

Ex: Variance in the hat problem.

Ex: Let X_1, X_2, X_3, \dots , be a sequence of IID Bernoulli(p) random variables (this “random sequence” is also called a “Bernoulli Process”). For concreteness, suppose that X_i stands for the result of the i^{th} trial in a sequence of independent trials, such that $X_i = 1$ if the trial is a success, and $X_i = 0$ if the trial does not result in a success.

1. Find $cov(X_i, X_j)$ for any i and j .
2. For every $i = 1, 2, \dots$ if trial i is successful, we toss fair a coin. If the coin comes up H, we let $Y_i = 1$. Otherwise, the value of Y_i is set to zero. Find $cov(Y_i, Y_j)$ and $cov(X_i, Y_j)$ for all i and j .
3. Now, let $Z_i = 1$ whenever $X_i = 1$ and the coin toss comes up T. Find $cov(Y_i, Z_i)$. What is sign of the correlation coefficient for Y and Z ?