has an $\Omega(n \log n)$ lower bound as well. Hence, the algorithm presented in this chapter is asymptotically optimal.

Section 1.5 EXERCISES

- 1.10 Let *S* be a set of *n* (possibly intersecting) unit circles in the plane. We want to compute the convex hull of *S*.
 - a. Show that the boundary of the convex hull of *S* consists of straight line segments and pieces of circles in *S*.
 - b. Show that each circle can occur at most once on the boundary of the convex hull.
 - c. Let S' be the set of points that are the centers of the circles in S. Show that a circle in S appears on the boundary of the convex hull if and only if the center of the circle lies on the convex hull of S'.
 - d. Give an $O(n \log n)$ algorithm for computing the convex hull of *S*.
 - e.* Give an $O(n \log n)$ algorithm for the case in which the circles in S have different radii.