In proving "convergence in probability" we often use Chebychev's Inequality, as in the following example.

Ex: Flip a fair coin n times, independently. Let Y_n be equal to the number of heads minus $n/2$. Does $\frac{Y_n}{n}$ converge?

The following result is a generalization of the previous example.

5.3 The Weak Law of Large Numbers

The Weak Law of Large Numbers (WLLN) is an important special case of convergence in probability. Consider $X_1, X_2, \ldots$ IID, with $E(X_i) = \mu$, and $\text{var}X_i = \sigma^2 < \infty$ for all i. The sample mean sequence $M_n = \frac{X_1+X_2+\ldots+X_n}{n}$ converges to μ in probability.

Proof: (Use the Chebychev Inequality on M_n.)

Ex: Polling: We want to estimate the fraction of the population that will vote for XYZ. Let X_i be equal to 1 if the i^{th} person votes in favor of XYZ, and 0 otherwise. How many people should we poll, to make sure our error will be less than 0.01 with 95% probability? (Answer: with Chebychev Inequality, we get n=50,000. However, this is too conservative. Using the Central Limit Theorem, we will get that a poll over a much smaller number of people will suffice.)