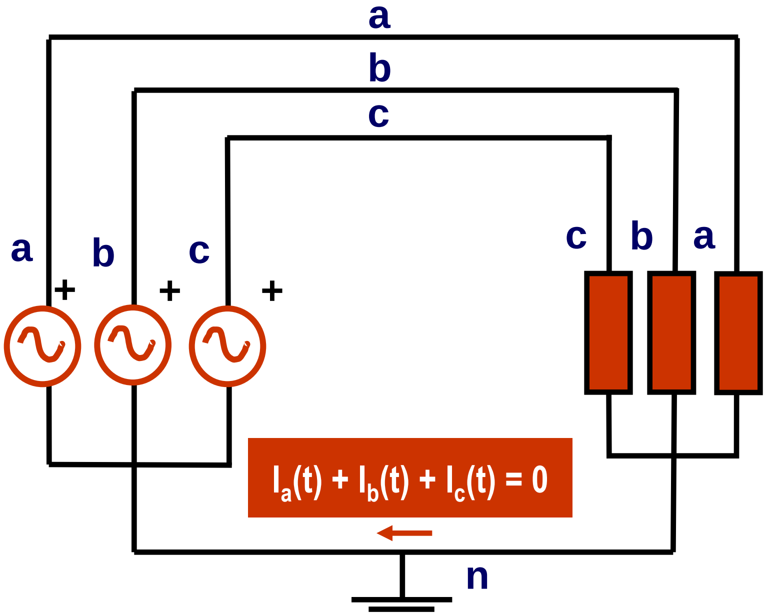
Parallel Operation of Plants (Double Bus Configuration)

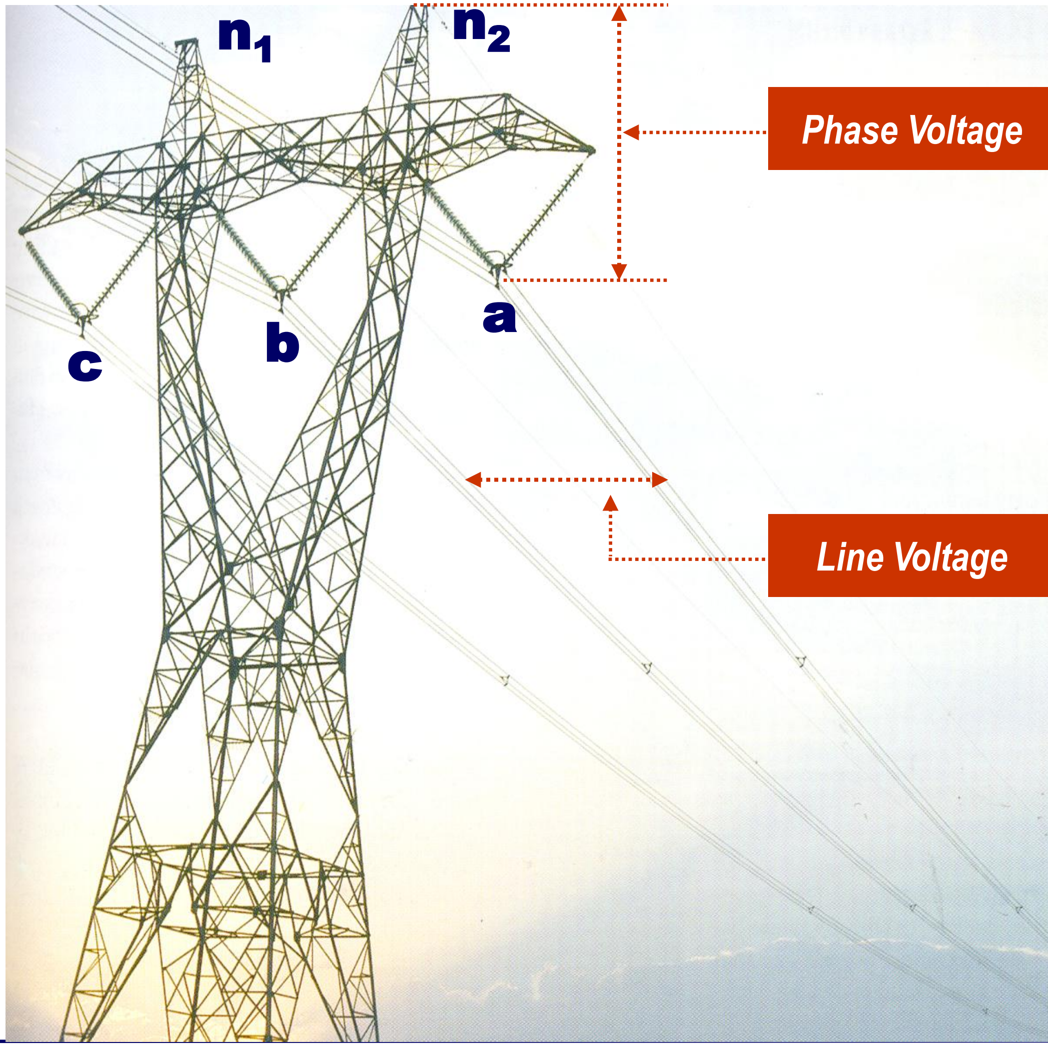


Phase and Line Voltages

Definition

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage





Three Phase Systems

Phase and Line Voltages

Definition

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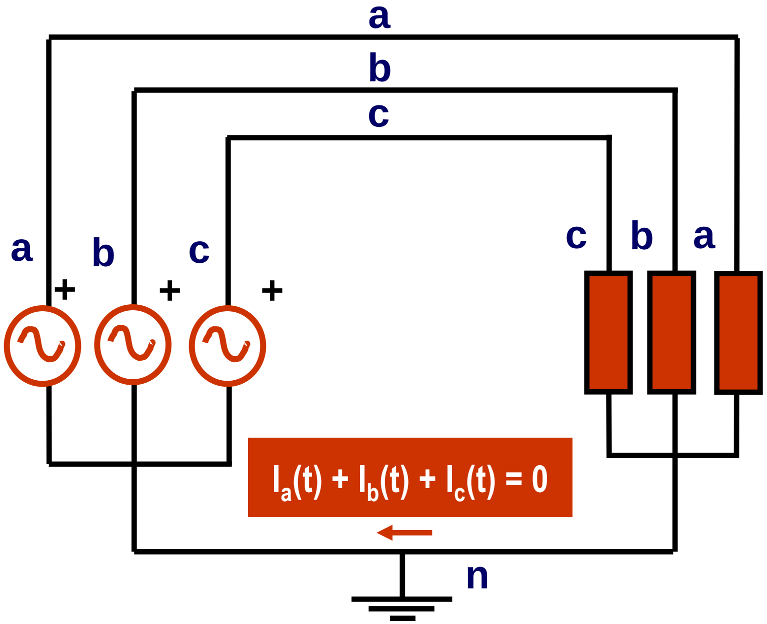


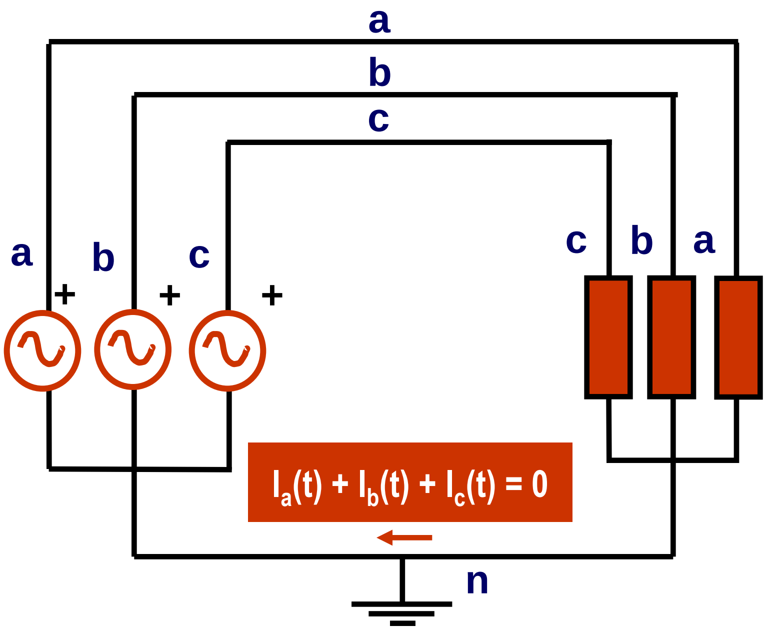
Birds prefer neutral wire since the voltage is zero

Three Phase Systems

Phase and Line Voltages

Definition

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage



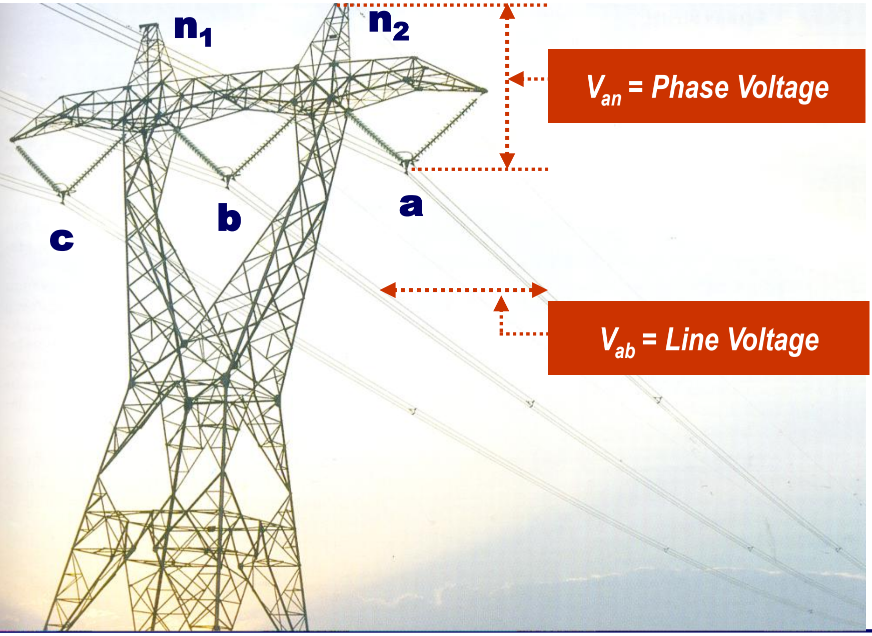
Netral wire acts as lightning arrestor

Three Phase Systems

Phase and Line Voltages

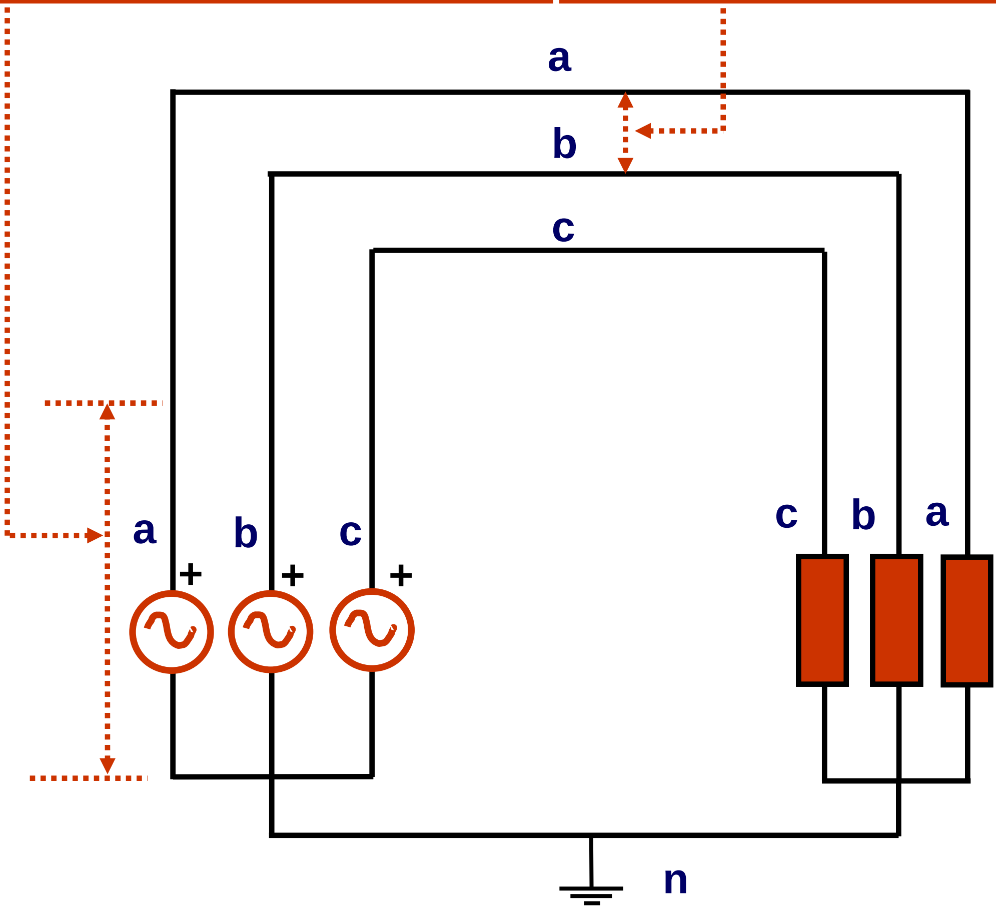
Definition

- Voltage between a phase conductor and ground is called phase voltage,
- Voltage between two phase conductors is called line voltage



V_{an} = Phase Voltage

V_{ab} = Line Voltage



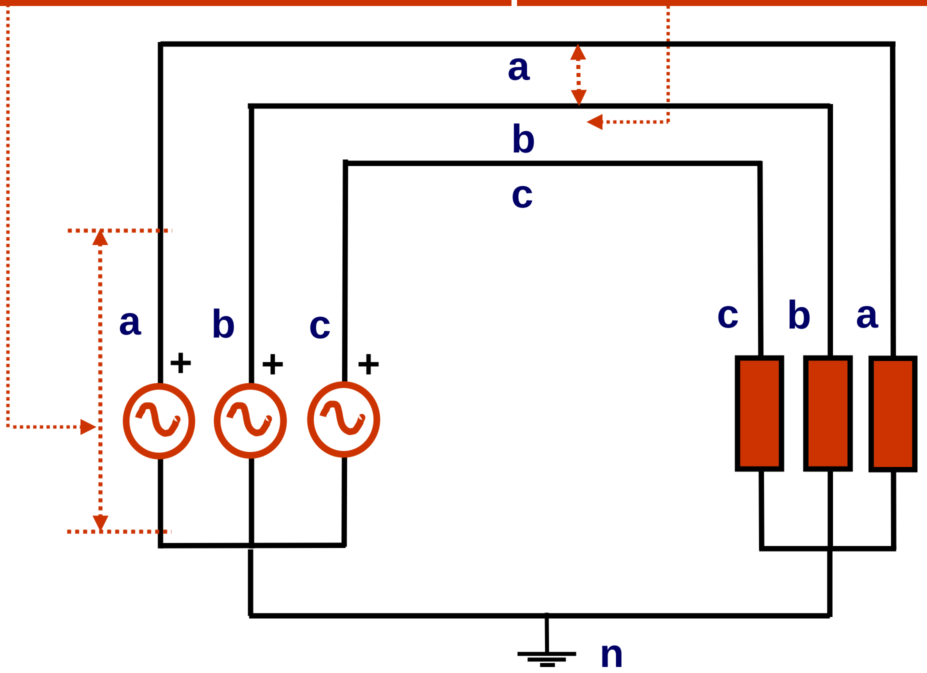
Three Phase Systems

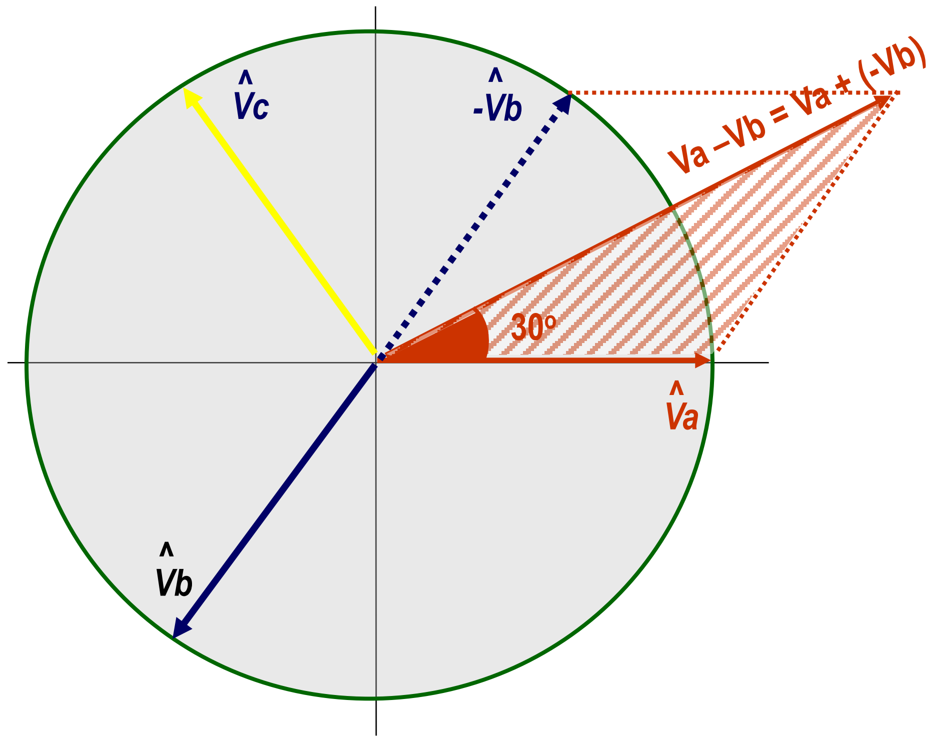
Relation between Phase and Line Voltages

Definition

$$V_{ab} = V_a - V_b = V_a + (-V_b)$$

V_{an} = Phase Voltage	V_{ab} = Line Voltage





Three Phase Systems

Relation between Phase and Line Voltages

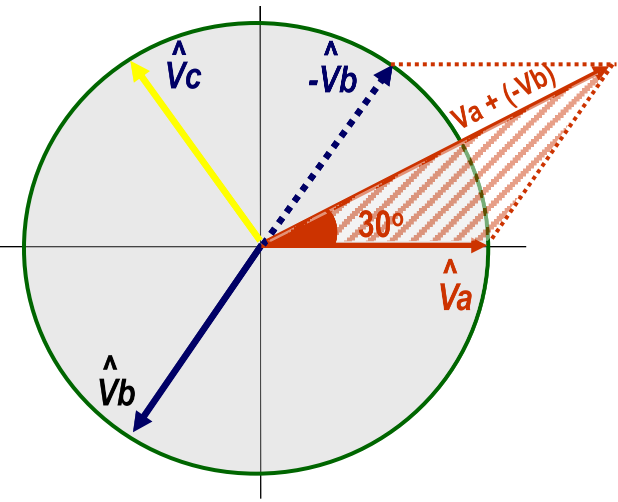
Definition

$$V_{ab} = V_a - V_b = V_a + (-V_b)$$

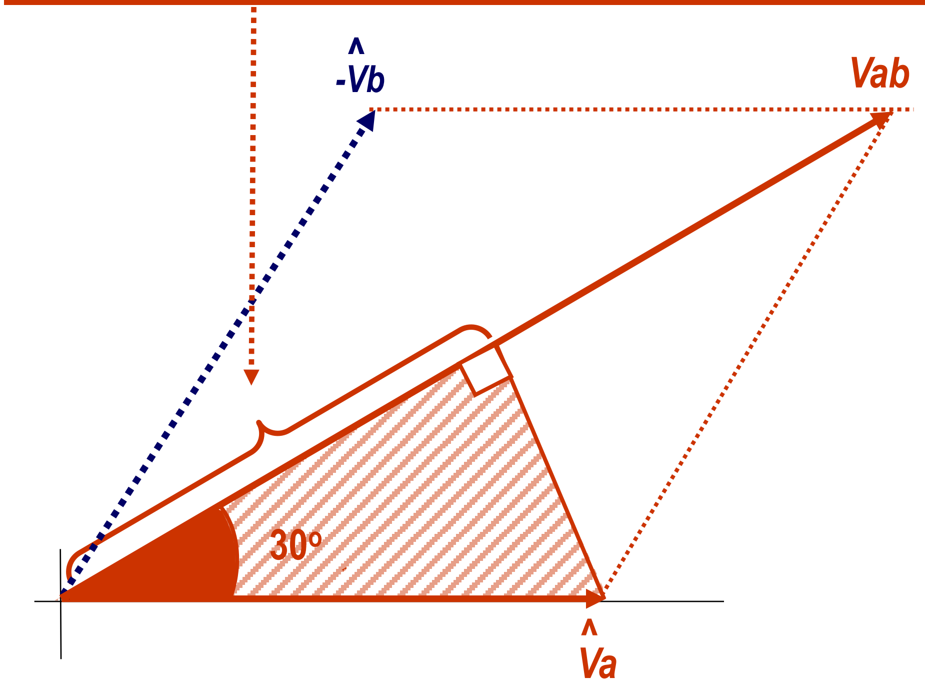
$$|V_{ab}| / 2 = |V_a| \cos 30 = |V_a| \sqrt{3}/2$$

$$|V_{ab}| = |V_a| \sqrt{3}$$

$$|\text{Line Voltage}| = \sqrt{3} \times |\text{Phase Voltage}|$$



$$|V_{ab}| / 2 = |V_a + (-V_b)| / 2$$



Three Phase Systems

Example - Turkish HV System

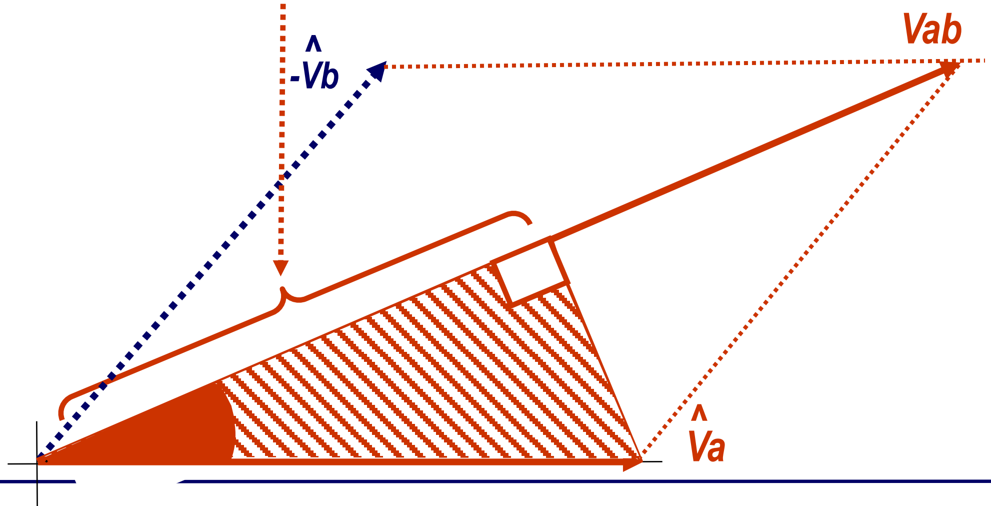
Example (Turkish HV System)

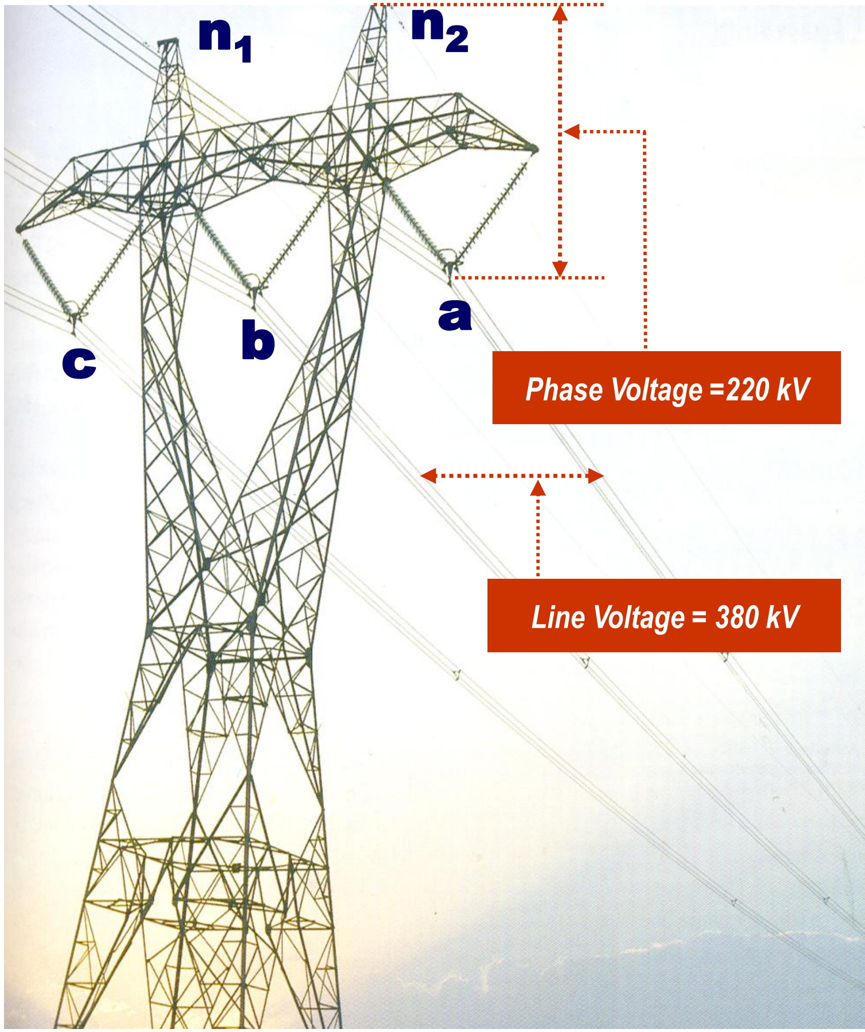
|Line Voltage| = $\sqrt{3}$ x |Phase Voltage|

Phase voltage (rms) = 220 kV

Line voltage (rms) = 220 x $\sqrt{3}$ = 380 kV

$|V_{ab}| / 2 = |V_a + (-V_b)| / 2$



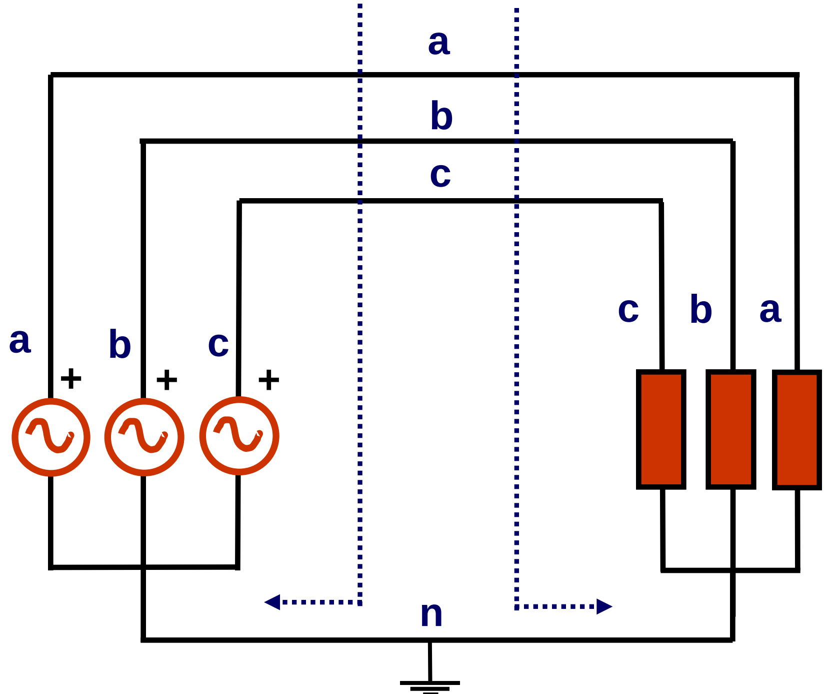


Three Phase Systems

Star (Y) Connection

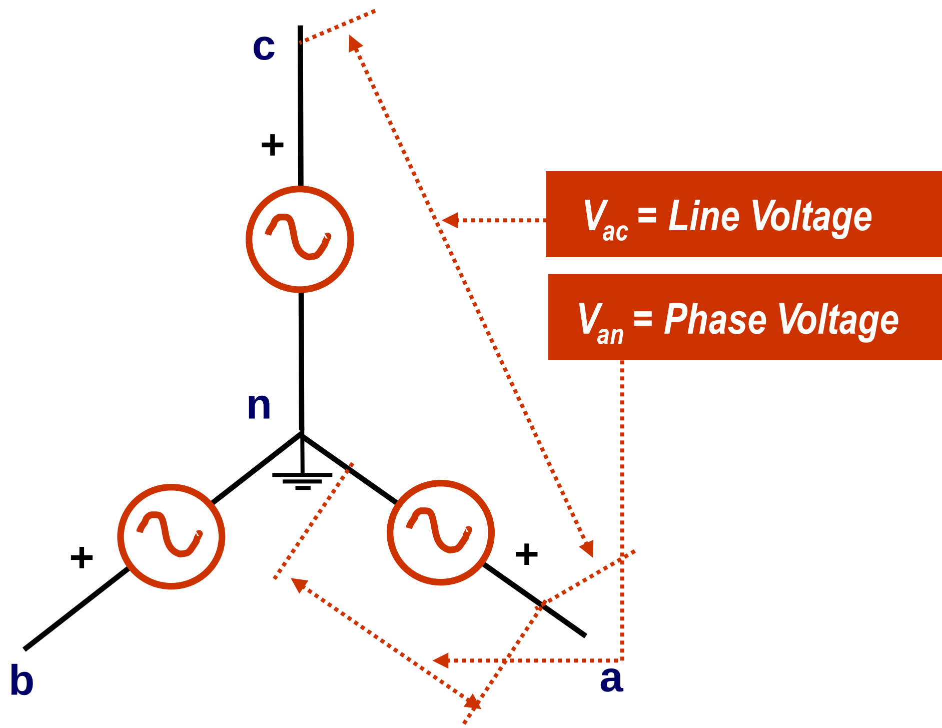
Definition

Star-connected configuration is the one with all the neutral points are connected to a common ground point



Alternative Representation

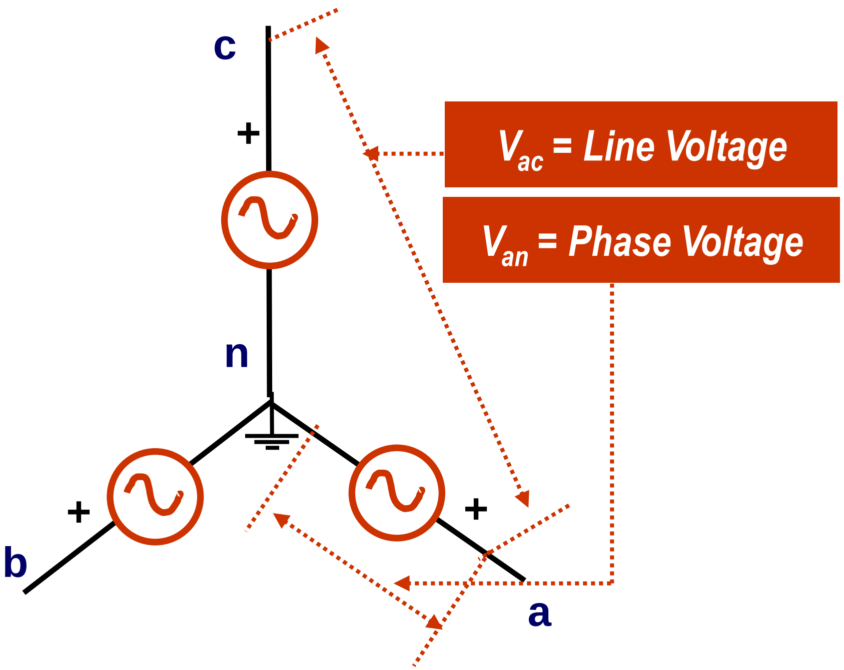
|Line Voltage| = $\sqrt{3}$ x |Phase Voltage|

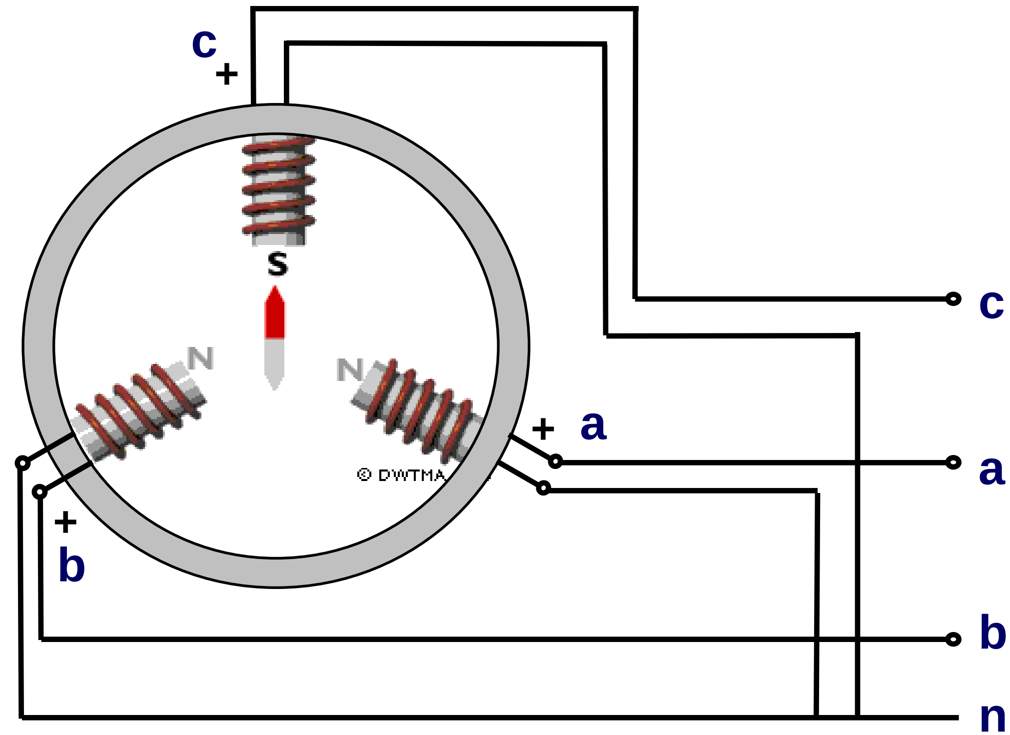


Three Phase Systems

Star (Y) Connected Generator

Definition	Alternative Representation
Star-connected generator is the one with all the generator neutral points are connected to a common ground point	|Line Voltage| = $\sqrt{3}$ x |Phase Voltage|



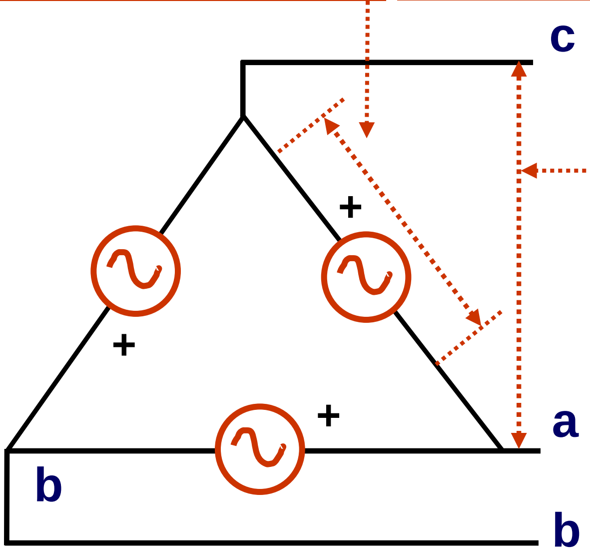


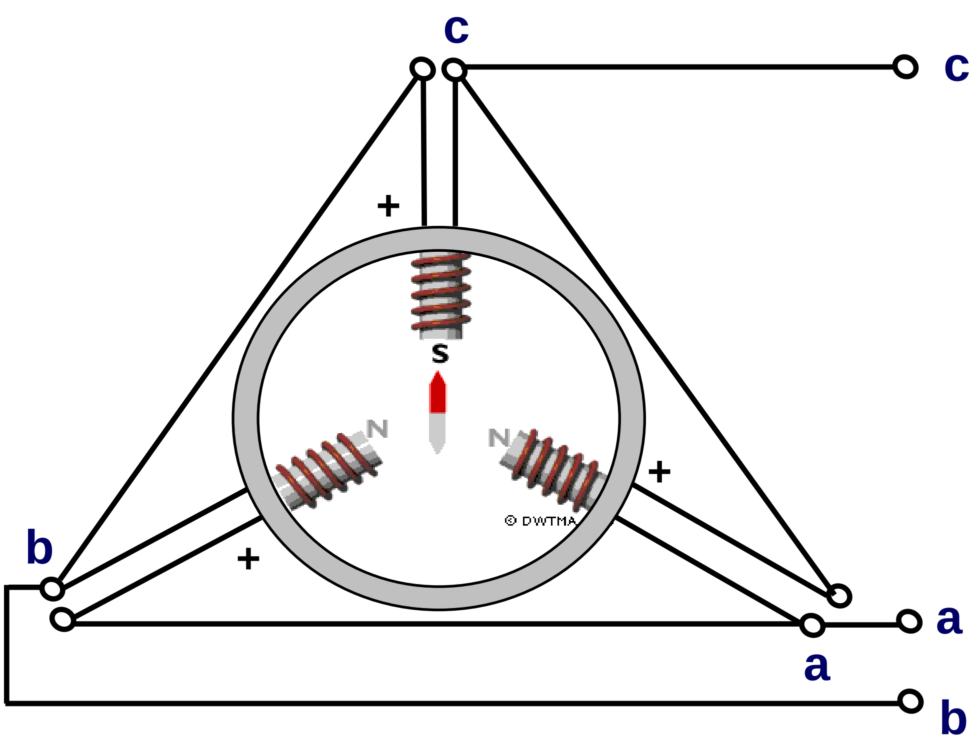
Three Phase Systems

Delta (Δ) Connected Generator

Definition	Configuration
Delta-connected generator is the one with the generator terminals are so connected that they form a triangle (delta)-configuration	| Line Voltage | = | Phase Voltage |

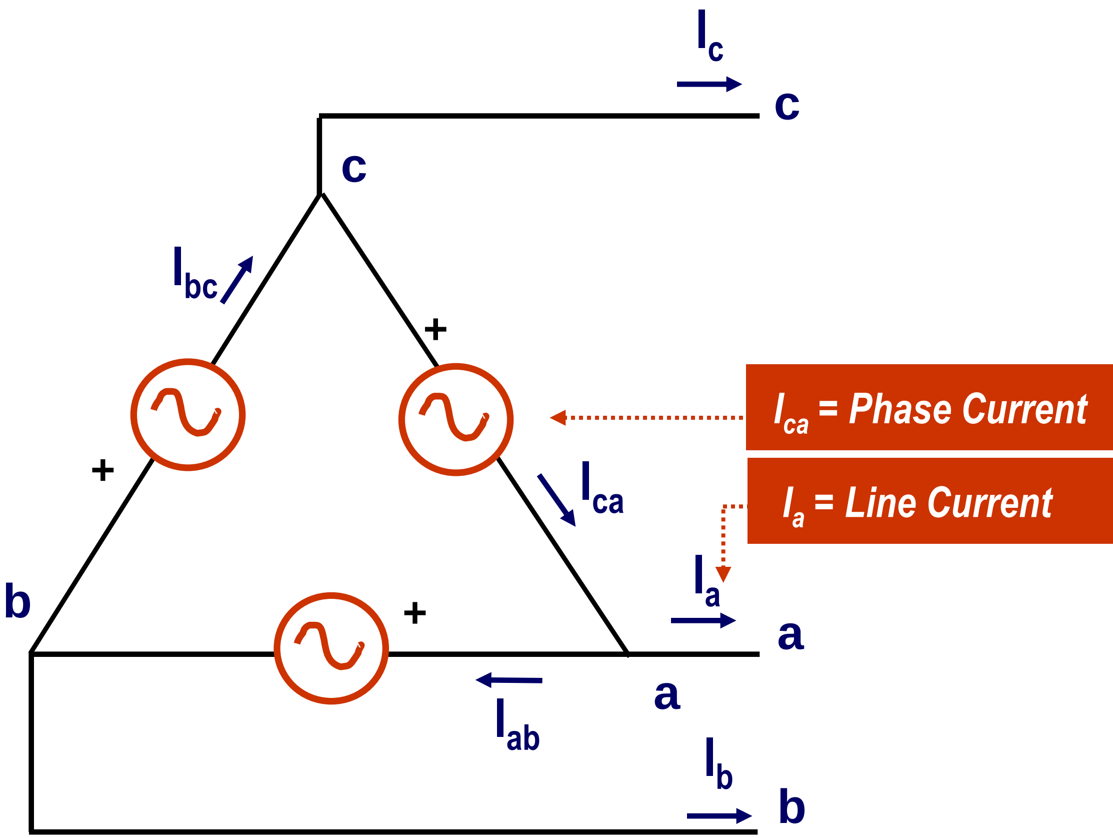
Phase Voltage | *Vac = Line Voltage*





Three Phase Systems

Currents in a Delta (Δ) Connected Generator

Definition	Configuration
$I_a = I_{ca} - I_{ab}$ $I_b = I_{ab} - I_{bc}$ $I_c = I_{bc} - I_{ca}$	

Three Phase Systems

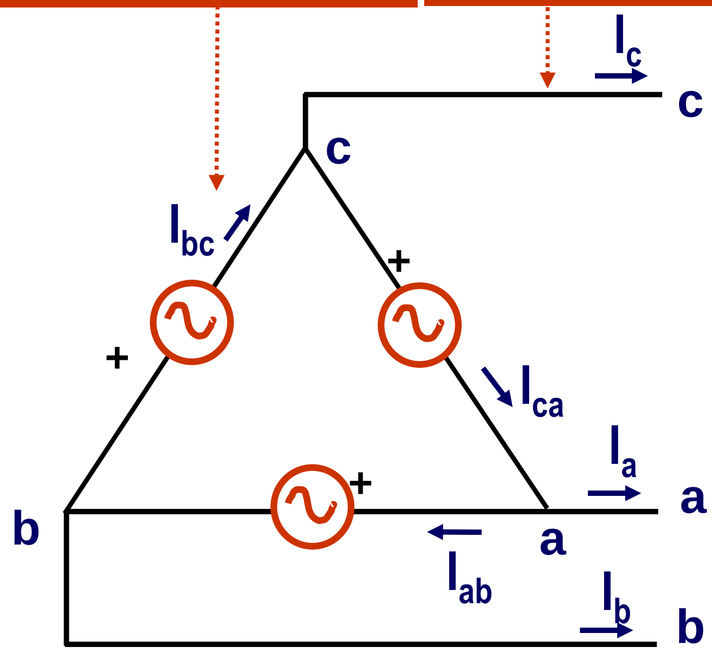
Currents in a Delta (Δ) Connected Generator

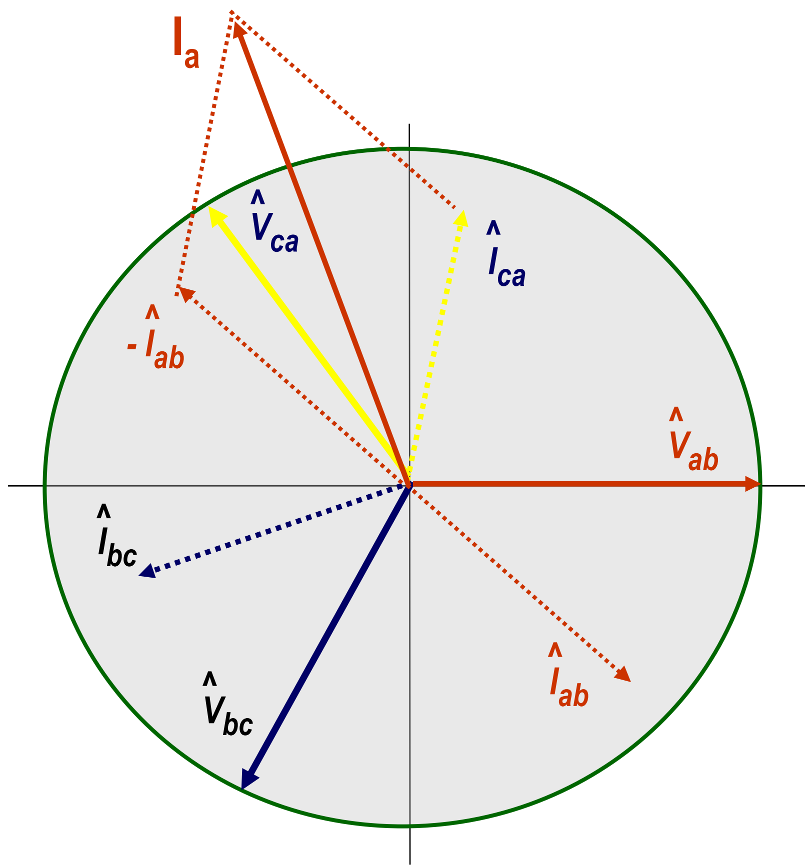
Relation between Line and Phase Currents

$I_a = I_{ca} + (-I_{ab})$

I_{ba} = Phase Current

I_a = Line Current





Currents in a Delta (Δ) Connected Generator

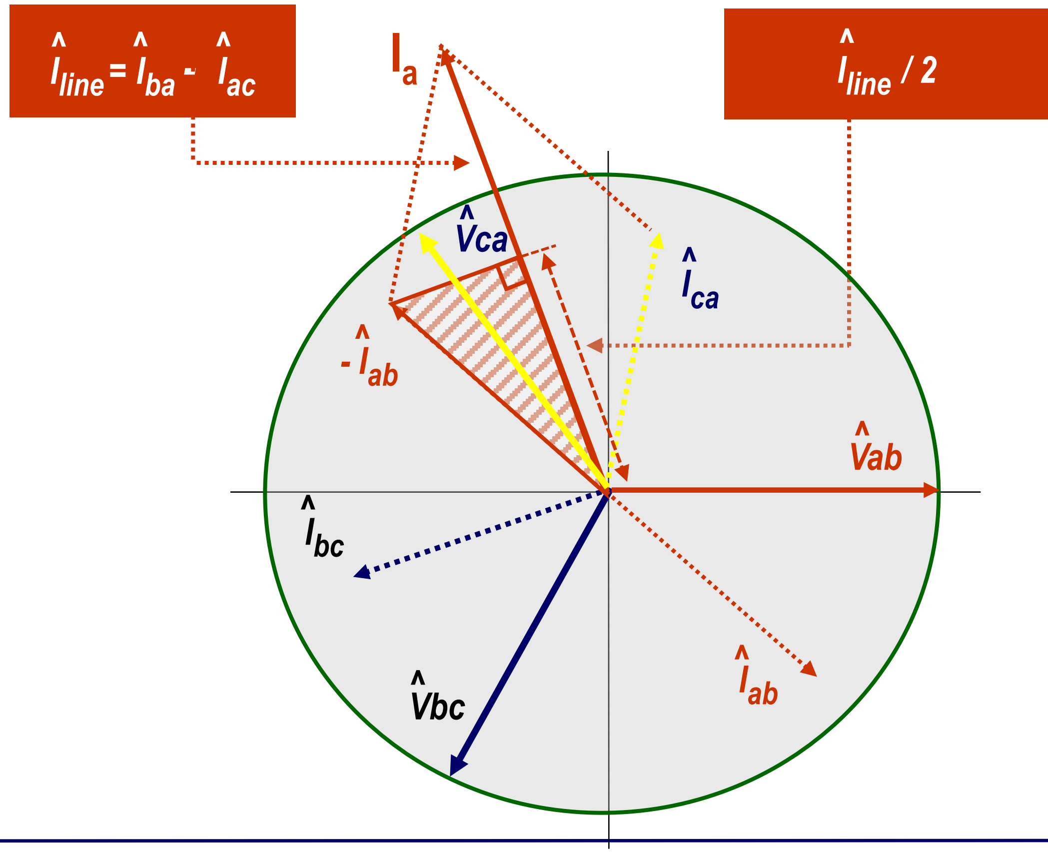
Definition

$$I_a = I_{ca} + (-I_{ab})$$

$$|I_{line}| / 2 = |I_{phase}| \times \cos 30^o$$
$$= |I_{phase}| \times \sqrt{3} / 2$$

|Line Current| = $\sqrt{3}$ x |Phase Current|

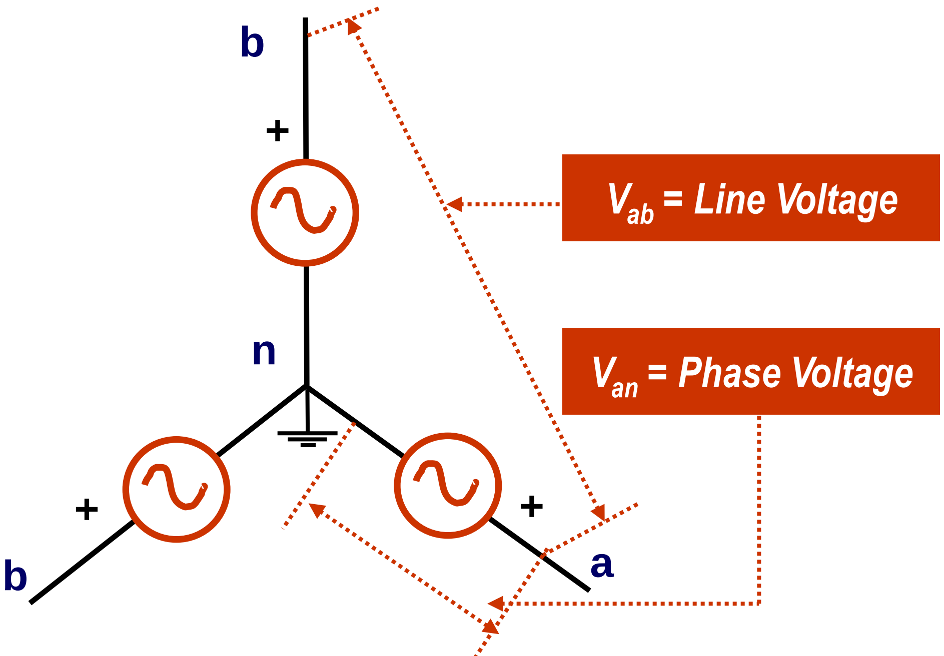
Relation between Line and Phase Currents



Three Phase Systems

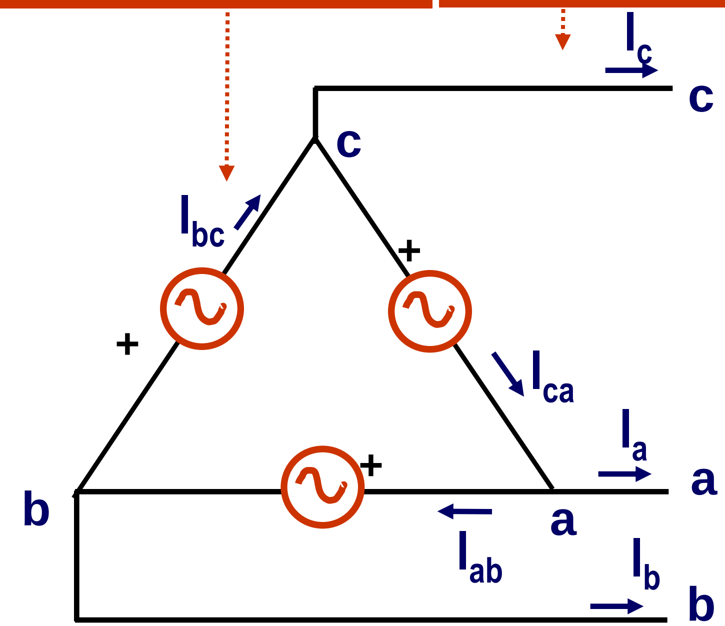
Summary

Star (Y) Connected System	Delta (Δ) Connected System
$\lvert \text{Line Voltage} \rvert = \sqrt{3} \times \lvert \text{Phase Voltage} \rvert$ Line Current = Phase Current	Line Voltage = Phase Voltage $\lvert \text{Line Current} \rvert = \sqrt{3} \times \lvert \text{Phase Current} \rvert$



I_{ba} = Phase Current

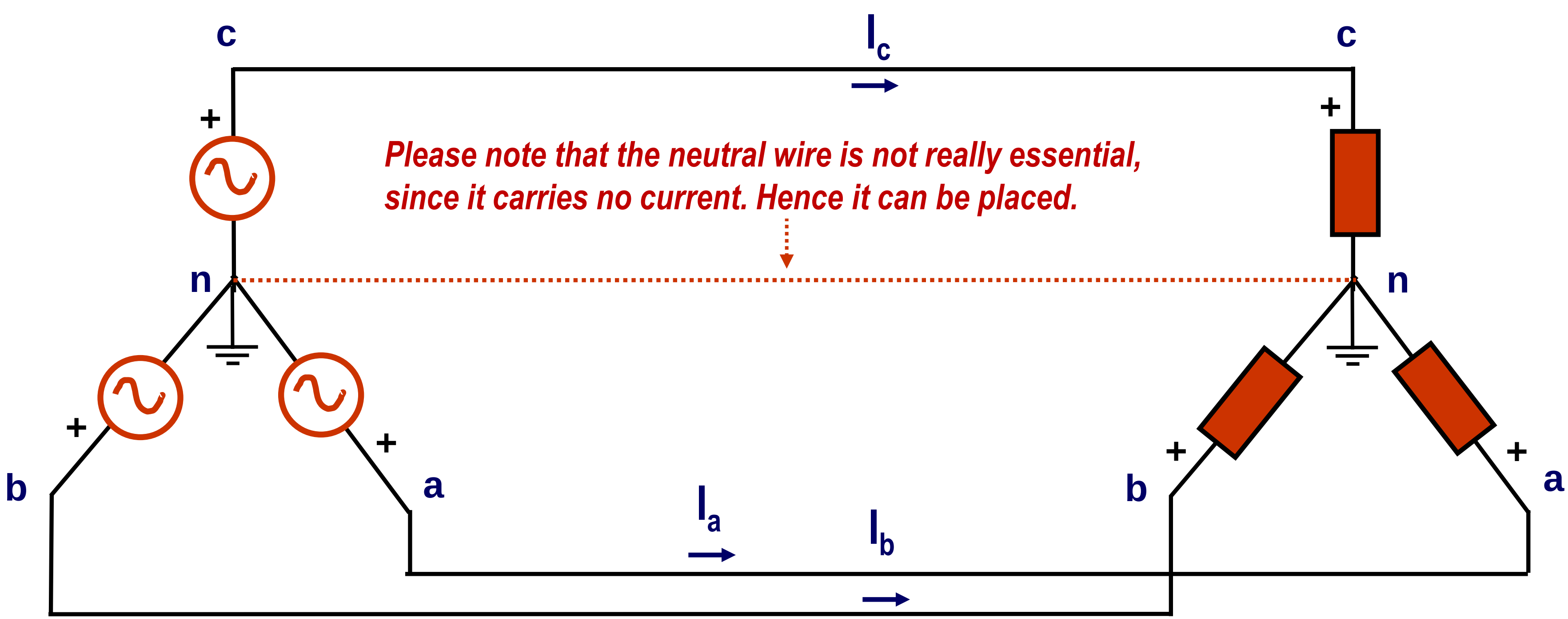
I_a = Line Current



Three Phase Systems

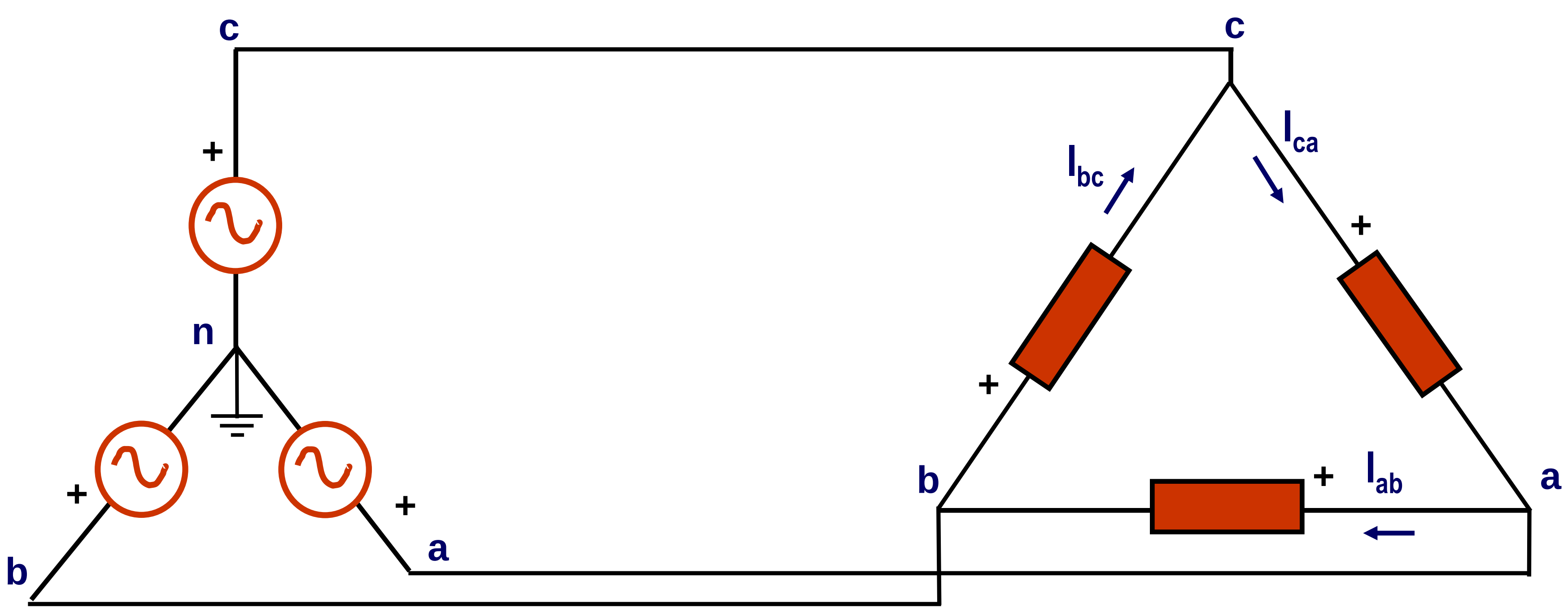
Star-Star (Y-Y) Connected Systems

Definition	Configuration
A star-star connected system is the one with both ends are star connected	Please note that neutral wire does not carry any current for a balanced Y-Y Syatem



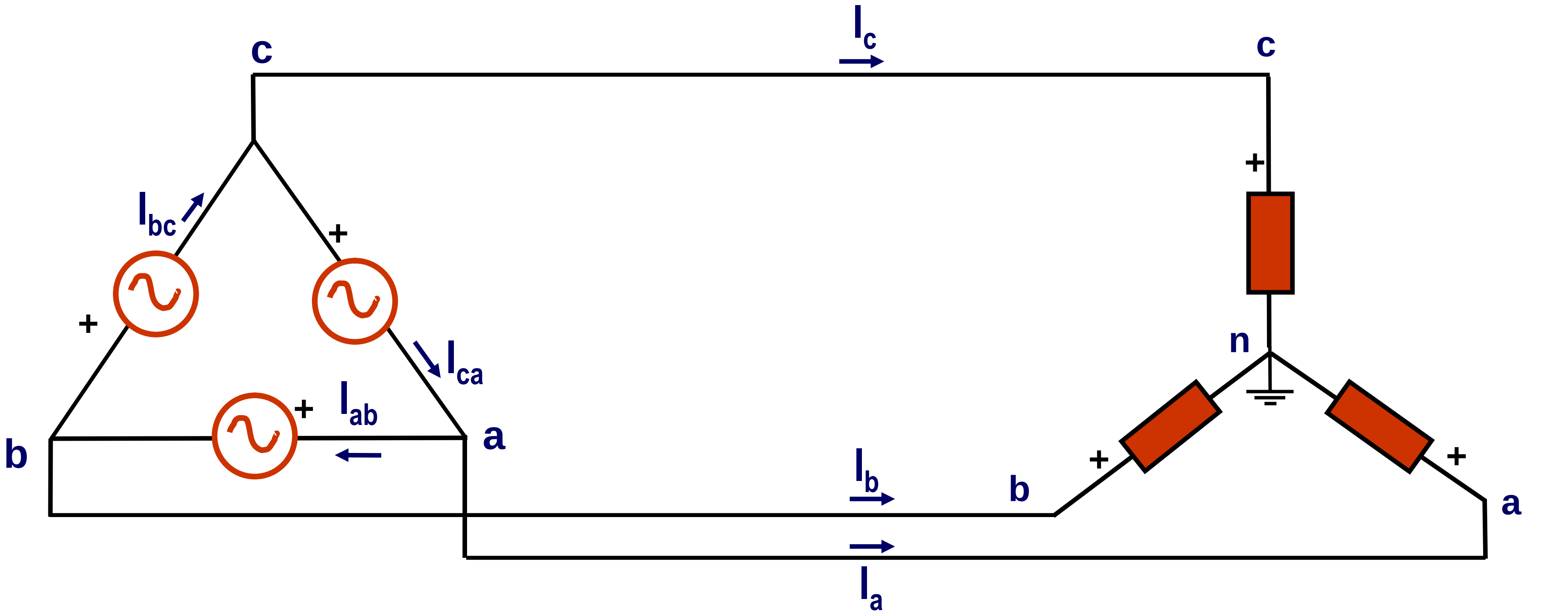
Star-Delta (Y- Δ) Connected Systems

Definition	Configuration
A star-delta connected system is the one with the source is star and load is delta connected	Please note that neutral wire does not exist in a system with one or both ends delta



Delta-Star (Δ -Y) Connected Systems

Definition	Configuration
A delta-star connected system is the one with the source is delta and load is star connected	Please note that neutral wire does not exist in a system with one or both ends delta



Three Phase Systems

Three Phase Power in Star Connected Loads

Star (Y) Connected System

$|\text{Line Voltage}| = \sqrt{3} \times |\text{Phase Voltage}|$

Line Current = Phase Current

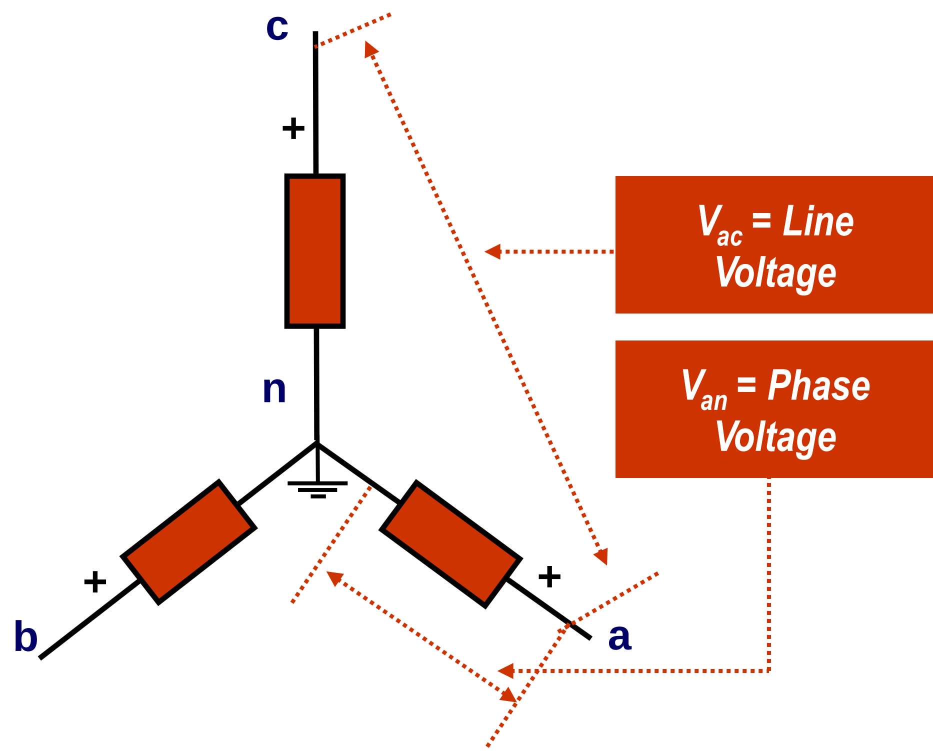
Power Per Phase

$$S_{ph} = P_{ph} + j\,Q_{ph} = V_{ph}\, I_{ph}^{*}$$

Since we have three phases with identical power consumption, total power consumption becomes

$$S_{3\text{-}ph} = 3 \times S_{ph} = 3\, V_{ph}\, I_{ph}^{*} = 3\,(V_{line} / \sqrt{3}\,)\, I_{line}$$

$$S_{3\text{-}ph} = \sqrt{3}\; V_{line}\, I_{line}$$



Three Phase Systems

Three Phase Active Power in Star Connected Loads

Star (Y) Connected System

|Line Voltage| = $\sqrt{3}$ x |Phase Voltage|

Line Current = Phase Current

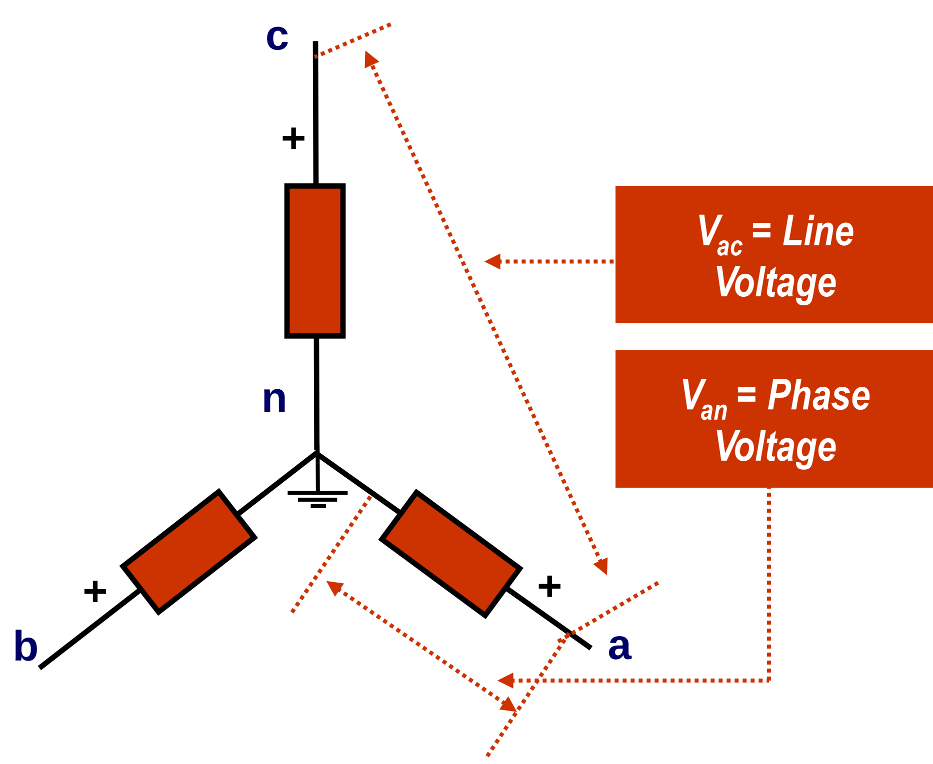
Remember that;

$$P_{ph} = S_{ph} \cos\theta$$

Since we have three phases with identical power consumption, total power consumption becomes

$$P_{3\text{-}ph} = 3 \times S_{ph} \cos\theta$$
$$= 3\, V_{ph}\, I_{ph} \cos\theta$$
$$= 3\, (V_{line} / \sqrt{3}\,)\, I_{line} \cos\theta$$

$$P_{3\text{-}ph} = \sqrt{3}\, V_{line}\, I_{line} \cos\theta$$



Three Phase Reactive Power in Star Connected Loads

Star (Y) Connected System

$|\text{Line Voltage}| = \sqrt{3} \times |\text{Phase Voltage}|$

Line Current = Phase Current

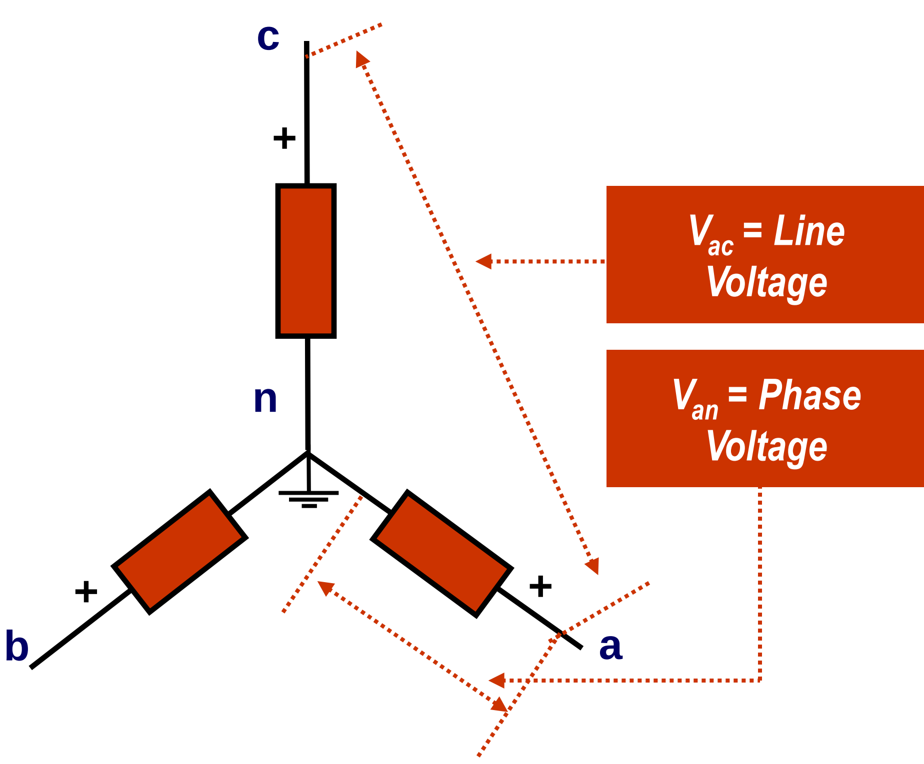
Remember that;

$$Q_{ph} = S_{ph} \sin\theta$$

Since we have three phases with identical power consumption, total power consumption becomes

$$Q_{3\text{-}ph} = 3 \times S_{ph} \sin\theta$$
$$= 3\, V_{ph}\, I_{ph} \sin\theta$$
$$= 3\, (V_{line} / \sqrt{3}\,)\, I_{line} \sin\theta$$

$$Q_{3\text{-}ph} = \sqrt{3}\, V_{line}\, I_{line} \sin\theta$$



Three Phase Systems

Three Phase Reactor

Three Phase Reactor

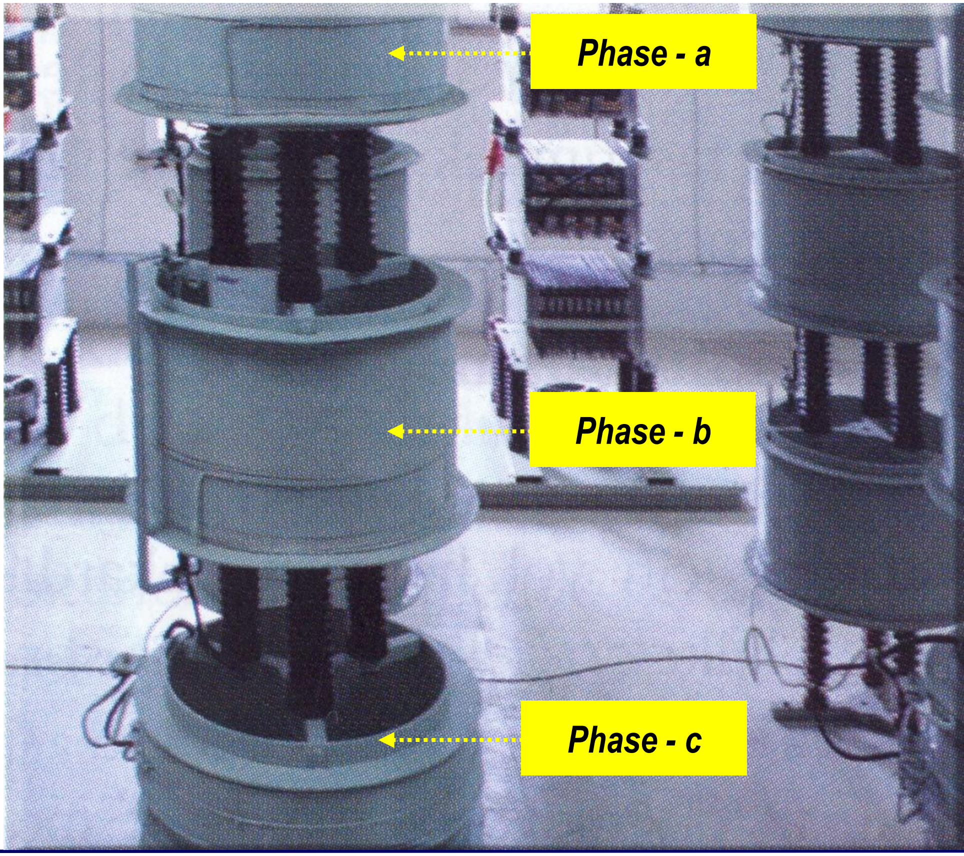
Remember that;

$$Q_{3\text{-}ph} = 3 \times S_{ph} \sin\theta$$
$$= 3\, V_{ph}\, I_{ph} \sin\theta$$
$$= 3\, (V_{line} / \sqrt{3}\,)\, I_{line} \sin\theta$$

$$Q_{3\text{-}ph} = \sqrt{3}\; V_{line}\, I_{line} \sin\theta$$

$$|\text{Line Voltage}| = \sqrt{3} \times |\text{Phase Voltage}|$$

Line Current = Phase Current



Three Phase Systems

Three Phase Reactor

Small-Size Three Phase Reactor

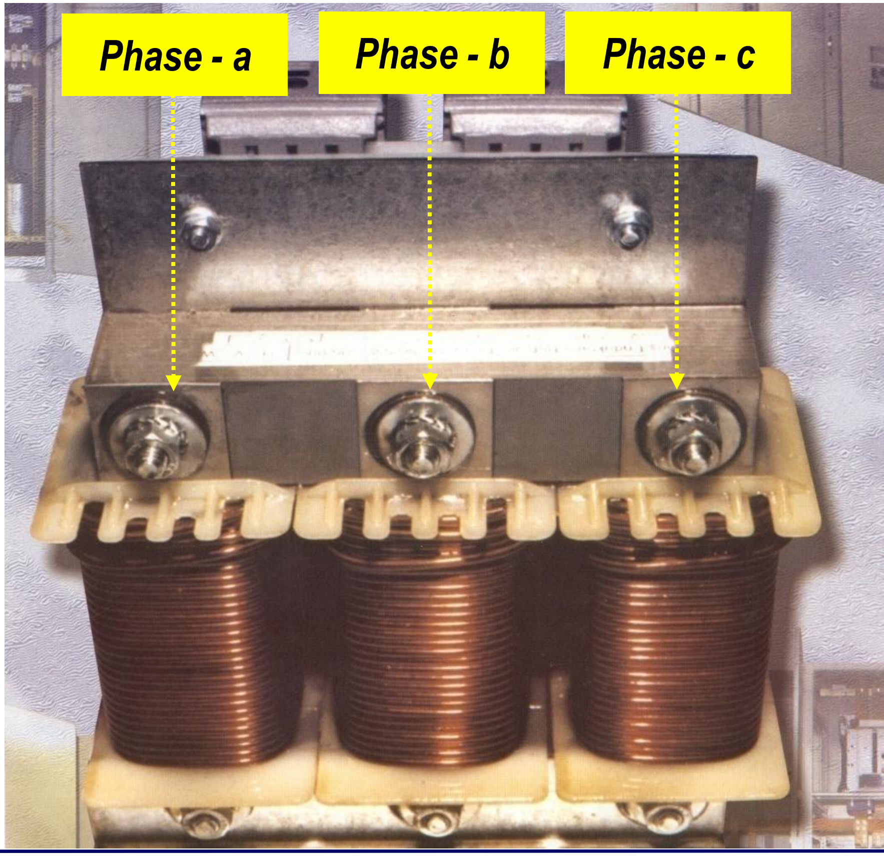
$|\text{Line Voltage}| = \sqrt{3} \times |\text{Phase Voltage}|$

Line Current = Phase Current

Remember that;

$$Q_{3\text{-}ph} = 3 \times S_{ph} \sin\theta$$
$$= 3\, V_{ph}\, I_{ph} \sin\theta$$
$$= 3\,(V_{line} / \sqrt{3}\,)\, I_{line} \sin\theta$$

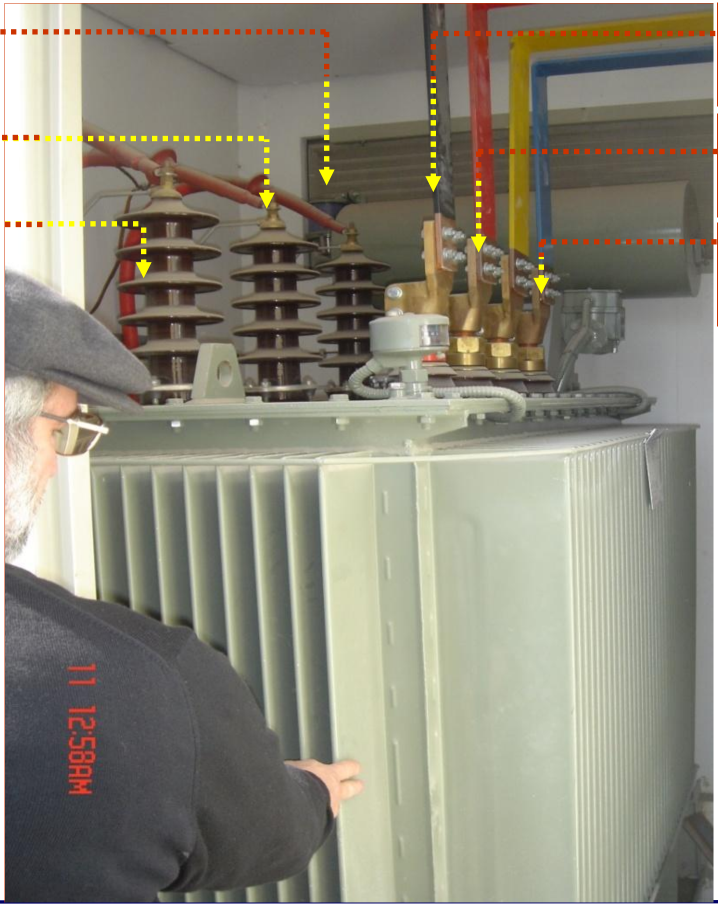
$$Q_{3\text{-}ph} = \sqrt{3}\; V_{line}\, I_{line} \sin\theta$$



Three Phase Systems

Three Phase Transformer

Primary Side (Delta)	Secondary Side (Star)
Primary Side Phase - a	Secondary Side Phase - a
Primary Side Phase - b	Secondary Side Phase - b
Primary Side Phase - c	Secondary Side Phase - c



Voltages

Primary Side

$V_{line} = 34500$ Volts

$V_{phase} = V_{line} = 34500$ Volt

Secondary Side

$V_{line} = 380$ Volts

$V_{phase} = 380 / \sqrt{3} = 220$ Volts

Three Phase Systems

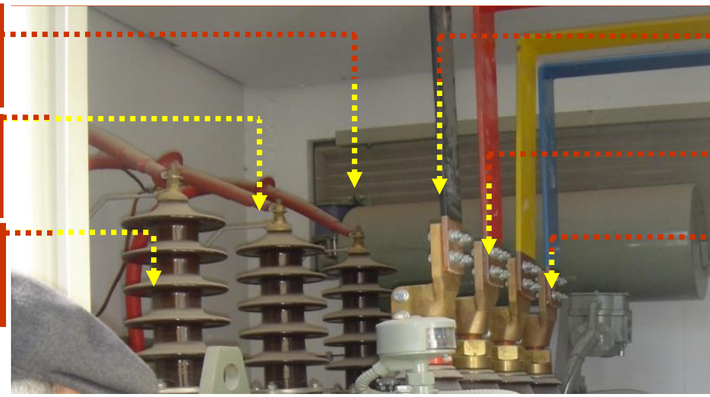
Three Phase Transformer

Primary Side (Delta)	Secondary Side (Star)

Primary Side Phase - a

Primary Side Phase - b

Primary Side Phase - c

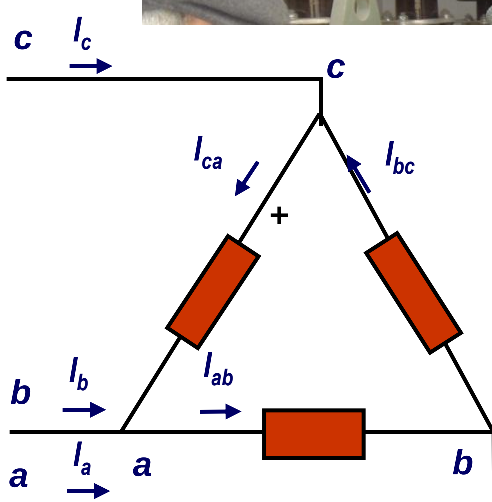


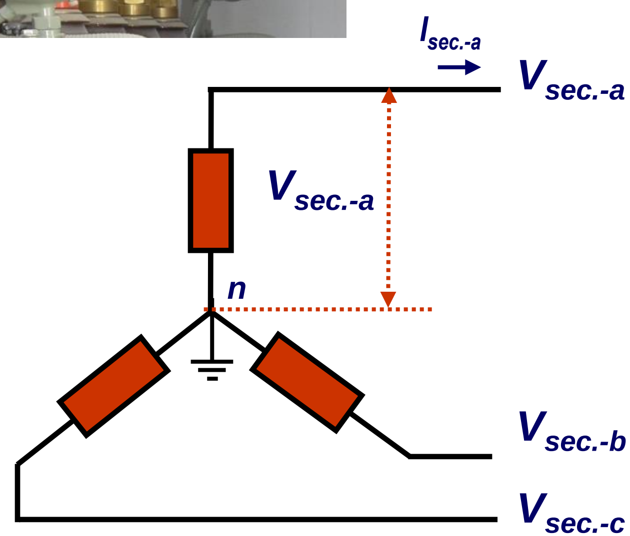
Secondary Side Phase - a

Secondary Side Phase - b

Secondary Side Phase - c

Please note that almost all distribution transformers are delta-star connected





Three Phase Transformer

Three-Phase Power (Overview)

$$P_{prim.-a} = V_a I_a \cos\theta$$

$$P_{prim.-b} = V_b I_b \cos\theta$$

$$P_{prim.-c} = V_c I_c \cos\theta$$

+

$$P_{prim.-Total} = V_a I_a \cos\theta + V_b I_b \cos\theta + V_c I_c \cos\theta$$

$$= 3\, V_{phase}\, I_{phase} \cos\theta$$

$$= 3\, V_{line}\, I_{line} / \sqrt{3} \cos\theta$$

$$= \sqrt{3}\, V_{line}\, I_{line} \cos\theta$$

Three Phase Transformer

Three-Phase Power (Overview)

Power on the Primary Side

$$P_{Prim.\text{ - }Total} = \sqrt{3}\, V_{Prim.\text{- }line}\, I_{Prim.\text{-}line} \cos\theta$$

$$Q_{Prim.\text{ - }Total} = \sqrt{3}\, V_{Prim.\text{- }line}\, I_{Prim.\text{-}line} \sin\theta$$

$$S_{Prim.\text{ - }Total} = \sqrt{3}\, V_{Prim.\text{- }line}\, I_{Prim.\text{-}line}$$

Power on the Secondary Side

$$P_{Sec.\text{ - }Total} = \sqrt{3}\, V_{Sec.\text{- }line}\, I_{Sec.\text{-}line} \cos\theta$$

$$Q_{Sec.\text{ - }Total} = \sqrt{3}\, V_{Sec.\text{- }line}\, I_{Sec.\text{-}line} \sin\theta$$

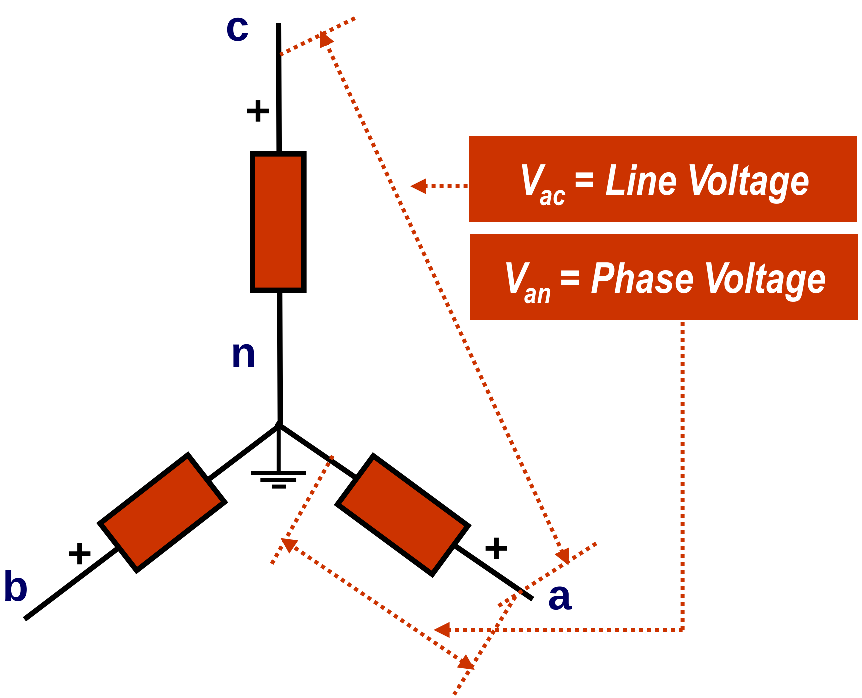
$$S_{Sec.\text{ - }Total} = \sqrt{3}\, V_{Sec.\text{- }line}\, I_{sec.\text{-}line}$$

Three Phase Systems

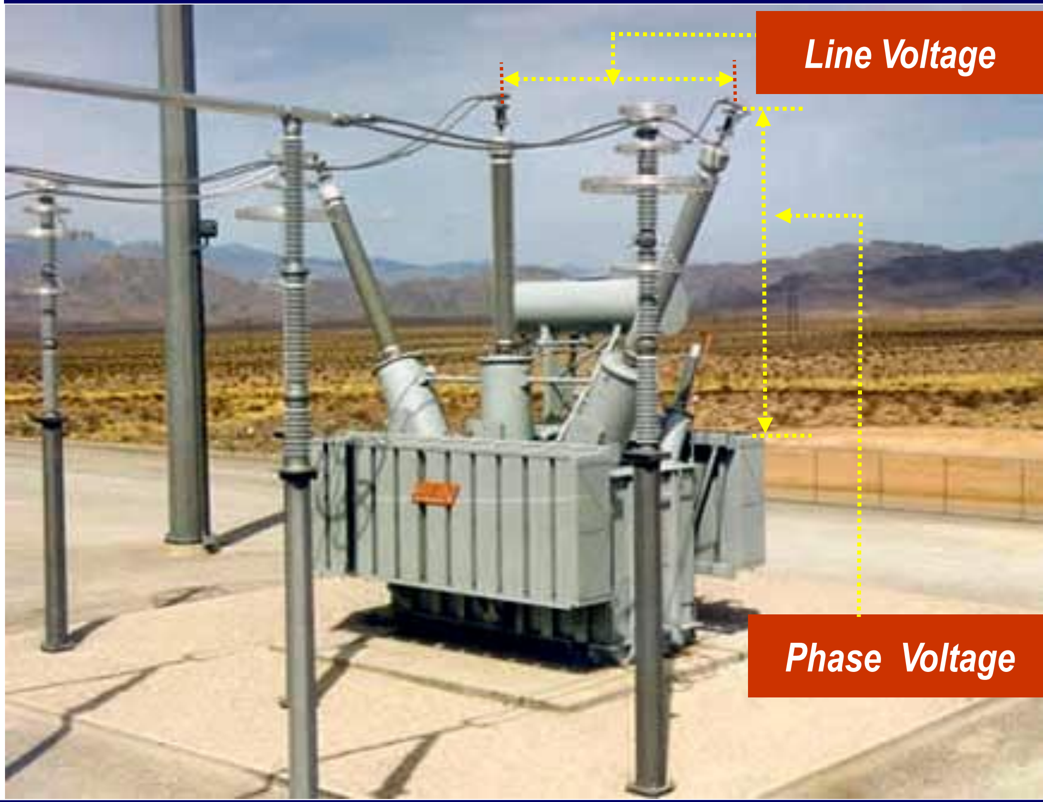
Example: Star (Y) Connected Load

|Line Voltage| = $\sqrt{3}$ x |Phase Voltage|

Line Current = Phase Current



135 MVA shunt reactor delivered to Nevada Power Company by VA TECH ELIN



Three Phase Systems

You do not seem to understand the line voltage in a Y- Connected Load ...

Three Phase Systems

Three Phase Power in Delta Connected Loads

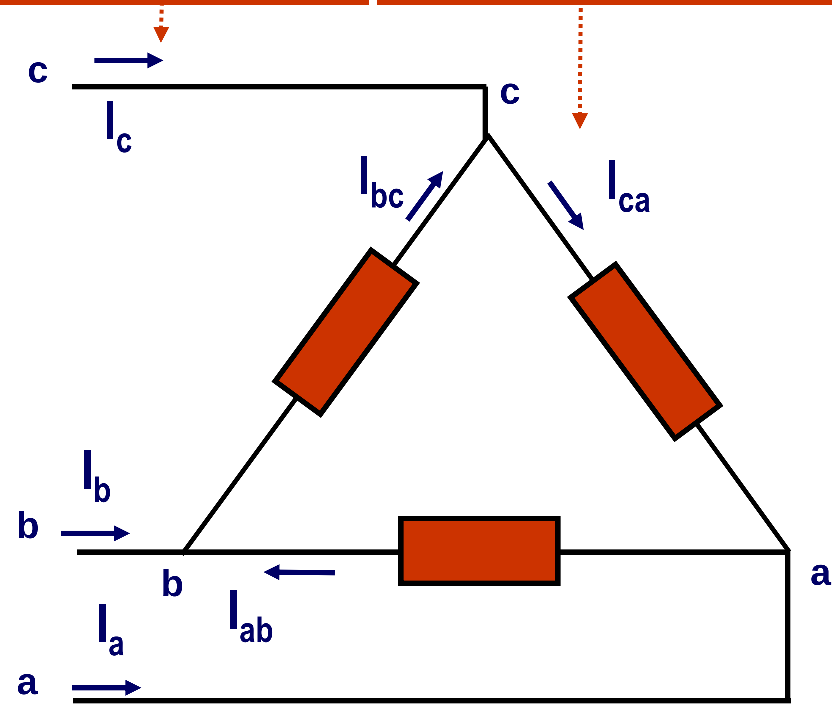
Delta (Δ) Connected System

Line Voltage = Phase Voltage

|Line Current| = $\sqrt{3}$ x |Phase Current|

I_a = Line Current

I_{ba} = Phase Current



Power Per Phase

$$S_{ph} = P_{ph} + j\,Q_{ph}$$
$$= V_{ph}\,I_{ph}^*$$

Since we have three phases with identical power consumptions, three phase (total) power consumption becomes

$$S_{3\text{-}ph} = 3 \times S_{ph}$$
$$= 3\,V_{ph}\,I_{ph}^*$$
$$= 3\,V_{line}\,(I_{line} / \sqrt{3}\,)$$

$$S_{3\text{-}ph} = \sqrt{3}\;V_{line}\,I_{line}$$

Three Phase Power in Delta Connected Loads

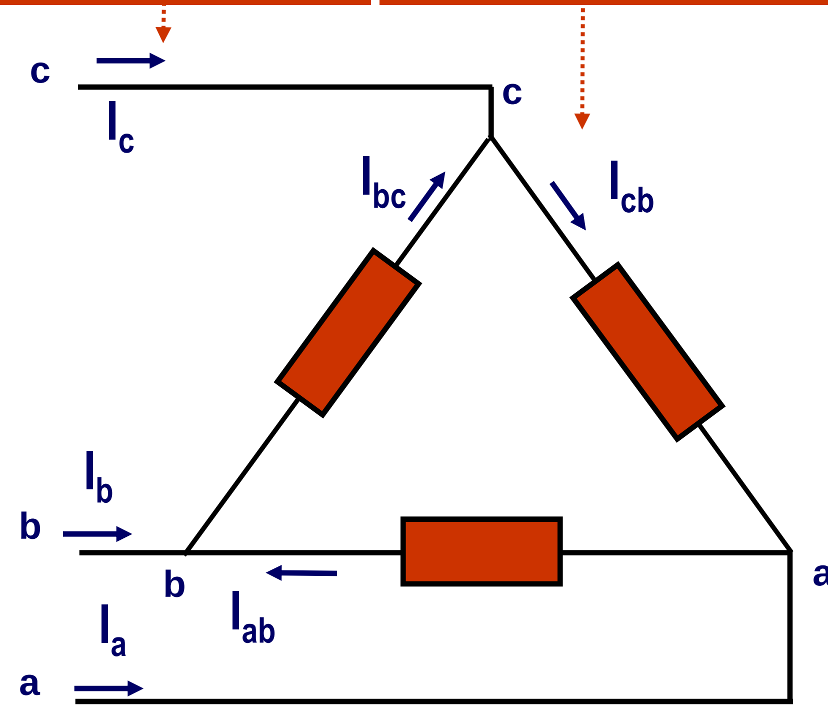
Delta (Δ) Connected System

Line Voltage = Phase Voltage

$|\text{Line Current}| = \sqrt{3} \times |\text{Phase Current}|$

I_a = *Line Current*

I_{ba} = *Phase Current*



Power Per Phase

$$P_{ph} = V_{ph}\, I_{ph} \cos\theta$$

Since we have three phases with identical power consumptions, three phase (total) power consumption becomes

$$P_{3\text{-}ph} = 3 \times P_{ph}$$
$$= 3\, V_{ph}\, I_{ph} \cos\theta$$
$$= 3\, V_{line}\, (\, I_{line} / \sqrt{3}\,) \cos\theta$$

$$P_{3\text{-}ph} = \sqrt{3}\; V_{line}\, I_{line} \cos\theta$$

Three Phase Systems

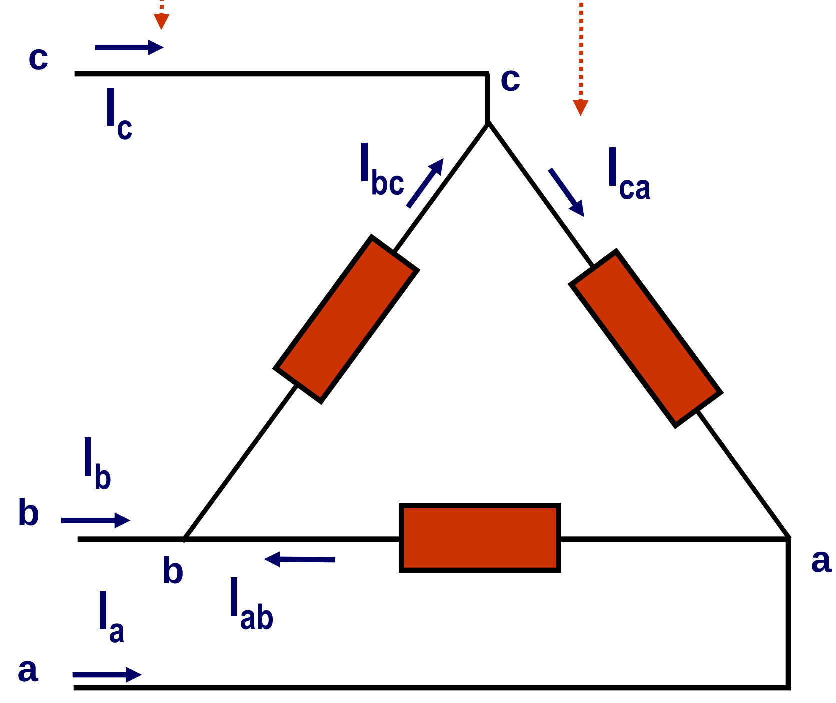
Three Phase Power in Delta Connected Loads

Delta (Δ) Connected System

Line Voltage = Phase Voltage

$|\text{Line Current}| = \sqrt{3} \times |\text{Phase Current}|$

I_a = Line Current *I_{ba} = Phase Current*



Power Per Phase

$$Q_{ph} = V_{ph} I_{ph} \sin\theta$$

Since we have three phases with identical power consumptions, three phase (total) power consumption becomes

$$Q_{3\text{-}ph} = 3 \times Q_{ph}$$
$$= 3 V_{ph} I_{ph} \sin\theta$$
$$= 3 V_{line} (I_{line} / \sqrt{3}) \sin\theta$$

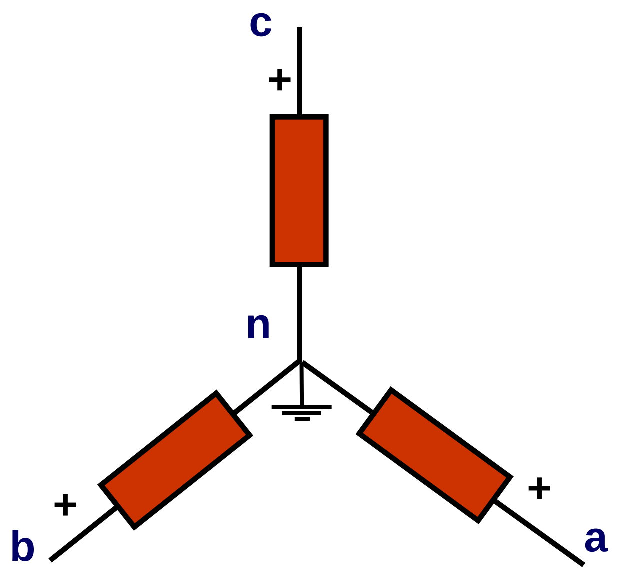
$$Q_{3\text{-}ph} = \sqrt{3}\, V_{line} I_{line} \sin\theta$$

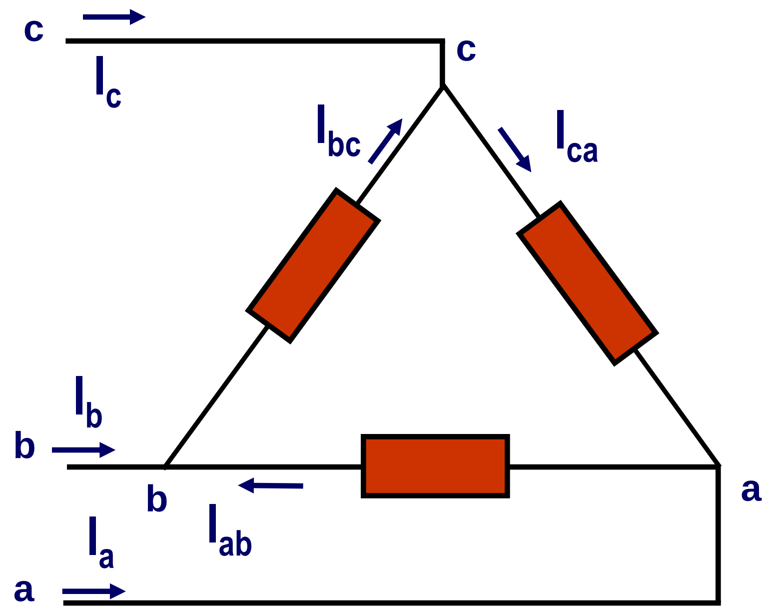
Three Phase Power - Summary

Star (Y) Connected System	Delta (Δ) Connected System

Three phase power expressions are identical for star and delta connections

$S_{3\text{-}ph} = \sqrt{3}\ V_{line}\ I_{line}$	$P_{3\text{-}ph} = \sqrt{3}\ V_{line}\ I_{line} \cos\theta$	$Q_{3\text{-}ph} = \sqrt{3}\ V_{line}\ I_{line} \sin\theta$





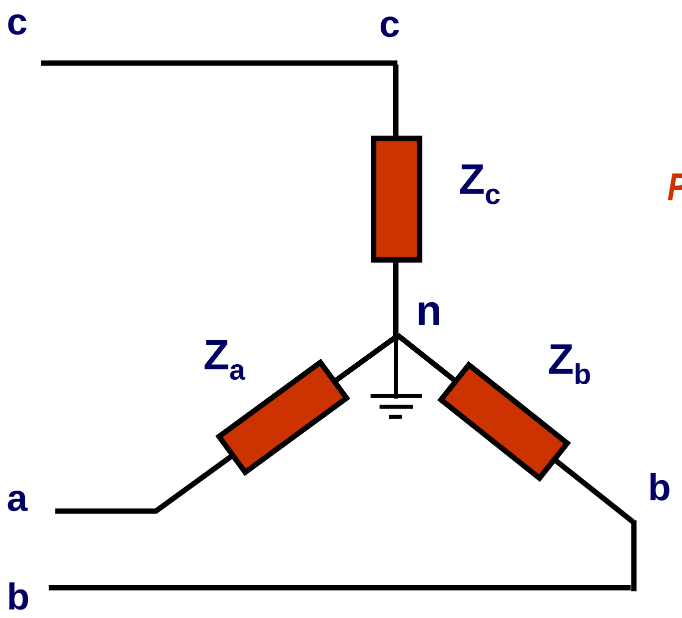
Three Phase Systems

Star - Delta Conversion

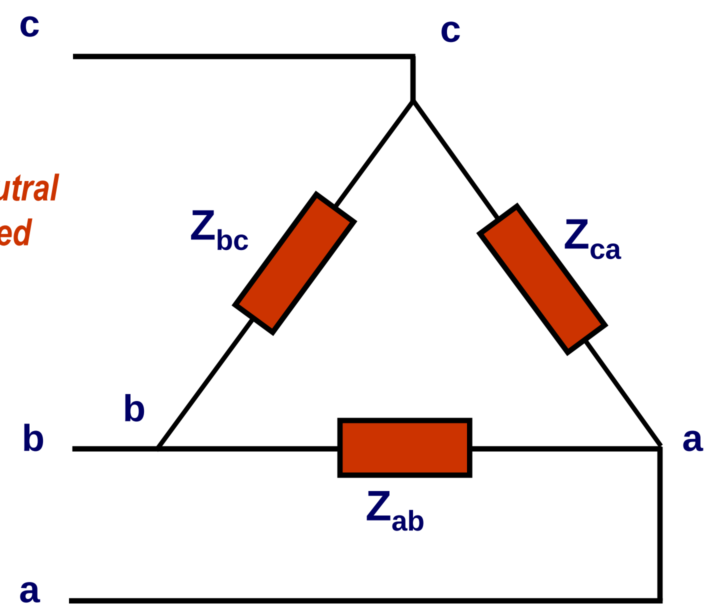
Formulation

A star connected load can be converted to a delta connected load as follows

$$z_{ab} = (z_a z_b + z_b z_c + z_c z_a) / z_c$$
$$z_{ca} = (z_a z_b + z_b z_c + z_c z_a) / z_b$$
$$z_{bc} = (z_a z_b + z_b z_c + z_c z_a) / z_a$$



Please note that the neutral node is now eliminated



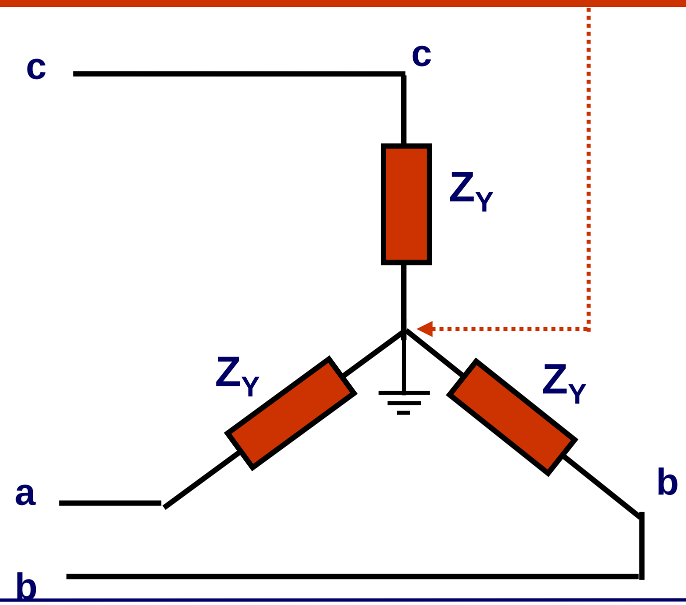
Star - Delta Conversion

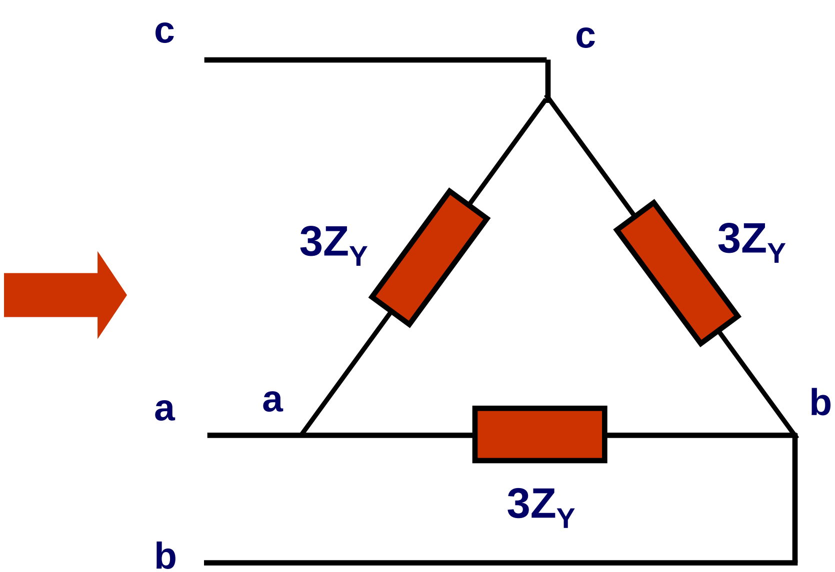
If all impedances of star are identical, then the formula reduces to the following simple form

Simplification

$$z_{\Delta} = (z_Y^2 + z_Y^2 + z_Y^2) / z_Y = 3 z_Y$$

Please note that the neutral node is eliminated by transformation, i.e. no. of nodes is reduced by one.





Delta - Star Conversion

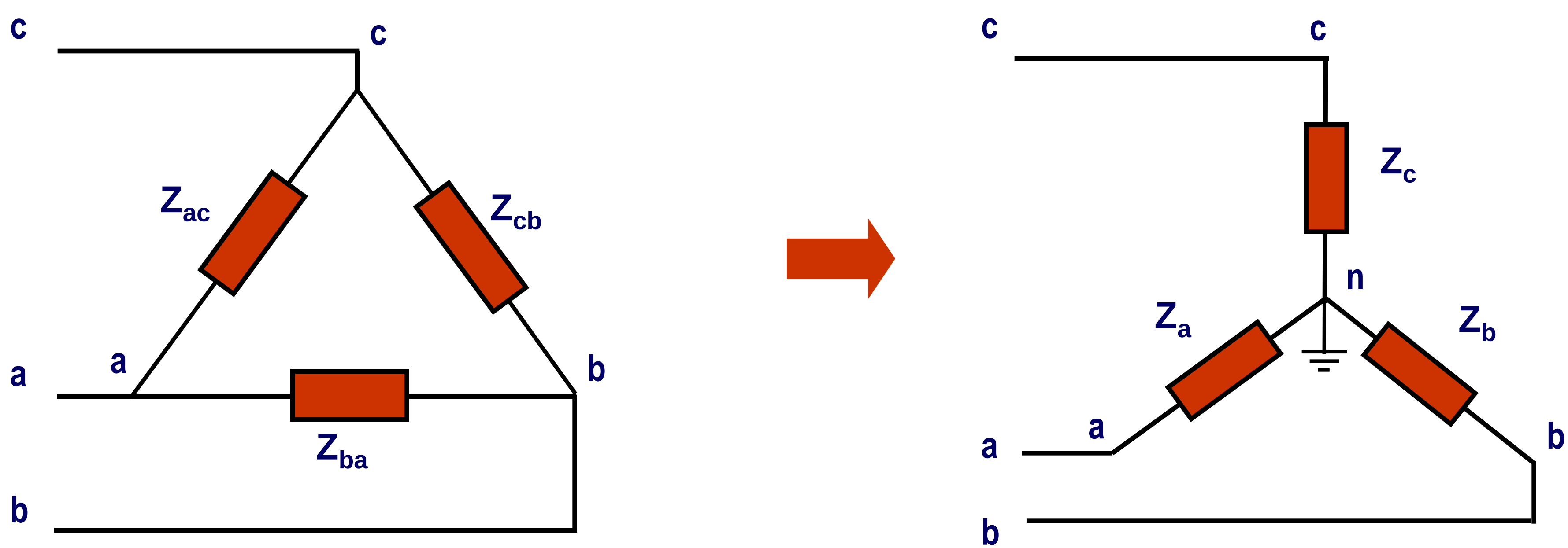
Formulation

A delta connected load can be converted to a star connected load as follows

$$z_a = z_{ba}\, z_{ac} \,/\, (\, z_{ba} + z_{ac} + z_{cb}\,)$$

$$z_b = z_{cb}\, z_{ba} \,/\, (\, z_{ba} + z_{ac} + z_{cb}\,)$$

$$z_c = z_{ac}\, z_{cb} \,/\, (\, z_{ba} + z_{ac} + z_{cb}\,)$$



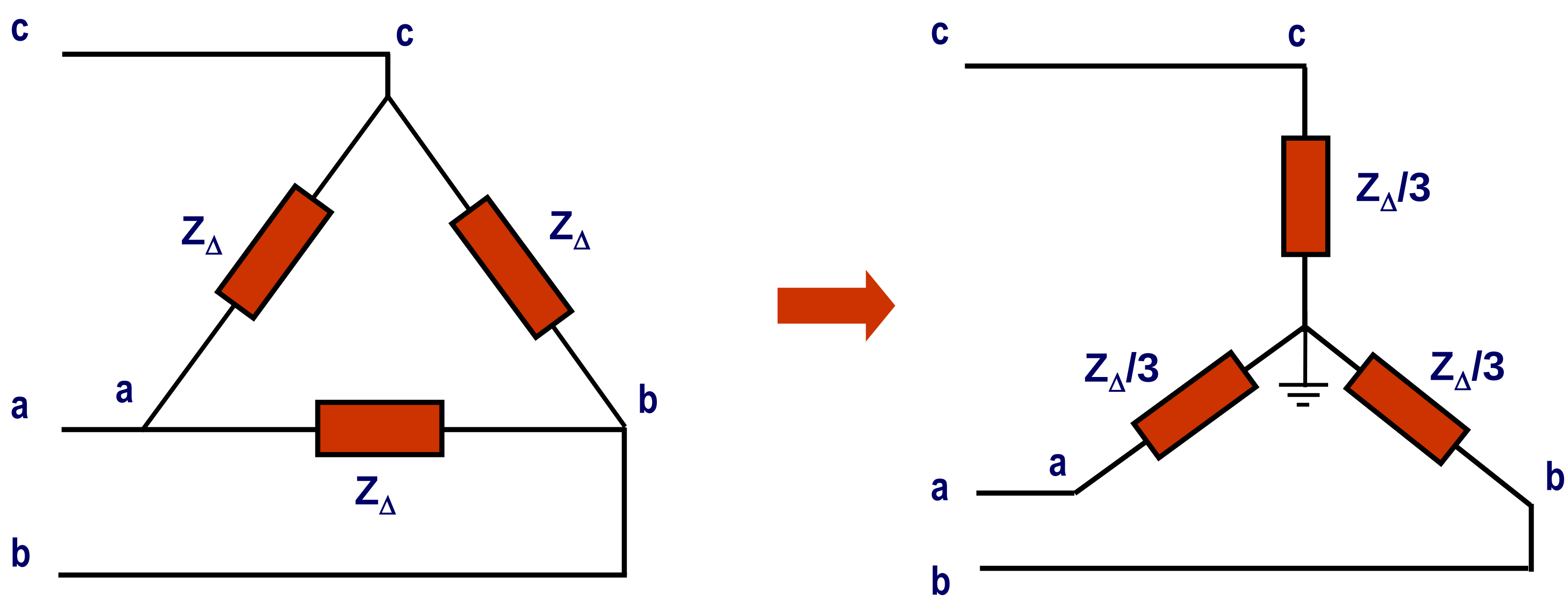
Three Phase Systems

Delta - Star Conversion

If all impedances of delta are identical, then the formula reduces to the following special simple form

Simplification

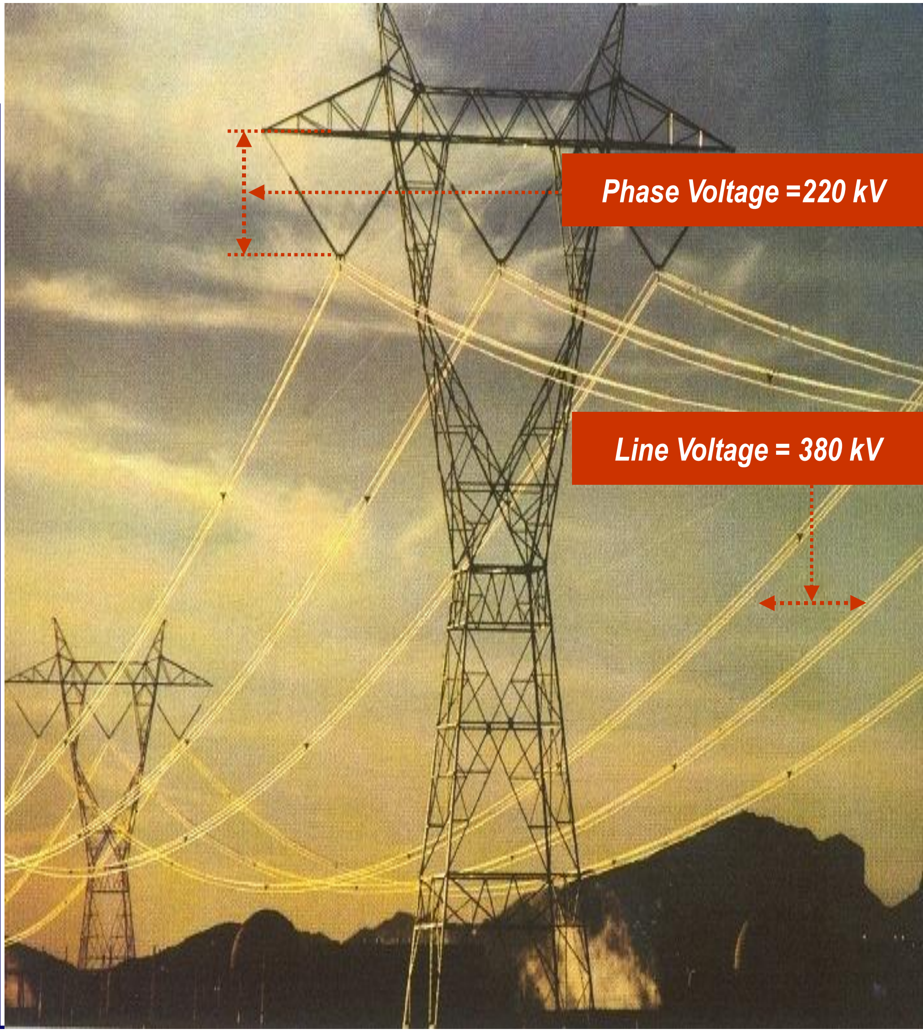
$$z_Y = z_\Delta^2 / (z_\Delta + z_\Delta + z_\Delta) = z_\Delta / 3$$



Solution Procedure for Three Phase Problems

Procedure

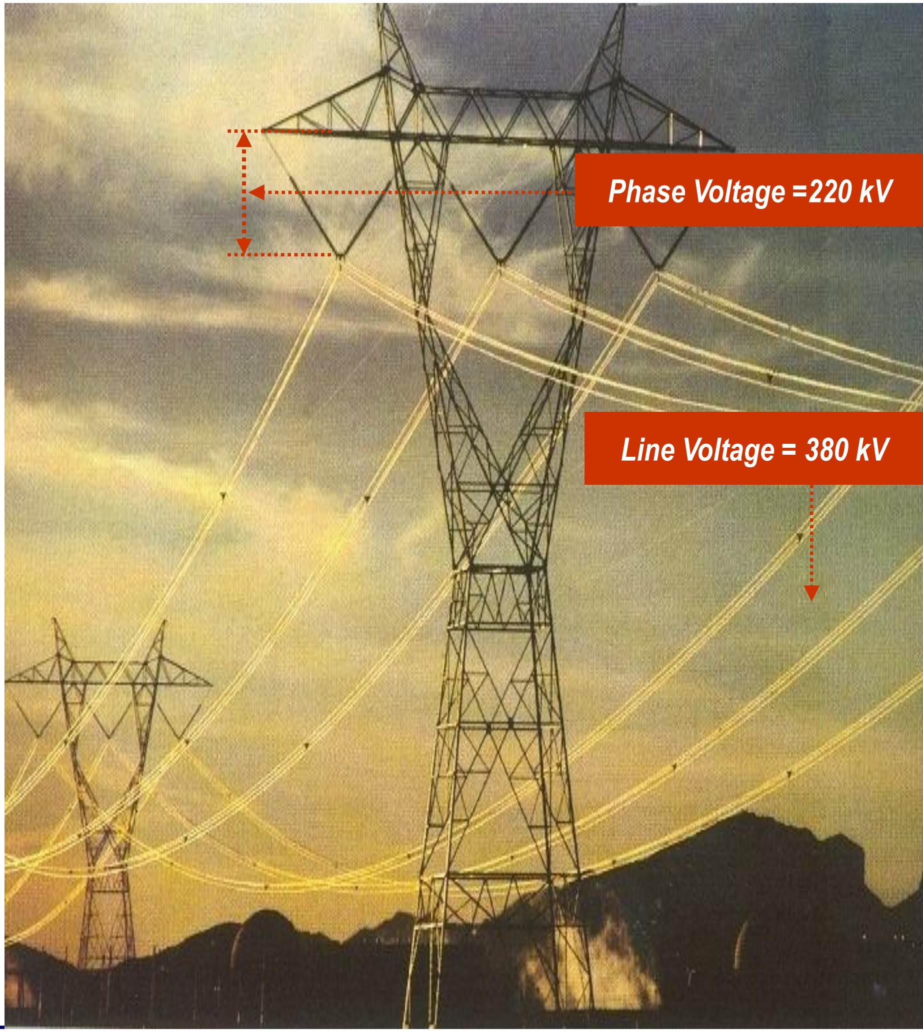
1. First convert all the Δ-connected loads, if any, to Y- connected loads by employing the Delta - Star Conversion Technique given in the previous section,
2. Find the source voltages / phase by dividing all the line voltages of the sources by $\sqrt{3}$ for the Y-connected sources
3. Decompose the given three phase system into three independent (electrically unconnected) single phase systems



Solution Procedure for Three Phase Problems

Procedure (Continued)

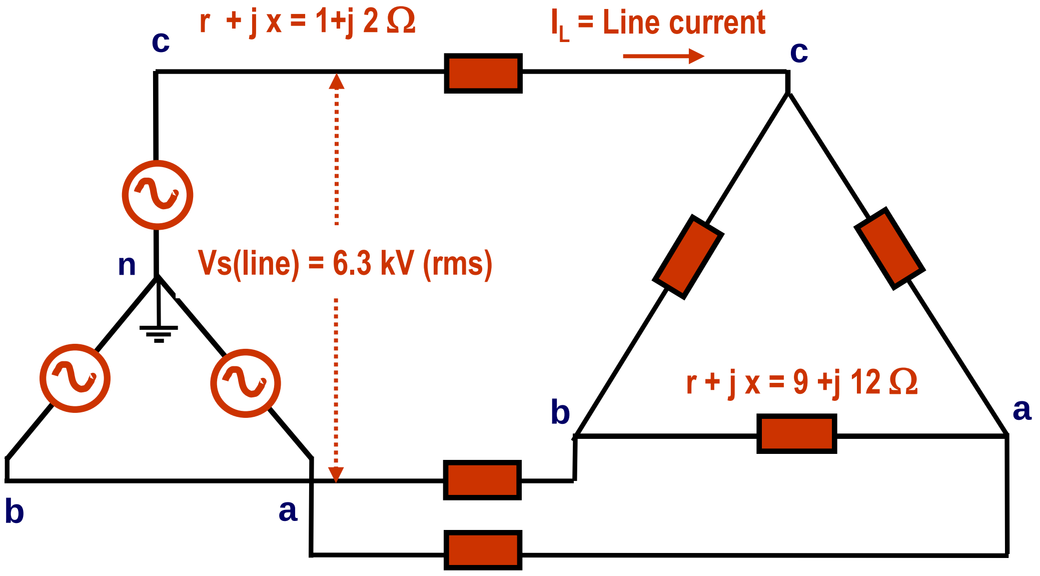
4. Then solve one of these single phase systems, i.e. in particular, the one which corresponds to phase-a,
5. Calculate active and reactive powers and power losses per phase,
6. Finally, multiply;
   a) all these active and reactive powers by three in order to find the three phase powers,
   b) all voltages by $\sqrt{3}$ in order to find the resulting line voltages

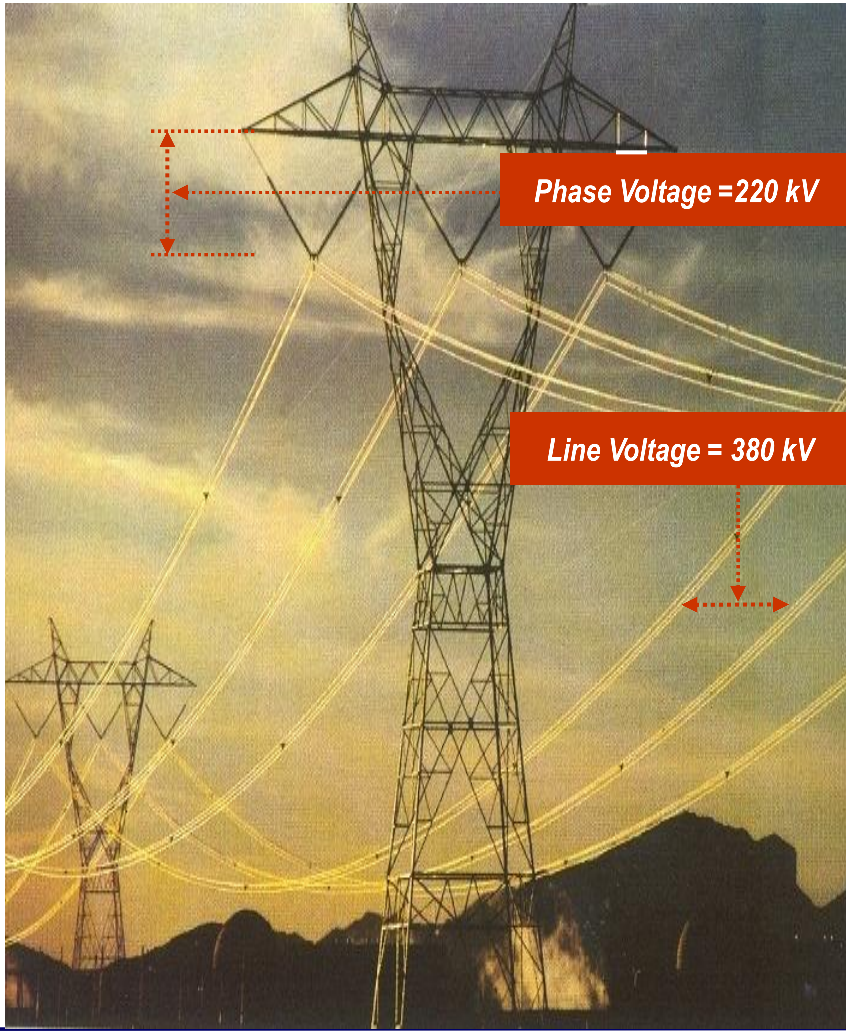


Example

Problem

Solve the following three phase system for line currents and line losses

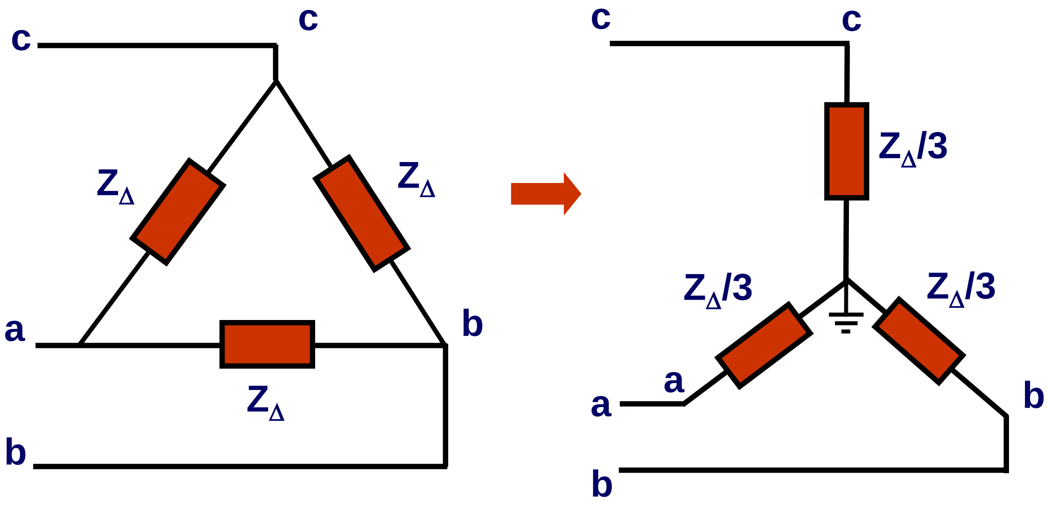




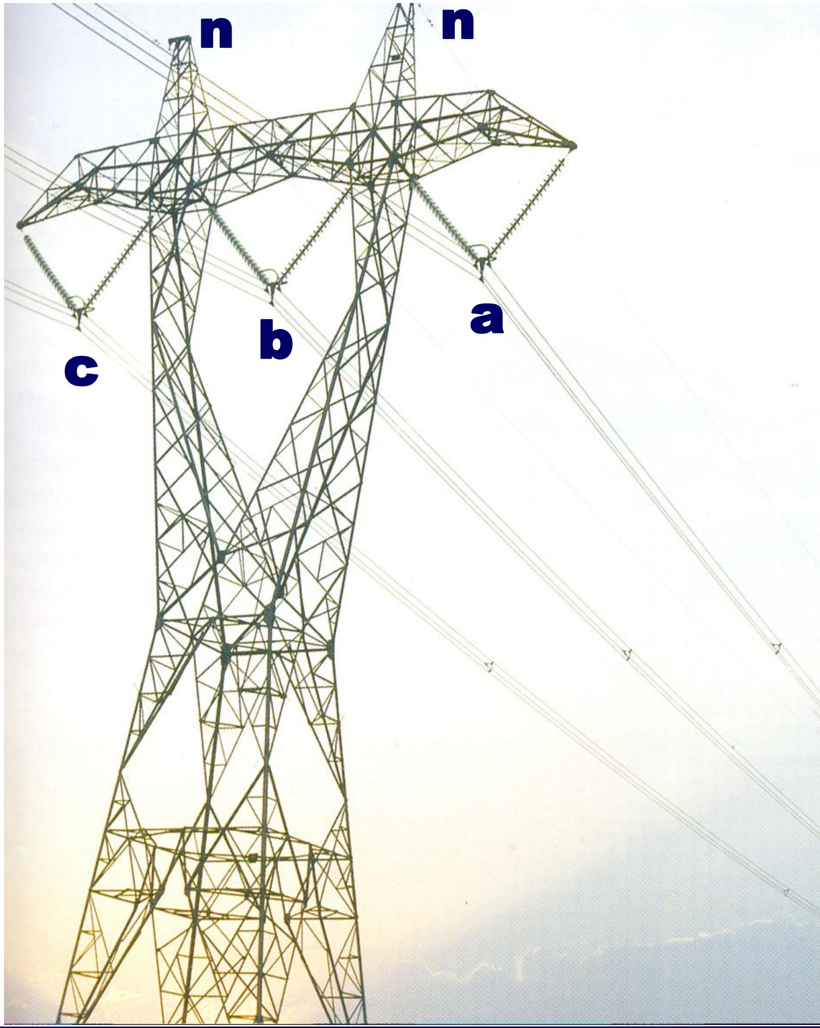
Example

Solution

1. First convert all the Δ-connected loads to Y- connected loads, if any, by employing the Delta - Star Conversion Technique given in the previous section



$$Z_\Delta / 3 = (9 + j\,12) / 3 = 3 + j\,4\ \Omega$$

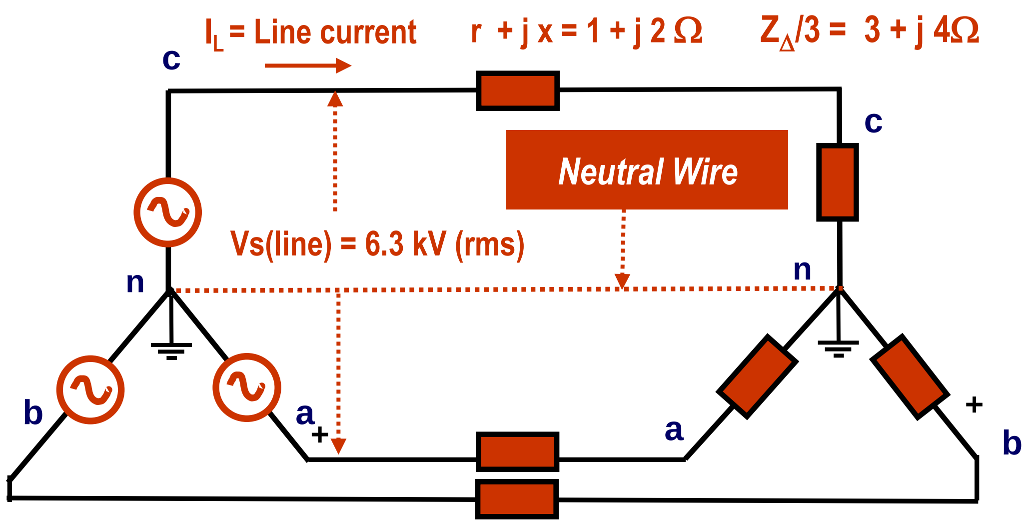


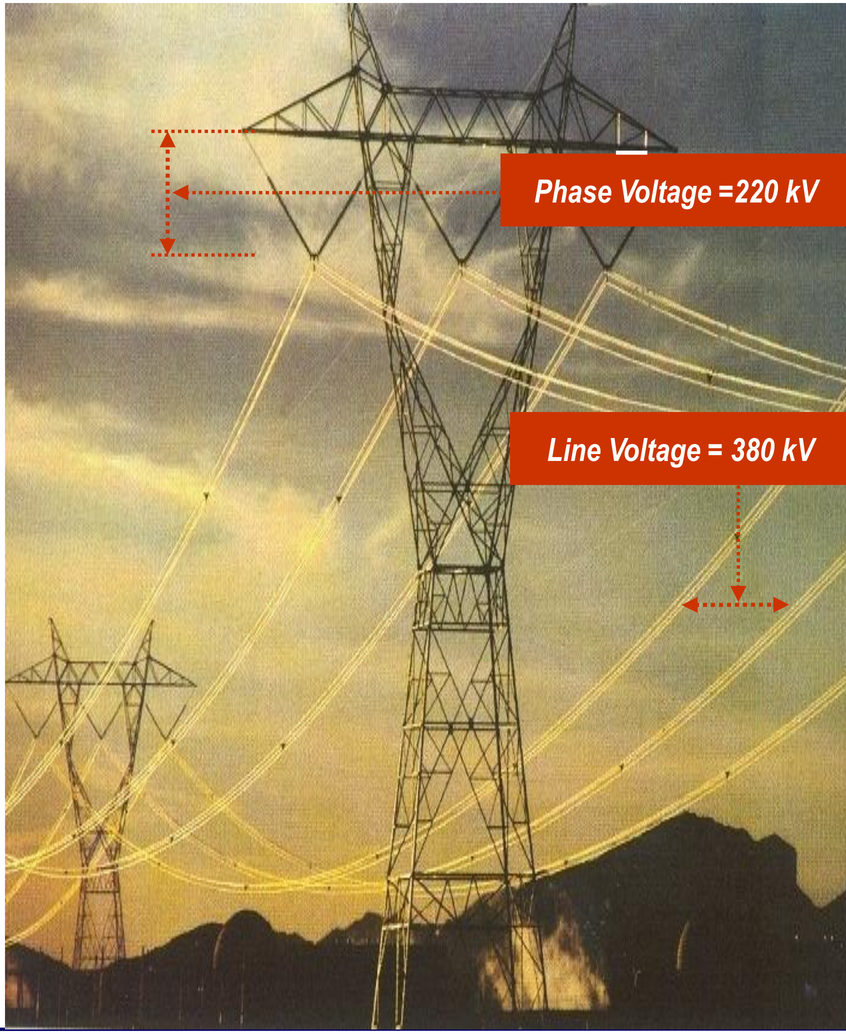
Three Phase Systems

Example

Problem

Solve the following three phase system for line currents and line losses





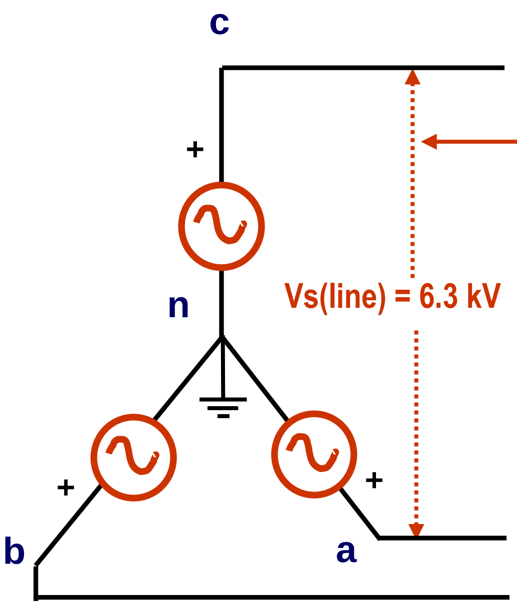
Three Phase Systems

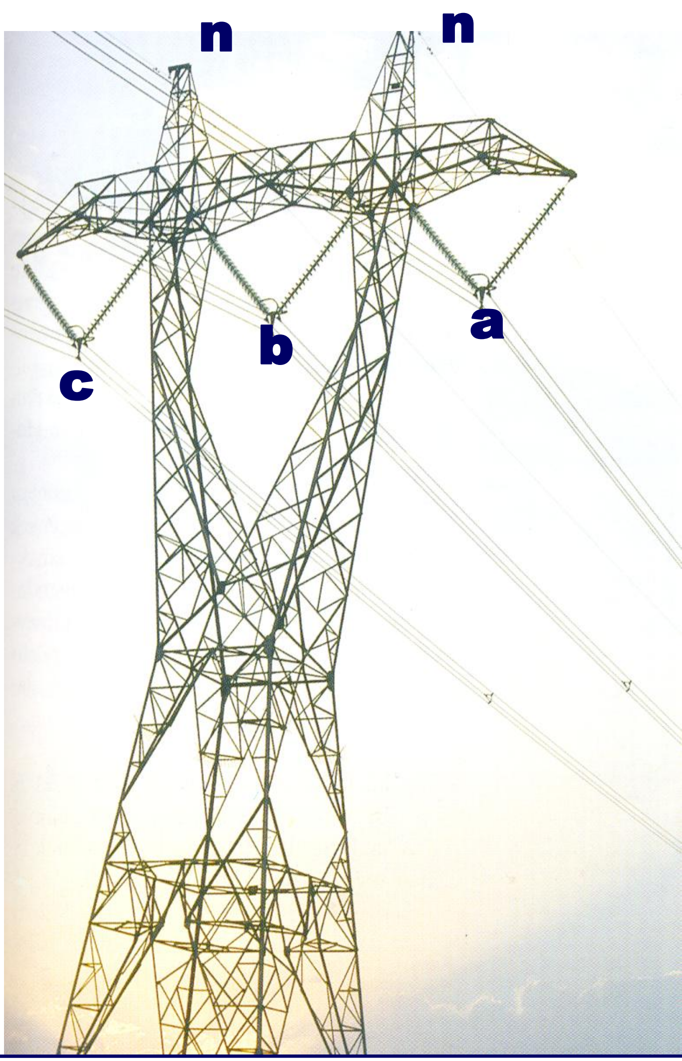
Example

Solution

2. find the source voltages / phase
3. Divide all the line voltages by $\sqrt{3}$ for Y-connected sources in order to

$$V_{s\ phase} = V_{s\ line} / \sqrt{3}$$
$$= 6300 / 1.732$$
$$= 3637.41 \text{ Volts (rms)}$$

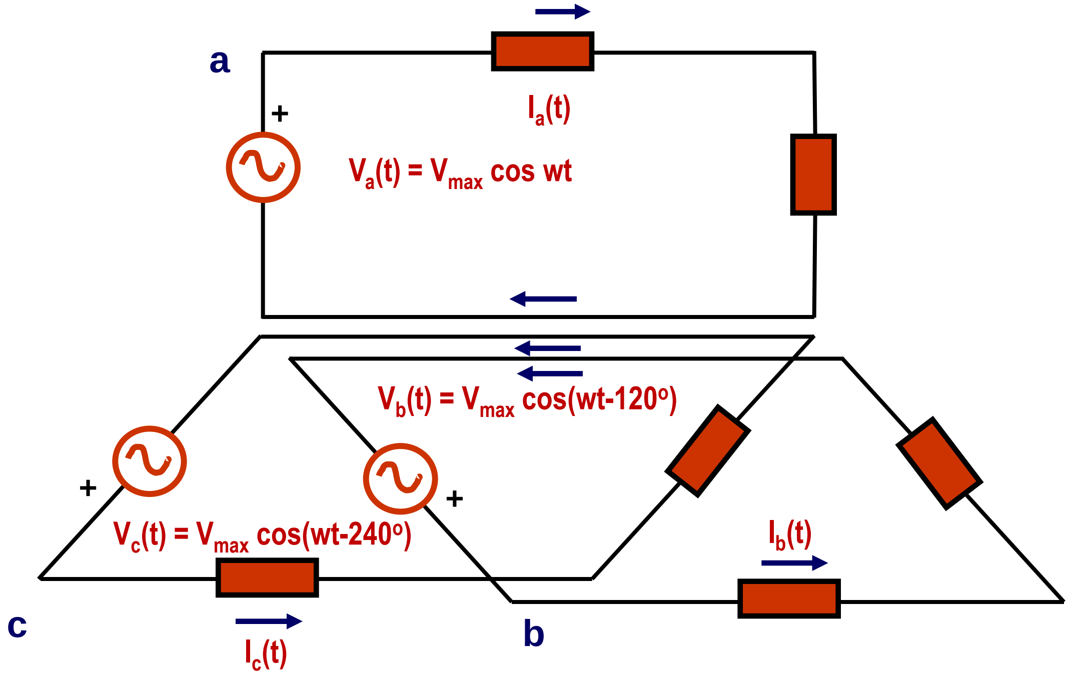


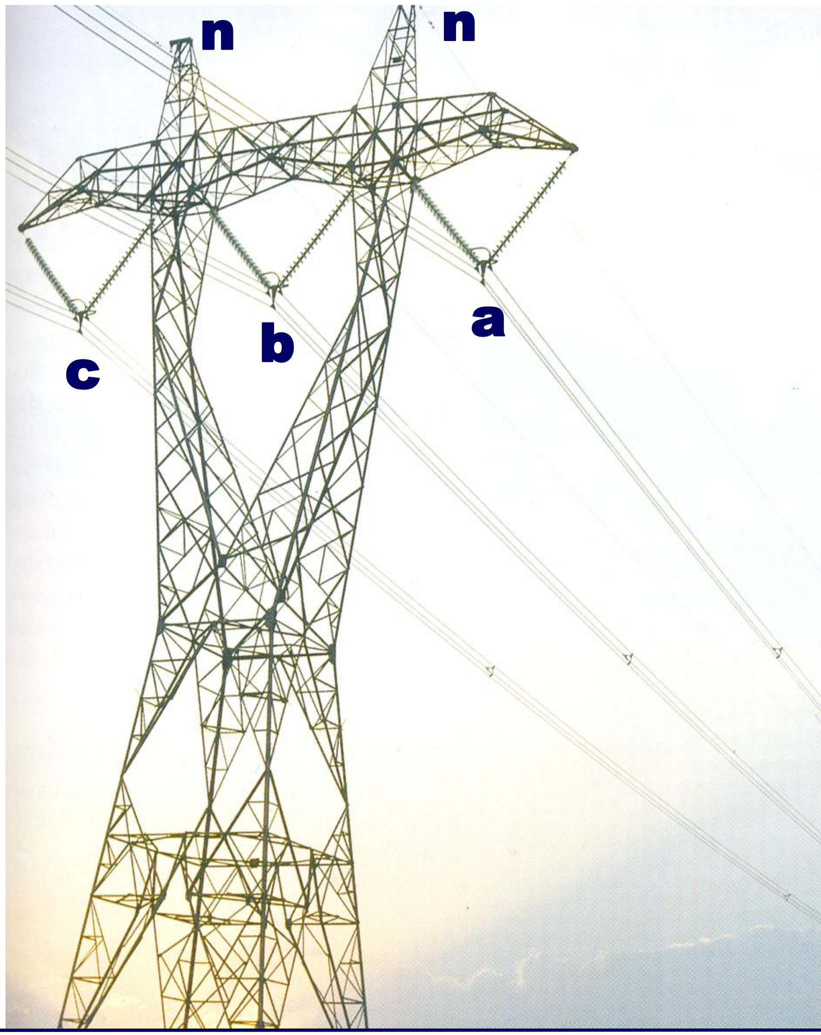


Example

Solution

3. Decompose the given three phase system into three independent (electrically unconnected) single phase systems

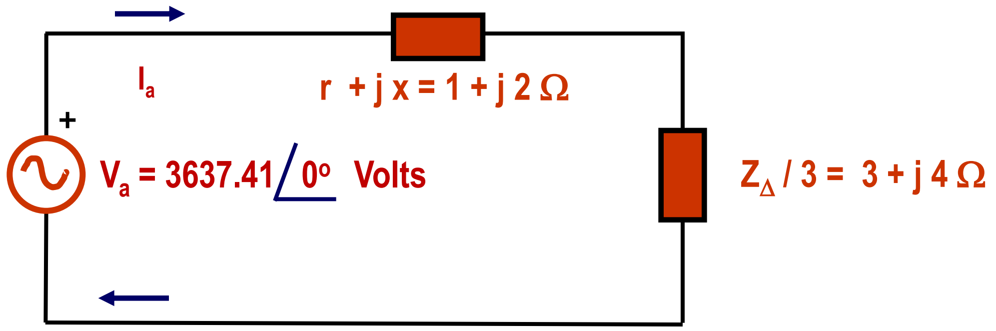


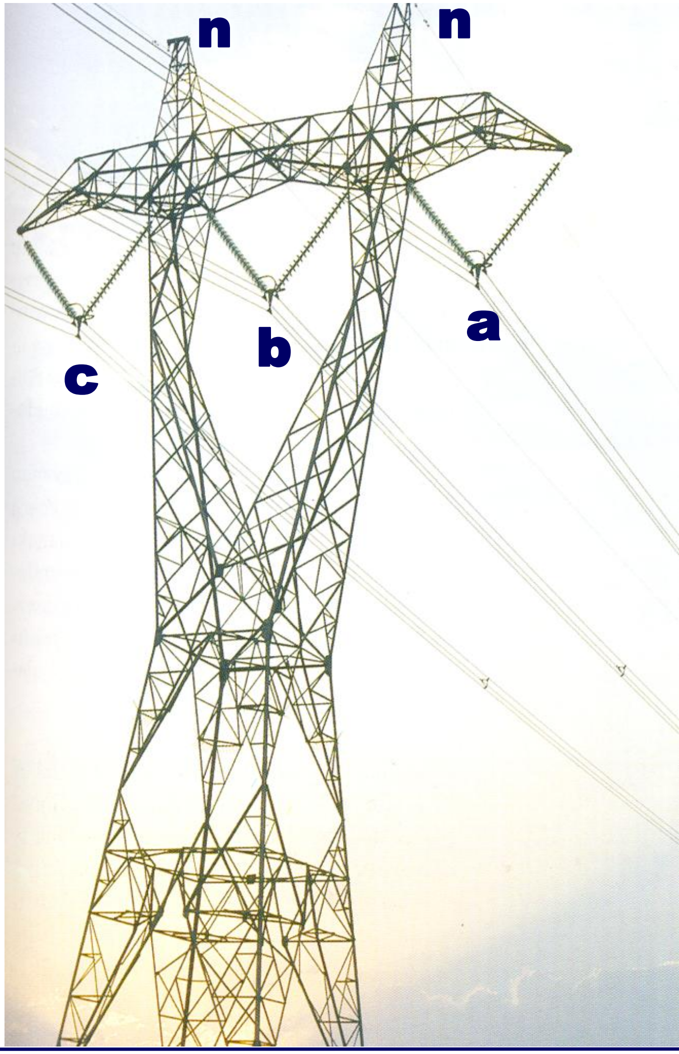


Example

Solution

4. Then, solve one of the resulting three single phase systems, i.e. in particular, the one which corresponds to phase-a, with zero phase angle, due to its simplicity





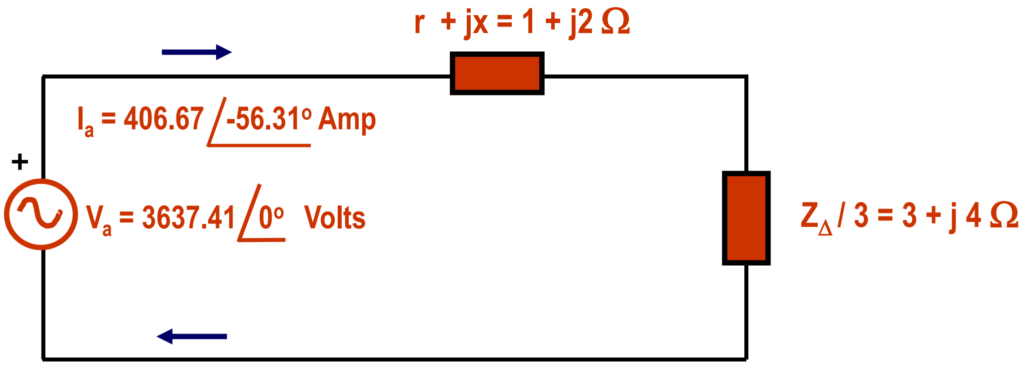
$$I_{line} = V_{a\text{ - }phase} / Z_{tot}$$
$$= 3637.41 \angle 0^o \;/\; [(1 + j2) + (3 + j4)]$$
$$= 3637.41 \angle 0^o / (4 + j6) = 3637.41 \angle 0^o \;/\; 8.9442 \angle 56.31^o$$
$$= 406.67 \angle -56.31^o \text{ Amp}$$

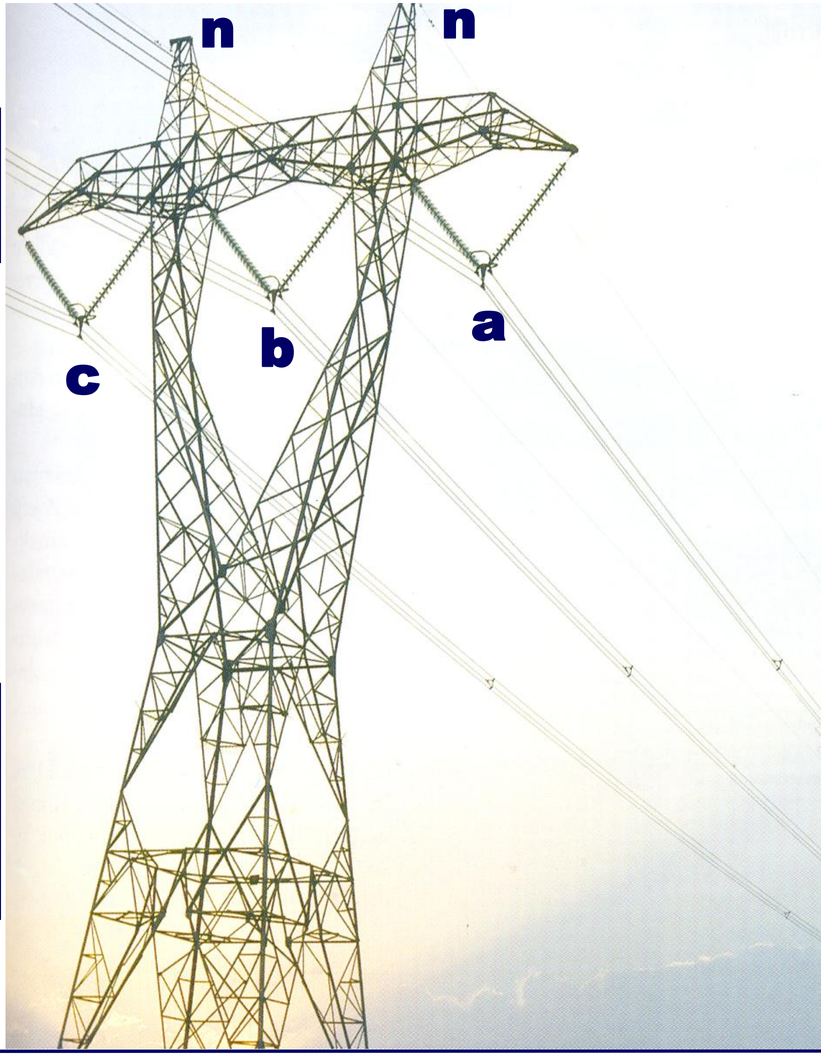
Three Phase Systems

Example

Solution

5. Now, calculate;
   a) Active and reactive power losses





Line Losses;

$r_{line}\, I^2 = 1 \times 406.67^2 = 165380$ Watts / phase

$= 165.38$ kWs / phase

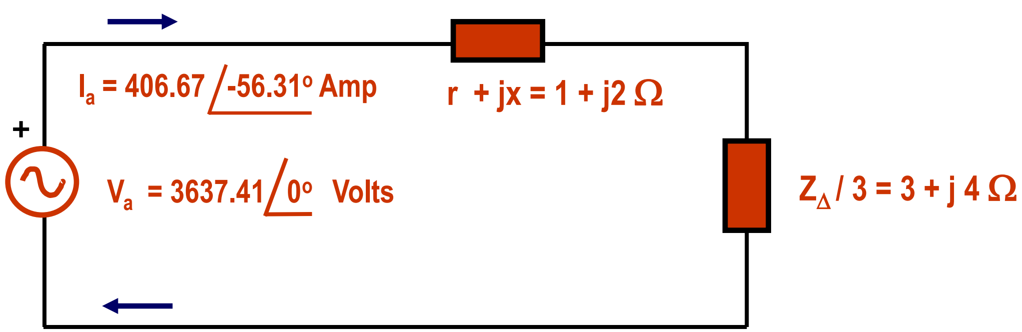
$x_{line}\, I^2 = 2 \times 406.67^2 = 330760$ Vars / phase

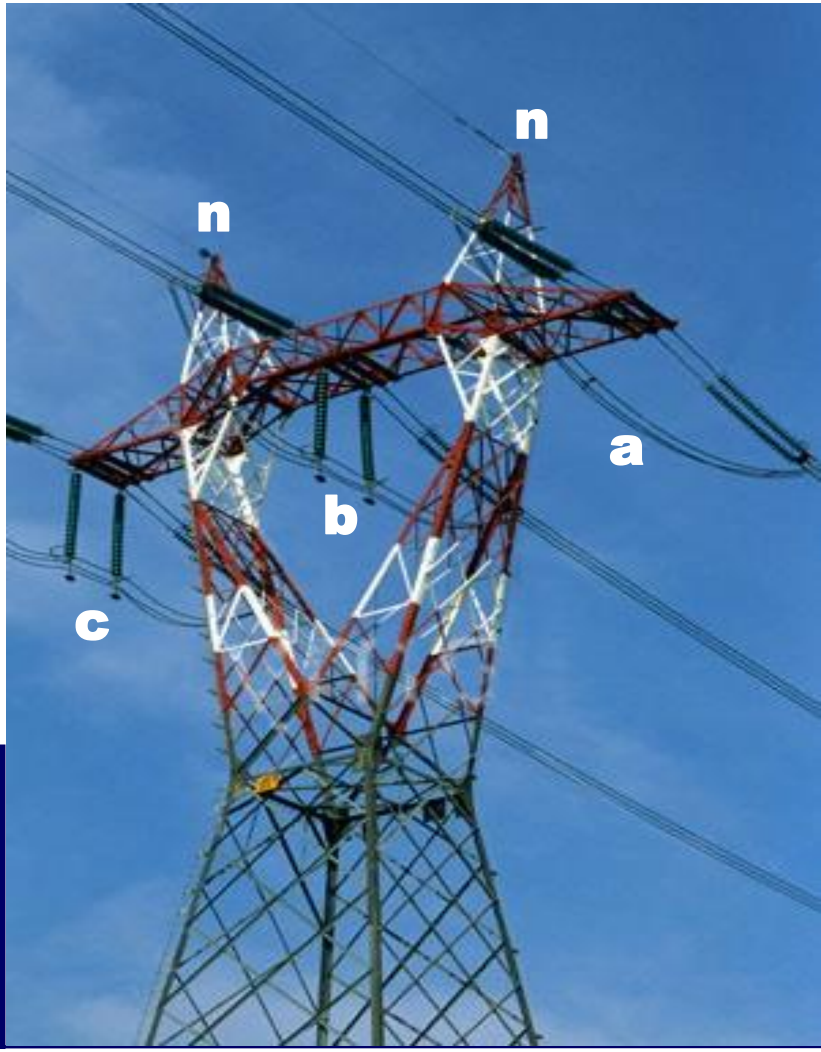
$= 330.76$ kVARs / phase

Example

Solution

6. Finally, multiply;
   a) Active and reactive power losses by three in order to find the three phase power losses





<u>Three phase power losses;</u>

Active power loss = 3 x active power loss /phase

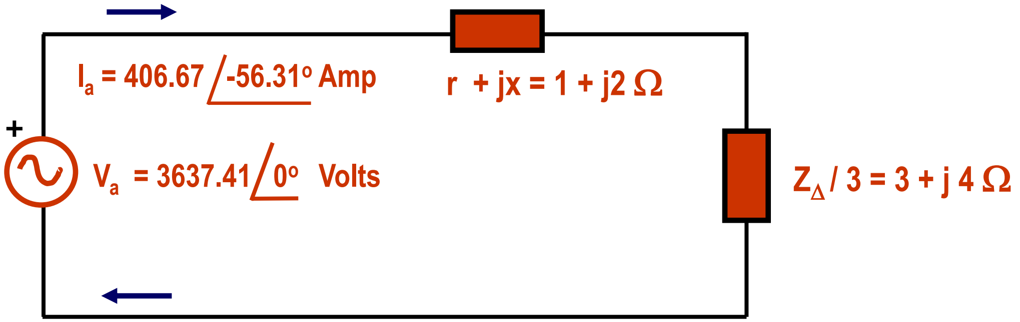
= 3 x 165.38 = 496.14 kWs

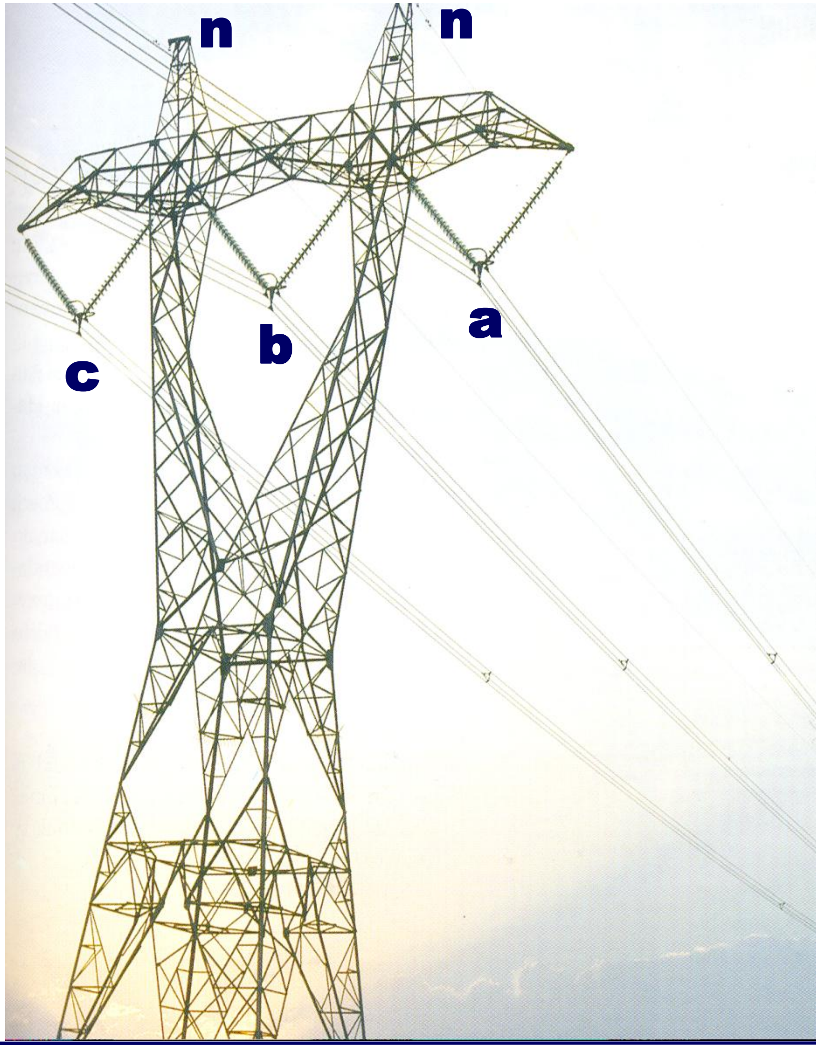
Reactive Power loss = 3 x 330.76 = 992.28 kVARs

Example

Solution

7. Now, calculate the active and reactive powers consumed by the load;





Active and reactive power consumptions;

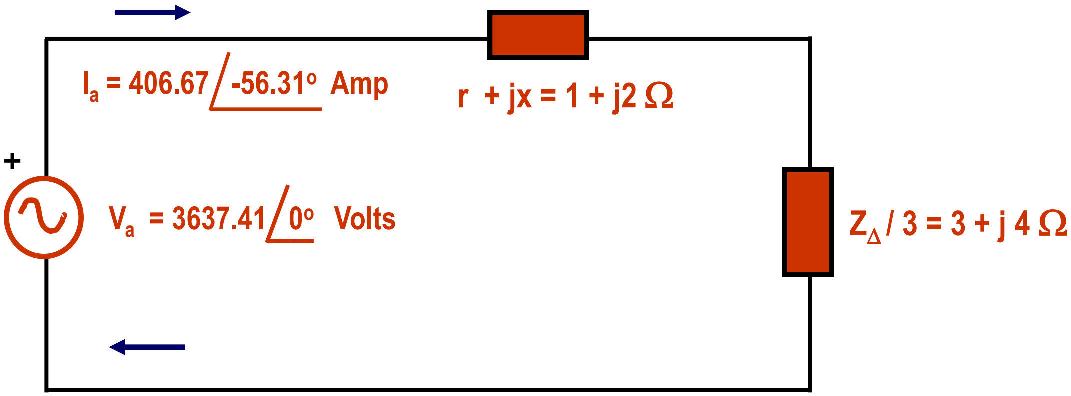
$$r_{load}\, I^2 = 3 \times 406.67^2 = 496140 \text{ Watts / phase}$$
$$= 496.14 \text{ kWs / phase}$$

$$x_{load}\, I^2 = 4 \times 406.67^2 = 661520 \text{ Vars / phase}$$
$$= 661.52 \text{ kVARs / phase}$$

Example

Solution

8. Now, calculate the three phase active and reactive powers consumed by the load;



Three phase active and reactive power consumptions;

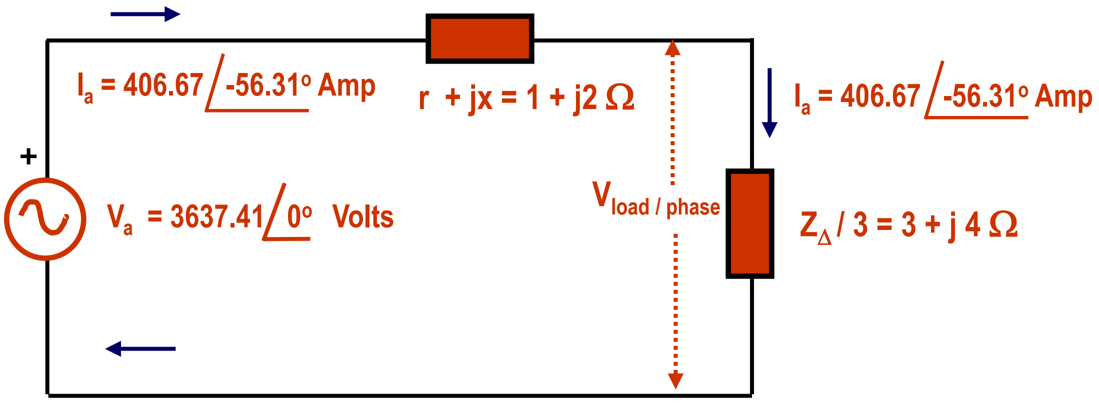
$3 \times r_{load} I^2 = 3 \times 496.14 = 1488.42$ kWs

$3 \times x_{load} I^2 = 3 \times 661.52 = 1984.56$ kVARs

Example

Solution

9. Now, calculate the load voltage / phase



Load Voltage/phase (Y-connected load)

$$V_{Load/phase} = Z_\Delta / 3 \times I_a$$
$$= (3 + j4) \times 406.67 \angle -56.31^o$$
$$= 5 \angle 53.13^o \times 406.67 \angle -56.31^o$$
$$= 2033.35 \angle -3.18^o \text{ Volts/phase}$$

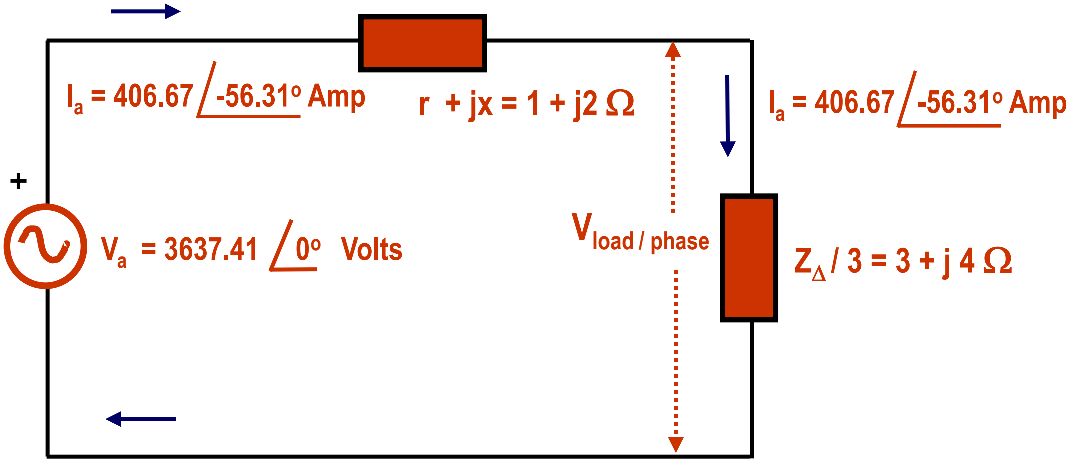
Three Phase Systems

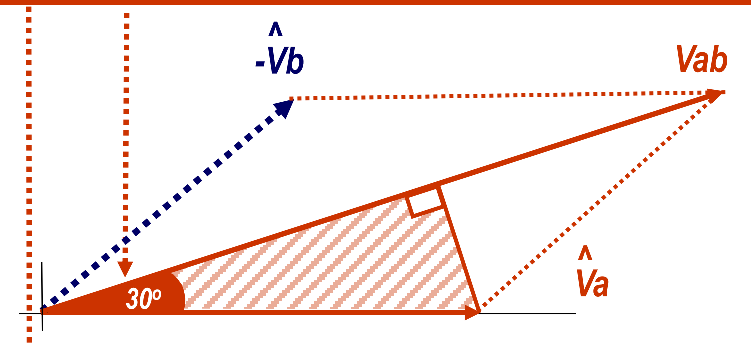
Example

Solution

$\angle V_{ab} = \angle V_a + 30^o$

10. Now, calculate the load voltage / line





Load Voltage / line (Y-connected load)

$$V_{Load/line} = \sqrt{3} \times 2033.35 \angle -3.18^o + 30^o$$

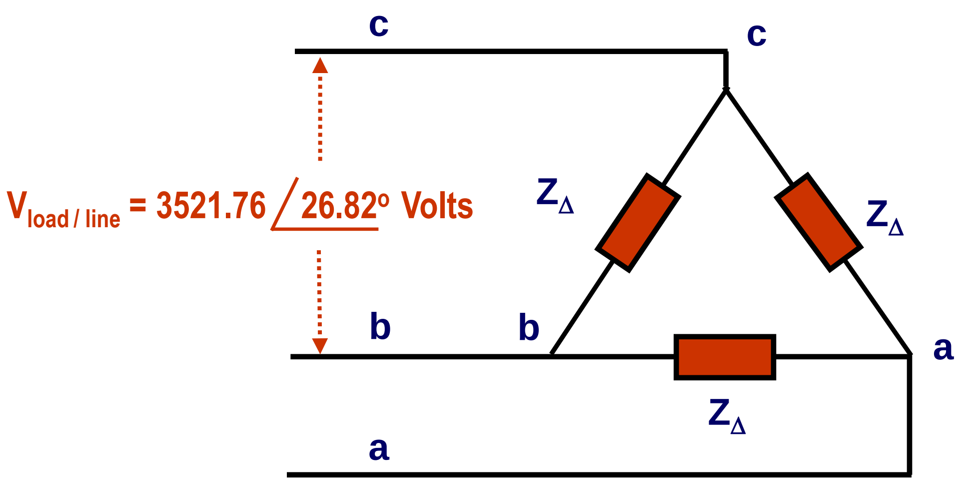
$$= 3521.76 \angle 26.82^o \text{ Volts / line}$$

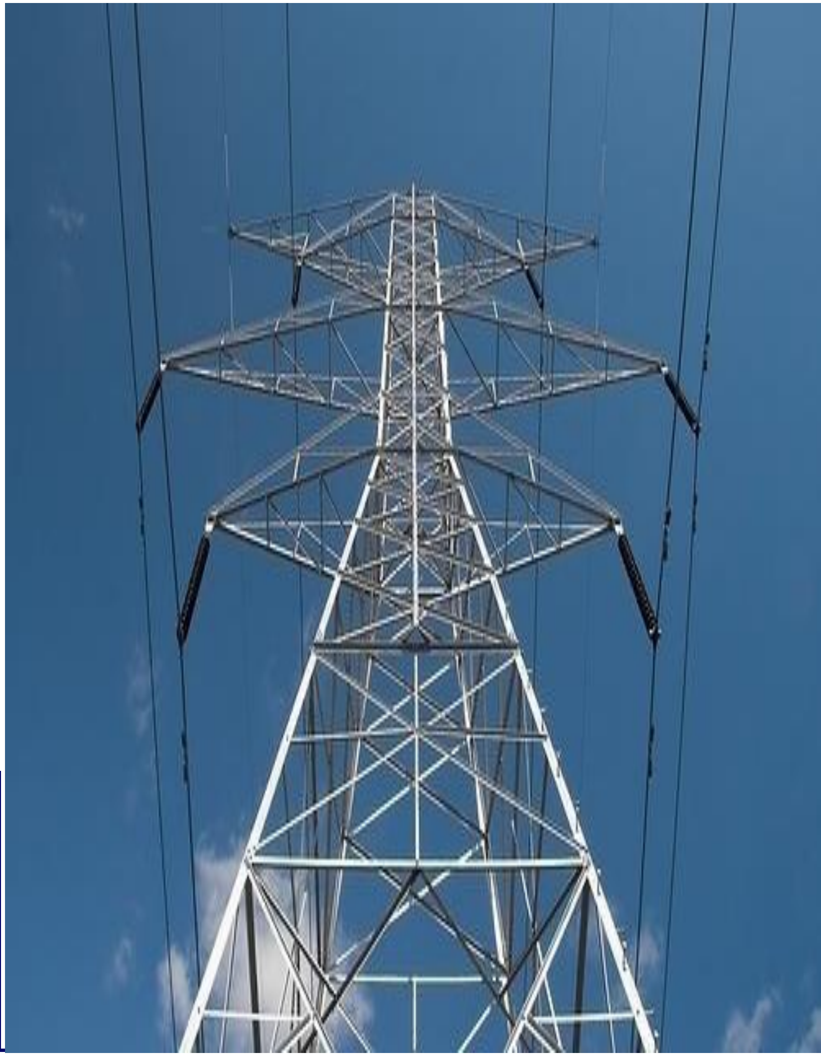
Three Phase Systems

Example

Solution

Please note that phase voltage across the delta is the same as line voltage, i.e.





<u>*Load Voltage / line (Y-connected load)*</u>

$$V_{Load/line} = V_{Load/phase}$$
$$= 3521.76 \angle 26.82^{o} \text{ Volts/line}$$

METU

Three Phase Systems

Did eveybody understand three phase systems ?