METU Informatics Institute

Min720 Pattern Classification with Bio-Medical Applications

Homework 1 Due Date: 30.03.2011

1- The class-conditional densities for a 2-class, single feature problem is given by the Cauchy distribution.

$$P(f\backslash c_i) = \frac{1}{\pi}\frac{1}{1+(f-a_i)^2}\ , i = 1,2$$

(a) Find the decision boundary and the decision regions when $P(c_1)=P(c_2)$. Show the probability of error geometrically (on sketch) when $a_1=3$, $a_2=5$. Write an expression for probability of error and find the result.

(b) Examine the behavior of the decision boundary with changing $P(c_1)$ and $P(c_2)$.

2- Consider a three-dimensional normal distribution $N(M_1, \sum_1)$ where

$$M_1=\begin{pmatrix}1\\2\\2\end{pmatrix} \qquad \sum_1=\begin{pmatrix}1&0&0\\0&5&2\\0&2&5\end{pmatrix}$$

a) Find the probability density at point $x_0= (.5, 0, 1)^t$

b) For the 2-category problem with $M_1= M_2$ and

$$\sum_2 = \begin{pmatrix}5&0&0\\0&10&5\\0&5&10\end{pmatrix}$$

Find the decision boundary. What is its shape?(linear, circle, parabola etc.)

3. Let x have 1-d uniform density

$$p(x \mid \Theta) \sim U(0, \Theta) = \begin{cases} 1/\ \Theta & 0 <= x <= \\ \\ 0 \text{ otherwise} \end{cases}$$

a) Suppose n samples D= (x_1, x_2, …, x_n) are drawn independently According to p(x | Θ). Show that the maximum likelihood estimate for Θ is max(D)- that is, the value of the maximum element of d.