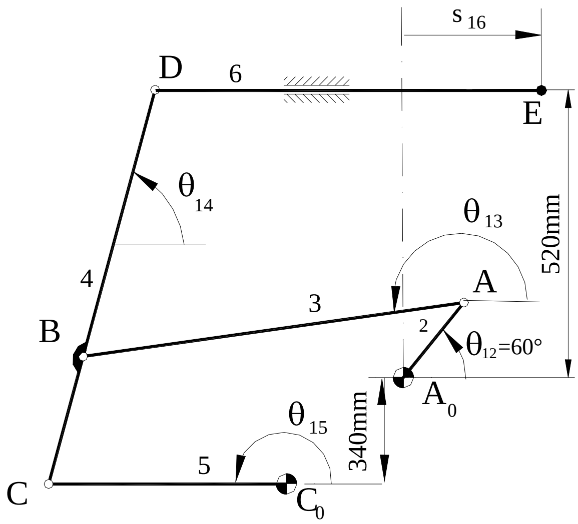
ME 310
NUMERICAL METHODS
Homework 3

Program Due: 07/11/2007 @ 24:00
Report Due: 08/11/2007 @ 13:30

Figure shows a six link mechanism used in textile machines. The link lengths are:

$A_0A = a_2 = 210$ mm;
$C_0C = a_5 = 600$ mm;
$CB = a_4 = 350$ mm;
$BD = b_4 = 600$ mm;
$A_0C_0 = c_1 = 710$ mm;
$AB = a_3 = 1225$ mm and
$DE = a_6 = 1100$ mm.
$a_1 = 340$ mm, ($b_1 = \sqrt{c_1^2 - a_1^2}$)
$d_1 = 520$ mm

You are to determine s_{16} when $\theta_{12} = 60^0$.



The loop equations, the equations that determine the relation between variable parameters due to the geometry of the mechanism, are given in terms of four nonlinear equations as:

$$a_2\cos\theta_{12} + a_3\cos\theta_{13} + b_1 - a_5\cos\theta_{15} - a_4\cos\theta_{14} = f_1(\theta_{13},\theta_{14},\theta_{15}) = 0$$
$$a_2\sin\theta_{12} + a_3\sin\theta_{13} + a_1 - a_5\sin\theta_{15} - a_4\sin\theta_{14} = f_2(\theta_{13},\theta_{14},\theta_{15}) = 0$$
$$a_2\cos\theta_{12} + a_3\cos\theta_{13} + b_4\cos\theta_{14} + a_6 - s_{16} = f_3(\theta_{13},\theta_{14},s_{16}) = 0$$
$$a_2\sin\theta_{12} + a_3\sin\theta_{13} + b_4\sin\theta_{14} - d_1 = f_4(\theta_{13},\theta_{14}) = 0$$

The unknowns are θ_{12}, θ_{14}, θ_{15} and s_{16}.

As an initial guess for the position variables assume $\theta_{13} = 200^0$, $\theta_{15} = 180^0$; $\theta_{14} = 80^0$ and $s_{16} = 400$ mm when $\theta_{12} = 60^0$ (please remember that you will be working in radians rather than degrees when solving the equations).

Using Newton Raphson method for several variables we obtain iteration formula:

$$\begin{bmatrix} \frac{\partial f_1}{d\theta_{13\,i}} & \frac{\partial f_1}{d\theta_{14\,i}} & \frac{\partial f_1}{d\theta_{15\,i}} & \frac{\partial f_1}{ds_{16\,i}} \\ \frac{\partial f_2}{d\theta_{13\,i}} & \frac{\partial f_2}{d\theta_{14\,i}} & \frac{\partial f_2}{d\theta_{15\,i}} & \frac{\partial f_2}{ds_{16\,i}} \\ \frac{\partial f_3}{d\theta_{13\,i}} & \frac{\partial f_3}{d\theta_{14\,i}} & \frac{\partial f_3}{d\theta_{15\,i}} & \frac{\partial f_3}{ds_{16\,i}} \\ \frac{\partial f_4}{d\theta_{13\,i}} & \frac{\partial f_4}{d\theta_{14\,i}} & \frac{\partial f_4}{d\theta_{15\,i}} & \frac{\partial f_4}{ds_{16\,i}} \end{bmatrix} \begin{bmatrix} \delta\theta_{13i} \\ \delta\theta_{14i} \\ \delta\theta_{15i} \\ \delta s_{16i} \end{bmatrix} = \begin{bmatrix} -f_{1i} \\ -f_{2i} \\ -f_{3i} \\ -f_{4i} \end{bmatrix}$$

The new value for the variables is determined as:

$$\theta_{13_{i+1}} = \theta_{13_i} + \delta\theta_{13i}; \qquad \theta_{14_{i+1}} = \theta_{14_i} + \delta\theta_{14i}; \qquad \theta_{15_{i+1}} = \theta_{15_i} + \delta\theta_{15i}; \qquad s_{16_{i+1}} = s_{16_i} + \delta s_{16i}$$

A) Write function routines with which each of the functions can be evaluated for a set of variable parameter values.

B) Write a subroutine in which the elements of the coefficient matrix are determined.

C) Write another subroutine (which in itself may contain other subroutines) to solve a set of linear equations using Gauss elimination method. The subroutine must be able to handle any number of equations.

D) In your main program read in the initial estimates of the variables from an input file and perform Newton Raphson method to find the roots of the nonlinear set of equations. As a stopping rule, you can end the program if:

i) $\delta\theta_{13}$, $\delta\theta_{14}$, $\delta\theta_{15}$ and δs_{16} values are all less than 10^{-5} radians or 10^{-4} mm.

ii) If the number of iterations is greater than 50.

Solution is: θ_{13}=190.5125^0, θ_{15}= 182.795^0; θ_{14}=69.400^0 and s_{16}=211.668 mm

E) (**Extra Credit**) When the position variables for a particular crank angle (θ_{12}=60^0) using the given initial guess is found, one can increment the crank angle (by 5^0 in CCW direction, say). Now, you can use the solution found in the previous step as the new guess value for the new crank angle incremented by 5^0. Thus the input crank angle can be rotated for the whole cycle and the value of the position variables can be found for all positions using one initial guess. Write a main program for the complete analysis of the mechanism.

What can you say about the number of iterations? Discuss your result.

Note: The input file should be created in “C:/temp/” directory as “idno-hw03-input.txt”.