Lecture Notes for EE230
Probability and Random Variables

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Chapter 1

Elementary Concepts

1.1 Introduction

Applied probability is an extremely useful tool in engineering as well as other fields. Specific examples from our field, electrical engineering, where probability is heavily used are communication theory, networking, detection and estimation. Examples from other disciplines that rely on probabilistic models are statistics, operations research, finance, genetics, games of chance, etc. A working knowledge of applied probability is useful in understanding and interpreting many phenomena in everyday life.

In applied probability, we learn to construct and analyze probabilistic models, using which we can solve interesting problems. It is important to distinguish probability from statistics: probabilistic models that we construct do not belong to the "real world". Rather, they live inside a *probability space*, which is a mathematical construction. Probability Theory is a mathematical theory, based on axioms. Generally, the *three axioms* we will introduce in Section 1.3 are used to define probability theory (due to Kolmogorov in his 1933 book). Probability theory is heavily based on the theory of sets, so we will start by reviewing them.

1.2 Set Theory

Definition 1 *A set is a collection of objects, which are called the elements of the set.*

Ex: $A = \{1, 2, 3, \ldots, \}$, $B = \{$Monday, Wednesday, Friday$\}$, $C = \{$real numbers $(x, y) : \min(x, y) \leq 2\}$. (Finite, Countably Infinite, Uncountably Infinite)

Null set=empty set=$\emptyset$=$\{\}$.

The universal set (Ω): The set which contains all the elements under investigation

Some relations

- A is a subset of B ($A \subset B$) if every element of A is also an element of B.
- A and B are equal ($A = B$) if they have exactly the same elements.

1.2.1 Set Operations

1. UNION

2. INTERSECTION

3. COMPLEMENT of a set

4. DIFFERENCE

Two sets are called disjoint or mutually exclusive if $A \cap B = \emptyset$.

A collection of sets is said to be a partition of a set S if the sets in the collection are disjoint and their union is S.

1.2.2 Properties of Sets and Operations

- Commutative: $A \cup B = B \cup A$
- Associative: $A \cup (B \cup C) = (A \cup B) \cup C$

  $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap \emptyset =$ $\quad A \cup \emptyset =$ $\quad \emptyset^c =$ $\quad \Omega^c =$
- $A \cup A^c =$ $\quad A \cap A^c =$ $\quad A \cup \Omega =$ $\quad A \cap \Omega =$
- De Morgan's Laws
  - $(A \cup B)^c = A^c \cap B^c$
  - $(A \cap B)^c = A^c \cup B^c$

The cartesian product of two sets A and B is the set of all ordered pairs such that $A \times B = \{(a, b) | a \in A, b \in B\}$.

Ex:

$|A|=$ Cardinality of set A (The number of elements in A)

The power set $\mathcal{P}(A)$ of a set A: the set of all subsets of A $|\mathcal{P}(A)| =?$